MECHANICAL ENGINEERING SERIES

Wodek K. Gawronski

Advanced Structural Dynamics and Active Control of Structures



Mechanical Engineering Series

Frederick F. Ling Series Editor

Springer New York

New York Berlin Heidelberg Hong Kong London Milan Paris Tokyo This page intentionally left blank

Wodek K. Gawronski

Advanced Structural Dynamics and Active Control of Structures

With 157 Figures



Wodek K. Gawronski Jet Propulsion Laboratory California Institute of Technology Pasadena, CA 91109, USA wodek.k.gawronski@jpl.nasa.gov

Series Editor Frederick F. Ling Ernest F. Gloyna Regents Chair in Engineering, Emeritus Department of Mechanical Engineering The University of Texas at Austin Austin, TX 78712-1063, USA and William Howard Hart Professor Emeritus Department of Mechanical Engineering, Aeronautical Engineering and Mechanics Rensselaer Polytechnic Institute Troy, NY 12180-3590, USA

Library of Congress Cataloging-in-Publication Data Gawronski, Wodek, 1944– Advanced structural dynamics and active control of structures/Wodek Gawronski. p. cm. — (Mechanical engineering series) ISBN 0-387-40649-2 (alk. paper) 1. Structural dynamics. 2. Structural control (Engineering) I. Title. II. Mechanical engineering series (Berlin, Germany) TA654.G36 2004 624.1'71—dc22 2003058443

Based on *Dynamics and Control of Structures: A Modal Approach*, by Wodek K. Gawronski, © 1998 Springer-Verlag New York, Inc.

ISBN 0-387-40649-2

Printed on acid-free paper.

© 2004 Springer-Verlag New York, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed in the United States of America.

9 8 7 6 5 4 3 2 1 SPIN 10943243

www.springer-ny.com

Springer-Verlag New York Berlin Heidelberg A member of BertelsmannSpringer Science+Business Media GmbH

To my friends—

Jan Kruszewski and Hans Günther Natke —scholars of dedication and imagination

Although this may seem a paradox, all exact science is dominated by the idea of approximation. —Bertrand Russell This page intentionally left blank

Mechanical Engineering Series

Frederick F. Ling *Series Editor*

The Mechanical Engineering Series features graduate texts and research monographs to address the need for information in contemporary mechanical engineering, including areas of concentration of applied mechanics, biomechanics, computational mechanics, dynamical systems and control, energetics, mechanics of materials, processing, production systems, thermal science, and tribology.

| Advisory Board | |
|-------------------------------|---|
| Applied Mechanics | F.A. Leckie University of California, Santa Barbara |
| Biomechanics | V.C. Mow Columbia University |
| Computational Mechanics | H.T. Yang University of California, Santa Barbara |
| Dynamical Systems and Control | K.M. Marshek University of Texas, Austin |
| Energetics | J.R. Welty University of Oregon, Eugene |
| Mechanics of Materials | I. Finnie University of California, Berkeley |
| Processing | K.K. Wang Cornell University |
| Production Systems | GA. Klutke Texas A&M University |
| Thermal Science | A.E. Bergles Rensselaer Polytechnic Institute |
| Tribology | W.O. Winer Georgia Institute of Technology |

This page intentionally left blank

Preface

Science is for those who learn; poetry for those who know. —Joseph Roux

This book is a continuation of my previous book, *Dynamics and Control of Structures* [44]. The expanded book includes three additional chapters and an additional appendix: Chapter 3, "Special Models"; Chapter 8, "Modal Actuators and Sensors"; and Chapter 9, "System Identification." Other chapters have been significantly revised and supplemented with new topics, including discrete-time models of structures, limited-time and -frequency grammians and reduction, almostbalanced modal models, simultaneous placement of sensors and actuators, and structural damage detection. The appendices have also been updated and expanded. Appendix A consists of thirteen new Matlab programs. Appendix B is a new addition and includes eleven Matlab programs that solve examples from each chapter. In Appendix C model data are given.

