LESSON 8.4  

Rhombuses, Rectangles, and Squares.

Vocabulary
A rhombus is a parallelogram with four congruent sides.

A rectangle is a parallelogram with four right angles.

A square is a parallelogram with four congruent sides and four right angles.

Rhombus Corollary: A quadrilateral is a rhombus if and only if it has four congruent sides.

Rectangle Corollary: A quadrilateral is a rectangle if and only if it has four right angles.

Square Corollary: A quadrilateral is a square if and only if it is a rhombus and a rectangle.

Theorem 8.11: A parallelogram is a rhombus if and only if its diagonals are perpendicular.

Theorem 8.12: A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

Theorem 8.13: A parallelogram is a rectangle if and only if its diagonals are congruent.

EXAMPLE 1
Use properties of special quadrilaterals

For any rhombus $ABCD$, decide whether the statement is always, sometimes, or never true. Draw a sketch and explain your reasoning.

a. $AB > BC$

b. $m \angle A > m \angle B$

Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. So, $AB = BC$. The statement is never true.
b. By definition, a rhombus is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent. Because \( \angle A \) and \( \angle B \) are not opposite angles, they are not necessarily congruent, and \( m \angle A \) could be greater than \( m \angle B \). If rhombus \( ABCD \) is a square, then \( m \angle A = m \angle B = 90^\circ \). So, the statement is \textit{sometimes} true.

Exercises for Example 1

For any rhombus \( DEFG \), decide whether the statement is \textit{always}, \textit{sometimes}, or \textit{never} true. Draw a sketch and explain your reasoning.

1. \( \angle DEG \cong \angle FEG \)
2. \( \angle DEG \cong \angle EFD \)
3. \( DG \cong GF \)

Solutions:

1. always

2. sometimes

3. always
EXAMPLE 2  
Classify special quadrilaterals

Classify the special quadrilateral. *Explain* your reasoning.

![Parallelogram Diagram]

**Solution**
The quadrilateral is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent, so all four angles of the quadrilateral are right angles. By the Rectangle Corollary, the quadrilateral is a rectangle. Because the four sides are not congruent, the rectangle is not a square.

EXAMPLE 3  
List properties of special parallelograms

Sketch rhombus $ABCD$. List everything you know about it.

**Solution**
By definition, you need to draw a figure with the following properties:
- The figure is a parallelogram.
- The figure has four congruent sides.

![Rhombus Diagram]

Because $ABCD$ is a parallelogram, it also has these properties:
- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.11, the diagonals of $ABCD$ are also perpendicular.

**Exercises for Examples 2 and 3** Classify the special quadrilateral. *Explain* your reasoning.

4. 

![Square Diagram]

5. 

![Kite Diagram]
6. Sketch square $ABCD$. List everything you know about it.

SOLUTIONS TO #4, 5, AND 6

4. The quadrilateral has four congruent sides, so it is a rhombus. Because all four angles are congruent, by the Corollary to Theorem 8.1, the measure of each angle is $360^\circ \div 4 = 90^\circ$, and the quadrilateral is a rectangle. So, by the Square Corollary, the quadrilateral is a square.

5. The diagonals bisect each other, so by Theorem 8.10 the quadrilateral is a parallelogram. The diagonals are perpendicular, so by Theorem 8.11 the parallelogram is a rhombus.

6. 

By definition, square $ABCD$ is a parallelogram with four congruent sides and four right angles. Because $ABCD$ is a parallelogram, it also has these properties: Opposite sides are parallel and congruent; opposite angles are congruent; consecutive angles are supplementary; and diagonals bisect each other. Because squares are also rectangles, by Theorem 8.12, the diagonals of $ABCD$ are congruent.