Variances in Roulette

STAT 350

Payouts

Bet	Payout
Any single number	35 to 1
Red	1 to 1

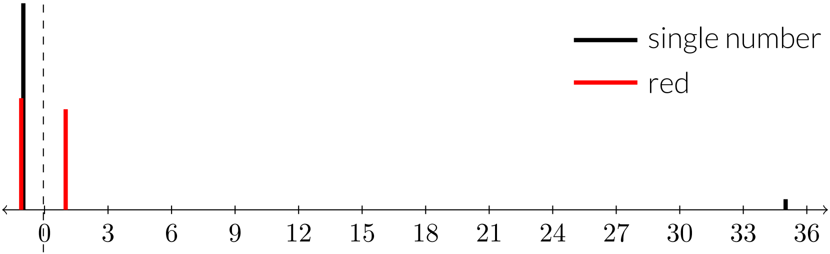
The two numbers in the payout indicate (1) how much money you win if you win and (2) how much money you lose if you lose.

Which bet is better for you?

Let's calculate the expected amount that you win.

Expected Value

Let's draw the p.m.f. of the net winnings from the two bets.



Both distributions have the same expected value of $-.053$.

So how do we quantify the difference between them?

Let's calculate the standard deviation of the amount that you win.

Bet on Red

Let R be the random variable representing the amount of money you win (or lose) when you make a \$1 bet on red. Find $\mathrm{SD}[R]$.

r	-1	1
$p[r]$	$20/38$	$18/38$

To calculate the standard deviation, we first calculate the variance. We use the expected value we calculated earlier: $\mathrm{E}[R] = -\$0.053$.

$$\begin{aligned}
\mathrm{Var}[R] &= E[(R - E[R])^2] \\
&= (-1 - (-.053))^2 \cdot \frac{20}{38} + (1 - (-.053))^2 \cdot \frac{18}{38} = .997. \\
\mathrm{Var}[R] &= E[R^2] - E[R]^2 \\
&= 1 - (-.053)^2 = .997
\end{aligned}$$

So $\mathrm{SD}[R] = \sqrt{.997} = 0.999$.

Bet on a Single Number

Let S represent the amount of money you win (or lose) when you make a \$1 bet on the number 23. Find $\mathrm{SD}[S]$.

s	-1	35
$p[s]$	$37/38$	$1/38$

To calculate the standard deviation, we first calculate the variance. Let's calculate the variance in a cute way. Consider the random variable X with p.m.f.

x	0	1
$p[x]$	$37/38$	$1/38$

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Then, X is Binomial($n = 1, p = 1/38$), so by the formula sheet,

$$\mathrm{Var}[X] = np(1-p) = \frac{1}{38}\left(1 - \frac{1}{38}\right) \approx .0256.$$

Note that $S = 36X - 1$. So $\mathrm{Var}[S] = 36^2 \mathrm{Var}[X] = 33.2$.
So $\mathrm{SD}[S] = \sqrt{33.2} = 5.8$.

So which is better?

If the expected value of two bets are equal, would you prefer the one with higher or lower variance?

If the expected value is positive, we might prefer a bet with a lower variance, to minimize the probability we lose money.

If the expected value is negative, we might prefer a bet with a higher variance, to maximize the probability we win money.