Several books on structural dynamics and control have been published. Meirovitch's textbook [108] covers methods of structural dynamics (virtual work, d'Alambert's principle, Hamilton's principle, Lagrange's and Hamilton's equations, and modal analysis of structures) and control (pole placement methods, LQG design, and modal control). Ewins's book [33] presents methods of modal testing of structures. Natke's book [111] on structural identification also contains excellent material on structural dynamics. Fuller, Elliot, and Nelson [40] cover problems of structural active control and structural acoustic control. Inman's book [79] introduces the basic concepts of vibration control, while Preumont in [120] presents modern approaches to structural control, including LQG controllers, sensors, and actuator placement, and piezoelectric materials with numerous applications in aerospace and civil engineering. The Junkins and Kim book [87] is a graduate-level textbook, while the Porter and Crossley book [119] is one of the first books on modal control. Skelton's work [125] (although on control of general linear systems) introduces methods designed specifically for the control of flexible structures. For example, the component cost approach to model or controller reduction is a tool frequently used in this field. The monograph by Joshi [83] presents developments on

dissipative and LQG controllers supported by numerous applications. Genta's book [65] includes rotor dynamics; the book by Kwon and Bang [92] is dedicated mainly to structural finite-element models, but a part of it is dedicated to structural dynamics and control. The work by Hatch [70] explains vibrations and dynamics problems in practical ways, is illustrated with numerous examples, and supplies Matlab programs to solve vibration problems. The Maia and Silva book [107] is a study on modal analysis and testing, while the Heylen, Lammens, and Sas book [71] is an up-to-date and attractive presentation of modal analysis. The De Silva book [26] is a comprehensive source on vibration analysis and testing. Clark, Saunders, and Gibbs [17] present recent developments in dynamics and control of structures; and Elliott [31] applies structural dynamics and control problems in balanced coordinates. The recent advances in structural dynamics and control can be found in [121].

This book describes comparatively new areas of structural dynamics and control that emerged from recent developments. Thus:

- State-space models and modal methods are used in structural dynamics as well as in control analysis. Typically, structural dynamics problems are solved using second-order differential equations.
- Control system methods (such as the state-space approach, controllability and observability, system norms, Markov parameters, and grammians) are applied to solve structural dynamics problems (such as sensor and actuator placement, identification, or damage detection).
- Structural methods (such as modal models and modal independence) are used to solve control problems (e.g., the design of LQG and H_∞ controllers), providing new insight into well-known control laws.
- The methods described are based on practical applications. They originated from developing, testing, and applying techniques of structural dynamics, identification, and control to antennas and radiotelescopes. More on the dynamics and control problems of the NASA Deep Space Network antennas can be found at http://tmo.jpl.nasa.gov/tmo/progress report/.
- This book uses approximate analysis, which is helpful in two ways. First, it simplifies analysis of large structural models (e.g., obtaining Hankel singular values for a structure with thousands of degrees of freedom). Second, approximate values (as opposed to exact ones) are given in closed form, giving an opportunity to conduct a parametric study of structural properties.

This book requires introductory knowledge of structural dynamics and of linear control; thus it is addressed to the more advanced student. It can be used in graduate courses on vibration and structural dynamics, and in control system courses with application to structural control. It is also useful for engineers who deal with structural dynamics and control.

Readers who would like to contact me with comments and questions are invited to do so. My e-mail address is <u>Wodek.K.Gawronski@jpl.nasa.gov.</u> Electronic versions

Preface

of Matlab programs from Appendix A, examples from Appendix B, and data from Appendix C can also be obtained from this address.

I would like to acknowledge the contributions of my colleagues who have had an influence on this work: Kyong Lim, NASA Langley Research Center (sensor/actuator placement, filter design, discrete-time grammians, and H_{∞} controller analysis); Hagop Panossian, Boeing North American, Inc., Rocketdyne (sensor/actuator placement of the International Space Station structure); Jer-Nan Juang, NASA Langley Research Center (model identification of the Deep Space Network antenna); Lucas Horta, NASA Langley Research Center (frequencydependent grammians for discrete-time systems); Jerzy Sawicki, Cleveland State University (modal error estimation of nonproportional damping); Abner Bernardo, Jet Propulsion Laboratory, California Institute of Technology (antenna data collection); and Angel Martin, the antenna control system supervisor at the NASA Madrid Deep Space Communication Complex (Spain) for his interest and encouragement. I thank Mark Gatti, Scott Morgan, Daniel Rascoe, and Christopher Yung, managers at the Communications Ground Systems Section, Jet Propulsion Laboratory, for their support of the Deep Space Network antenna study, some of which is included in this book. A portion of the research described in this book was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

