MatLab Project 1 - For-loop and Approximation of π (Due February 6)

MatLab (**Mat**rix **Lab**oratory) is an interactive software system for numerical computations and graphics. As the name suggests, **MatLab** is especially designed for matrix computations: solving systems of linear equations, computing eigenvalues and eigenvectors, factoring matrices, and so forth. In addition, it has a variety of graphical capabilities, and can be extended through programs written in its own programming language. Actually, **MatLab** has been called the 4th generation programming language.

MatLab is installed on PCs in Lab 220, Thompson Hall. To start the software, click the following sequence of icons: **Start, Programs, TH 220 Programs, MatLab6.5.1, MatLab icon**. Three windows are displayed by **MatLab.** For this project, you mainly use the "Command Window" on the right. So, you may close the other two windows. Now let us learn some simple **MatLab** commands.

1. **To Quit:**

   To quit from **MatLab**, type "**quit**" at the MatLab prompt, i.e.

   ```
     >> quit (enter)
   ```

2. **Using MatLab as a calculator**

   MatLab can be used as an expression evaluator. To do this you simply type a mathematical expression after the **MatLab** prompt >>. For example, we want to evaluate π :

   ```
     >> pi (enter)
   ```

   Now You see: ans = 3.1416. If you need to read more decimal digits, type **format long** and then press the return key. Now type pi again.

   ```
    >> formate long
    >> pi
     >> 3.14159265358979
   ```

   Type **format short** to return back to 5 decimal digits. **pi** is a built-in MatLab variable. Try the following examples:

   ```
    >> sin(pi/4)
    >> 2^(log2(4))
    >> sqrt(9)
   ```

3. **Assigning Values to Variables**

   MatLab allows you create variables. For example,

   ```
    >> x = 5;
    >> y = pi/4;
    >> z = y + sqrt(x)
        z =3.02146614089724
   ```

   From this point on, $x = 5$, $y = \frac{\pi}{4}$ and $z = \frac{\pi}{4} + \sqrt{5}$.

   Use the **who** command to list the currently active variables.

   ```
    >> who
        Your variables are:   ans x y z
   ```

4. **Built-in Functions**

   MatLab has many built-in functions. For example, (try them to see what you get)

   ```
     >>sin(pi/2)
    >>cos(pi/3)
    >>tan(pi/4)
    >>exp(0.2)
    >>atan(1)
    >>log(exp(1))
   ```

   Guess and try a function yourself. How does it work? After you use **MatLab** for a while, you can guess many commands.

5. **Operations:**

   Operations: $+, -, x$, /, ^. Try the following.

```
>>2^(1/2)-sqrt(2)
>>3*exp(-1/2)*sin(pi/3)+2*log(3)
```

6. **Programs: - for-loop**

   Suppose we need to compute the sum $s = \sum_{i=1}^{n} \frac{(-1)^i}{i+1}$ for a given positive integer n.

```
>>n=1000;  s=0;  for i=1:n,  s=s+(-1)^i/(i+1); end
>>s
```

   If we need to compute the sum with a different n now, we can use the up arrow key to bring up the previous command and then change the value n.

   Your assignment (1): Try the same computation with $n = 10000$, $n = 100000$, and $n = 1000000$. What are the values of s? Use **format long** and copy down **10 digits**.

Now let us use **MatLab** to approximate π. It is known that the Taylor series for $\tan^{-1}(x)$ is

$$\arctan(x) = \tan^{-1}(x) = \sum_{i=1}^{\infty} (-1)^{i-1} \frac{1}{2i-1} x^{2i-1}, \quad -1 < x \leq 1.$$

Since $\arctan(1) = \frac{\pi}{4}$,

$$\pi = 4\arctan(1) = 4\left(\sum_{i=1}^{\infty} (-1)^{i-1} \frac{1}{2i-1} (1)^{2i-1} \right) = 4\left(\sum_{i=1}^{\infty} (-1)^{i-1} \frac{1}{2i-1} \right).$$

Hence, π can be approximated by

$$4\left(\sum_{i=1}^{n} (-1)^{i-1} \frac{1}{2i-1} \right)$$

for some large positive integer n. Let $n = 100$.

```
>>s=0; for i=1:100, s=s+(-1)^(i-1)/(2*i-1); end, s=4*s
   s =3.13159290355855
```

Now we compute the relative error:

```
>>abs(s-pi)/pi
   ans =0.00318301929431
```

Your assignment (2): Approximate π with $n = 1000$, $n = 10000$ and $n = 100000$. Compute each relative error.