> Wodek K. Gawronski Pasadena, California January 2004

This page intentionally left blank

| eries Prefac | e | |
|--------------|--|--|
| reface . | ix | |
| st of Symbo | ols | |
| Introducti | on to Structures | |
| 1.1 Exa | mples | |
| 1.1.1 | A Simple Structure | |
| 1.1.2 | A 2D Truss | |
| 1.1.3 | A 3D Truss | |
| 1.1.4 | A Beam | |
| 1.1.5 | The Deep Space Network Antenna | |
| 1.1.6 | The International Space Station Structure 6 | |
| 1.2 Def | inition | |
| 1.3 Proj | perties | |
| Standard | Models | |
| 2.1 Mo | dels of a Linear System | |
| 2.1.1 | State-Space Representation | |
| 2.1.2 | Transfer Function | |
| 2.2 Sec | ond-Order Structural Models | |
| 2.2.1 | Nodal Models | |
| 2.2.2 | Modal Models | |
| 2.3 Stat | e-Space Structural Models | |
| 2.3.1 | Nodal Models | |
| 2.3.2 | Models in Modal Coordinates | |
| 2.3.3 | Modal Models | |
| | reface st of Symbol Introducti 1.1 Exa 1.1.1 1.1.2 1.1.3 1.1.4 1.1.5 1.1.6 1.2 Def 1.3 Proj Standard 2.1 Moo 2.1.1 2.2.2 2.3 Stat 2.3.1 2.3.2 2.3.3 | prices Prefaceviirefaceixist of SymbolsxixIntroduction to Structures11.1Examples11.1.1A Simple Structure11.1.2A 2D Truss21.1.3A 3D Truss21.1.4A Beam31.1.5The Deep Space Network Antenna31.1.6The International Space Station Structure61.2Definition61.3Properties7Standard Models142.1.1State-Space Representation142.1.2Transfer Function152.2Second-Order Structural Models162.2.1Nodal Models172.3State-Space Structural Models292.3.1Nodal Models292.3.2Models in Modal Coordinates312.3.3Modal Models35 |

| 3 | Special Models | . 41 | |
|---|---|------|--|
| | 3.1 Models with Rigid-Body Modes | . 41 | |
| | 3.2 Models with Accelerometers | . 45 | |
| | 3.2.1 State-Space Representation | . 45 | |
| | 3.2.2 Second-Order Representation | . 48 | |
| | 3.2.3 Transfer Function | . 49 | |
| | 3.3 Models with Actuators | 50 | |
| | 3 3 1 Model with Proof-Mass Actuators | 50 | |
| | 3.3.2 Model with Inertial Actuators | 53 | |
| | 3.4 Models with Small Nonproportional Damping | 54 | |
| | 3.5 Generalized Model | 58 | |
| | 3.5.1 State-Snace Representation | . 50 | |
| | 3.5.2 Transfer Function | . 59 | |
| | 3.6 Discrete-Time Models | . 57 | |
| | 3.6.1 State-Snace Representation | . 00 | |
| | 3.6.2 Transfer Function | . 63 | |
| | | . 05 | |
| 4 | Controllability and Observability | . 65 | |
| - | 4.1 Definition and Properties | 65 | |
| | 4.1.1 Continuous Time Systems | . 05 | |
| | 4.1.1 Continuous-Time Systems $1.1.1$ Continuous-Time Systems $4.1.2$ Discrete-Time Systems | . 00 | |
| | 4.1.2 District Time Systems | . 08 | |
| | 4.1.5 Relationship Between Continuous- and Discrete Time Grammians | 60 | |
| | 4.2 Delenard Depresentation | . 09 | |
| | 4.2 Datanced Representation | . /1 | |
| | 4.5 Balanced Structures with Rigid-Body Modes | . /3 | |
| | 4.4 Input and Output Gains | | |
| | 4.5 Controllability and Observability of a Structural Modal Model | . /0 | |
| | 4.5.1 Diagonally Dominant Grammians | | |
| | 4.5.2 Closed-Form Grammians | . /9 | |
| | 4.5.3 Approximately Balanced Structure in Modal Coordinates | . 80 | |
| | 4.6 Controllability and Observability of a Second-Order Modal Model . | . 85 | |
| | 4.6.1 Grammians \dots 1.6. \dots 1.6. \dots 1.6. | . 85 | |
| | 4.6.2 Approximately Balanced Structure in Modal Coordinates | . 8/ | |
| | 4./ Three Ways to Compute Hankel Singular Values | . 91 | |
| | 4.8 Controllability and Observability of the Discrete-Time | 0.1 | |
| | | . 91 | |
| | 4.9 Time-Limited Grammians | . 94 | |
| | 4.10 Frequency-Limited Grammians | . 99 | |
| | 4.11 Time- and Frequency-Limited Grammians | 103 | |
| | 4.12 Discrete-Time Grammians in Limited-Time and -Frequency Range | 107 | |
| 5 | Norms | 109 | |
| 5 | 5.1 Norms of the Continuous Time Systems | 100 | |
| | 5.1 The H-Norm | 109 | |
| | $5.1.1 \text{The H}_2 \text{ Norm}$ | 109 | |
| | 5.1.2 The Hambel Name | 111 | |
| | 3.1.3 The Hankel Norm | 112 | |

xiv

6

| 5.2 Nor | ms of the Discrete-Time Systems | . 113 |
|----------|---|-------|
| 5.2.1 | The H_2 Norm | . 113 |
| 5.2.2 | The H_{∞} Norm | . 114 |
| 5.2.3 | The Hankel Norm | . 114 |
| 5.3 Nor | ms of a Single Mode | . 115 |
| 5.3.1 | The H_2 Norm | . 115 |
| 5.3.2 | The H_{∞} Norm | . 117 |
| 5.3.3 | The Hankel Norm | . 118 |
| 5.3.4 | Norm Comparison | . 119 |
| 5.4 Nor | ms of a Structure | 120 |
| 5.4.1 | The H_2 Norm | 121 |
| 5.4.2 | The H_{∞} Norm | . 121 |
| 5.4.3 | The Hankel Norm | . 123 |
| 5.5 Nor | ms of a Structure with a Filter | . 124 |
| 5.5.1 | The H_2 Norm | . 124 |
| 5.5.2 | The H_{∞} Norm | . 126 |
| 5.5.3 | The Hankel Norm | . 127 |
| 5.6 Nor | ms of a Structure with Actuators and Sensors | . 127 |
| 5.6.1 | The H_2 Norm | . 128 |
| 5.6.2 | The H_{∞} Norm | . 130 |
| 5.6.3 | The Hankel Norm | . 132 |
| 5.7 Nor | ms of a Generalized Structure | . 135 |
| 5.8 Nor | ms of the Discrete-Time Structures | . 137 |
| 5.8.1 | The H_2 Norm | . 138 |
| 5.8.2 | The H_{∞} Norm | . 139 |
| 5.8.3 | The Hankel Norm | . 140 |
| 5.8.4 | Norm Comparison | . 140 |
| | | |
| Model De | duction | 1/2 |
| Model Ke | | . 143 |
| 6.1 Red | luction Through Truncation | . 143 |
| 6.2 Red | | . 145 |
| 6.2.1 | H_2 Model Reduction | . 145 |
| 6.2.2 | H_{∞} and Hankel Model Reduction | . 146 |
| 6.3 Red | luction in the Finite-Time and -Frequency Intervals | . 147 |
| 6.3.1 | Reduction in the Finite-Time Interval | . 148 |
| 6.3.2 | Reduction in the Finite-Frequency Interval | . 150 |
| 6.3.3 | Reduction in the Finite-Time and -Frequency Intervals | . 151 |
| 6.4 Stru | ictures with Rigid-Body Modes | . 155 |
| 6.5 Stru | ictures with Actuators and Sensors | . 159 |
| 6.5.1 | Actuators and Sensors in a Cascade Connection | . 159 |
| 6.5.2 | Structure with Accelerometers | . 101 |
| 6.5.3 | Structure with Proof-Mass Actuators | . 162 |
| 6.3.4 | Structure with inertial Actuators | . 165 |

| 7 | Actuator and Sensor Placement | 167 |
|----|---|-----|
| | 7.1 Problem Statement | 168 |
| | 7.2 Additive Property of Modal Norms | 168 |
| | 7.2.1 The H_2 Norm | 169 |
| | 7.2.2 The H_{∞} and Hankel Norms $\dots \dots \dots$ | 169 |
| | 7.3 Placement Indices and Matrices | 170 |
| | 7.3.1 H ₂ Placement Indices and Matrices | 170 |
| | 7.3.2 H ₂ and Hankel Placement Indices and Matrices | 172 |
| | 7.3.3 Actuator/Sensor Indices and Modal Indices | 173 |
| | 7.4 Placement for Large Structures | 180 |
| | 7.4.1 Actuator Placement Strategy | 182 |
| | 7.4.2 Sensor Placement Strategy | 182 |
| | 7.5 Placement for a Generalized Structure | 187 |
| | 7.5.1 Structural Testing and Control | 187 |
| | 7.5.2 Sensor and Actuator Properties | 189 |
| | 7.5.3 Placement Indices and Matrices | 192 |
| | 7.5.4 Placement of a Large Number of Sensors | 193 |
| | 7.6 Simultaneous Placement of Actuators and Sensors | 197 |
| | | 177 |
| 8 | Modal Actuators and Sensors | 203 |
| | 8.1 Modal Actuators and Sensors Through Modal Transformations | 204 |
| | 8.1.1 Modal Actuators | 204 |
| | 8.1.2 Modal Sensors | 208 |
| | 8.2 Modal Actuators and Sensors Through Grammian Adjustment | 213 |
| | | |
| 9 | System Identification | 219 |
| | 9.1 Discrete-Time Systems | 220 |
| | 9.2 Markov Parameters | 221 |
| | 9.3 Identification Algorithm | 221 |
| | 9.4 Determining Markov Parameters | 224 |
| | 9.5 Examples | 226 |
| | 9.5.1 A Simple Structure | 226 |
| | 9.5.2 The 2D Truss | 230 |
| | 9.5.3 The Deep Space Network Antenna | 232 |
| | | |
| 10 | Collocated Controllers | 235 |
| | 10.1 A Low-Authority Controller | 236 |
| | 10.2 Dissipative Controller | 237 |
| | 10.3 Properties of Collocated Controllers | 239 |
| | 10.4 Root-Locus of Collocated Controllers | 241 |
| | 10.5 Collocated Controller Design Examples | 245 |
| | 10.5.1 A Simple Structure | 245 |
| | 10.5.2 The 2D Truss | 246 |
| | | _ |
| 11 | LQG Controllers | 249 |
| | 11.1 Definition and Gains | 250 |
| | 11.2 The Closed-Loop System | 253 |
| | | |

xvi

| 11.3 The Balanced LQG Controller | | | | | | 254 |
|--|---|---|-------|---|---|-----|
| 11.4 The Low-Authority LQG Controller | | | | | | 255 |
| 11.5 Approximate Solutions of CARE and FARE | | | | | | 257 |
| 11.6 Root-Locus | | | | | | 260 |
| 11.7 Almost LQG-Balanced Modal Representation | | | | | | 262 |
| 11.8 Three Ways to Compute LQG Singular Values . | | | | | | 264 |
| 11.9 The Tracking LQG Controller | | | | | | 264 |
| 11.10 Frequency Weighting | | | | | | 266 |
| 11.11 The Reduced-Order LQG Controller | | | | | | 269 |
| 11.11.1 The Reduction Index | | | | | | 269 |
| 11.11.2 The Reduction Technique | | | | | | 271 |
| 11.11.3 Stability of the Reduced-Order Controller . | | | | | | 272 |
| 11.11.4 Performance of the Reduced-Order Controller | | • | | | | 274 |
| 11.11.5 Weights of Special Interest | | | | | | 275 |
| 11.12 Controller Design Procedure | | | | | | 276 |
| 11.13 Controller Design Examples | | | | | | 277 |
| 11.13.1 A Simple Structure | | | | | | 277 |
| 11.13.2 The 3D Truss | | | | | | 279 |
| 11.13.3 The 3D Truss with Input Filter | | | | | | 281 |
| 11.13.4 The Deep Space Network Antenna | | | | | | 283 |
| | | | | | | |
| 12 H_{∞} and H_2 Controllers $\ldots \ldots \ldots \ldots \ldots$ | • | • | • | • | • | 287 |
| 12.1 Definition and Gains | | | | | | 288 |
| 12.2 The Closed-Loop System | | | | | | 291 |
| 12.3 The Balanced H_{∞} Controller | | | | | | 292 |
| 12.4 The H_2 Controller | | | | | | 294 |
| 12.4.1 Gains | | | | | | 294 |
| 12.4.2 The Balanced H_2 Controller | | | | | | 296 |
| 12.5 The Low-Authority H_{∞} Controller | | | | | | 296 |
| 12.6 Approximate Solutions of HCARE and HFARE | | | | | | 298 |
| 12.7 Almost H_{∞} -Balanced Modal Representation | | | | | | 300 |
| 12.8 Three Ways to Compute H_{∞} Singular Values | | | | | | 301 |
| 12.9 The Tracking H_{∞} Controller | | | | | | 301 |
| 12.10 Frequency Weighting | | | | | | 301 |
| 12.11 The Reduced-Order H_{∞} Controller | | | | | | 304 |
| 12.11.1 The Reduction Index | | | | | | 304 |
| 12.11.2 Closed-Loop Poles | | | | | | 304 |
| 12.11.3 Controller Performance | | | | | | 306 |
| 12.12 Controller Design Procedure | | | | | | 307 |
| 12.13 Controller Design Examples | | | | | | 308 |
| 12.13.1 A Simple Structure | | | | | | 308 |
| 12.13.2 The 2D Truss | | | | | | 310 |
| 12.13.3 Filter Implementation Example | | | | | | 312 |
| 12.13.4 The Deep Space Network Antenna with | | | | | | |
| Wind Disturbance Rejection Properties | | | | | | 313 |

| Ap | opendi | ces | 317 |
|----|-------------|--|-----|
| A | Matl | ab Functions | 319 |
| | A.1 | Transformation from an Arbitrary State-Space Representation to | |
| | | the Modal 1 State-Space Representation | 320 |
| | A.2 | Transformation from an Arbitrary State-Space Representation to | |
| | | the Modal 2 State-Space Representation | 322 |
| | A.3 | Transformation from Modal Parameters to the Modal 1 State-Space | |
| | | Representation | 324 |
| | A.4 | Transformation from Modal Parameters to the Modal 2 State-Space | |
| | | Representation | 325 |
| | A.5 | Transformation from Nodal Parameters to the Modal 1 State-Space | |
| | | Representation | 326 |
| | A.6 | Transformation from Nodal Parameters to the Modal 2 State-Space | |
| | | Representation | 328 |
| | A.7 | Determination of the Modal 1 State-Space Representation and the | |
| | | Time- and Frequency-Limited Grammians | 329 |
| | A.8 | Open-Loop Balanced Representation | 331 |
| | A.9 | $H_2 \text{ Norm of a Mode} \qquad \dots \qquad $ | 332 |
| | A.10 | $H_\infty \text{ Norm of a Mode} \qquad \dots \qquad $ | 333 |
| | A.11 | Hankel Norm of a Mode | 333 |
| | A.12 | LQG-Balanced Representation | 334 |
| | A.13 | H_{∞} -Balanced Representation | 335 |
| B | Matl | ab Examples | 337 |
| | B.1 | Example 2.5 | 337 |
| | B.2 | Example 3.3 | 341 |
| | B.3 | Example 4.11 | 342 |
| | B.4 | Example 5.3 | 344 |
| | B.5 | Example 6.7 | 347 |
| | B.6 | Example 7.2 | 348 |
| | B. 7 | Example 8.1 | 353 |
| | B.8 | Example 9.1 | 356 |
| | B.9 | Example 10.4.2 | 359 |
| | B.10 | Example 11.13.1 | 361 |
| | B.11 | Example 12.13.2 | 365 |
| С | Struc | ctural Parameters | 371 |
| | C 1 | Mass and Stiffness Matrices of the 2D Truss | 371 |
| | C_{2} | Mass and Stiffness Matrices of the Clamped Beam Divided into | 571 |
| | 0.2 | 15 Finite Elements | 373 |
| | C.3 | State-Space Representation of the Deep Space Network Antenna | 376 |
| R¢ | ferenc | Pes | 379 |
| | | | 517 |
| In | dex | | 389 |

xviii

List of Symbols

Each equation in the book ... would halve the sales. -Stephen Hawking

| General | |
|------------------------|---|
| A^T | transpose of matrix A |
| A^* | complex-conjugate transpose of matrix A |
| A^{-1} | inverse of square nonsingular matrix A |
| tr(A) | trace of a matrix A, $tr(A) = \sum_{i} a_{ii}$ |
| $\left\ A\right\ _{2}$ | Euclidean (Frobenius) norm of a real-valued matrix A: |
| | $\left\ A\right\ _{2} = \sqrt{\sum_{i,j} a_{ij}^{2}} = \sqrt{\operatorname{tr}(A^{T} A)}$ |
| $diag(a_i)$ | diagonal matrix with elements a_i along the diagonal |
| eig(A) | eigenvalue of a square matrix A |
| $\lambda_i(A)$ | <i>i</i> th eigenvalue of a square matrix A |
| $\lambda_{\max}(A)$ | maximal eigenvalue of a square matrix A |
| $\sigma_i(A)$ | <i>i</i> th singular value of a matrix A |
| $\sigma_{\max}(A)$ | maximal singular value of a matrix A |
| I_n | identity matrix, $n \times n$ |
| $0_{n \times m}$ | zero matrix, $n \times m$ |

Linear Systems

| (A,B,C,D) | quadruple of the system state-space representation |
|--|--|
| (A,B,C) | triple of the system state-space representation |
| (A_d, D_d, C_d) | LOC controller state space representation |
| $(A_{lqg}, D_{lqg}, C_{lqg})$ | EQG controller state-space representation |
| $(A_{\infty}, B_{\infty}, C_{\infty})$ | H_{∞} controller state-space representation |
| (A_o, B_o, C_o) | closed-loop state-space representation |
| G | transfer function |
| G_d | discrete-time transfer function |
| H_1 | Hankel matrix |
| H_2 | shifted Hankel matrix |
| h_k | kth Markov parameter |
| U | input measurement matrix |
| Y | output measurement matrix |
| x | system state |
| x_e | system estimated state |
| <i>u</i> | system (control) input |
| y z | performance output |
| w | disturbance input |
| B_1 | matrix of disturbance inputs |
| <i>B</i> ₂ | matrix of control inputs |
| C_1 | matrix of performance outputs |
| C_2 | matrix of measured outputs |
| $\left\ G\right\ _{2}$ | continuous-time system H ₂ norm |
| $\ G\ _{\infty}$ | continuous-time system $H_{\! \infty}$ norm |
| $\left\ G\right\ _h$ | continuous-time system Hankel norm |
| $\left\ G_d\right\ _2$ | discrete-time system H ₂ norm |
| $\left\ G_d\right\ _{\infty}$ | discrete-time system H_{∞} norm |
| $\left\ G_d\right\ _h$ | discrete-time system Hankel norm |
| \mathcal{C} | controllability matrix |
| \mathcal{O} | observability matrix |
| W_c | controllability grammian |
| W_o | observability grammian |
| γ_i | <i>i</i> th Hankel singular value |
| $\gamma_{\rm max}$ | the largest Hankel singular value of a system |
| Γ | matrix of Hankel singular values |
| CARE | controller algebraic Riccati equation |
| FARE | filter (or estimator) algebraic Riccati equation |
| HCARE | H_{∞} controller algebraic Kiccati equation |

| HFARE | H_{∞} filter (or estimator) algebraic Riccati equation |
|------------------|--|
| S_c | solution of CARE |
| S_e | solution of FARE |
| $S_{\infty c}$ | solution of HCARE |
| $S_{\infty e}$ | solution of HFARE |
| μ_i | <i>i</i> th LQG singular value |
| $\mu_{\infty i}$ | <i>i</i> th H_{∞} singular value |
| М | matrix of the LQG singular values, $M = diag(\mu_i)$ |
| M_{∞} | matrix of the H_{∞} singular values, $M_{\infty} = \text{diag}(\mu_{\infty i})$ |
| ρ | parameter of the $H_{\!\infty}$ controller |
| K _c | controller gain |
| K _e | estimator gain |
| ε | tracking error |
| t | time sequence |
| Δt | sampling time |
| N | number of states |
| S | number of inputs |
| r | number of outputs |
| | |

Structures

| D | damping matrix |
|--------------|--|
| Κ | stiffness matrix |
| М | mass matrix |
| D_m | modal damping matrix |
| K_m | modal stiffness matrix |
| M_m | modal mass matrix |
| q | structural displacement (nodal) |
| q_m | structural displacement (modal) |
| q_{ab} | structural displacement (almost-balanced) |
| q_i | displacement of the <i>i</i> th degree of freedom |
| q_{mi} | displacement of the <i>i</i> th mode |
| q_{abi} | displacement of the <i>i</i> th almost-balanced mode |
| ϕ_i | <i>i</i> th structural mode |
| ϕ_{abi} | almost-balanced ith structural mode |
| Φ | modal matrix |
| Φ_{ab} | almost-balanced modal matrix |
| ω_i | <i>i</i> th natural frequency |
| Ω | matrix of natural frequencies |

| ζ_i | <i>i</i> th modal damping |
|---|---|
| Ζ | matrix of modal damping coefficients |
| B_o | nodal input matrix |
| C_{oq} | nodal displacement output matrix |
| C_{ov} | nodal velocity output matrix |
| B_m | modal input matrix |
| C_{mq} | modal displacement output matrix |
| C_{mv} | modal velocity output matrix |
| C_m | modal output matrix, $C_m = C_{mq} \Omega^{-1} + C_{mv}$ |
| b _{mi} | input matrix of the <i>i</i> th mode, <i>i</i> th row of B_m |
| C _{mi} | output matrix of the <i>i</i> th mode, <i>i</i> th column of C_m |
| $\left\ B_{m}\right\ _{2}$ | modal input gain |
| $\left\ C_{m}\right\ _{2}$ | modal output gain, $\ C_m\ _2^2 = \ C_{mq}\Omega^{-1}\ _2^2 + \ C_{mv}\ _2^2$ |
| $\left\ b_{mi}\right\ _{2}$ | input gain of the <i>i</i> th mode |
| $\left\ c_{mi}\right\ _{2}$ | output gain of the <i>i</i> th mode |
| $\Delta \omega_i$ | <i>i</i> th half-power frequency, $\Delta \omega_i = 2\zeta_i \omega_i$ |
| σ_{2ij} | H_2 placement index for the <i>i</i> th actuator (sensor) |
| | and the <i>k</i> th mode |
| $\sigma_{\scriptscriptstyle{\infty}ij}$ | H_{∞} placement index for the <i>i</i> th actuator (sensor) |
| | and the <i>k</i> th mode |
| Σ_2 | H ₂ placement matrix |
| Σ_{∞} | H_{∞} placement matrix |
| I(k) | membership index of the <i>k</i> th sensor |
| eta_i | pole shift factor |
| n _d | number of degrees of freedom |
| n | number of modes |
| IV S | number of states |
| r | number of outputs |
| S | number of candidate actuator locations |
| R | number of candidate sensor locations |