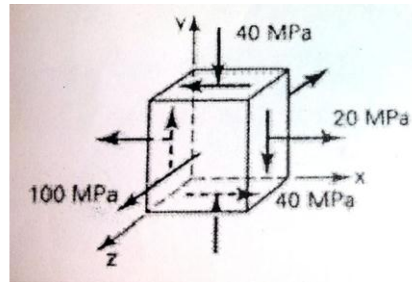
1. **Given the state of stress described in the figure below, determine the principal stresses and the angle of rotation for the principal stresses.**



Ans.

$\sigma_x = 20\ Mpa$, $\sigma_y = -40\ Mpa$, $\sigma_z = 100\ Mpa$ and $\tau_{xy} = 40\ Mpa$

Method (1) - Using Equation 3-15 $\rightarrow$ $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$$I_1 = \sigma_x + \sigma_y + \sigma_z = 20 - 40 + 100 = 80$$

$$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -800 + 2000 - 4000 - 40^2 - 0 - 0 = -4400$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_z\tau_{xy}^2 - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 = -80000 - 160000 = -240000$$

$$\rightarrow \quad \sigma^3 - (80)\sigma^2 + (-4400)\sigma - (-240000) = 0$$

This equation can be solved using softwares, calculator or method below:

(Knowing one of the principal stresses is $\sigma_z = 100\ Mpa$ - because $\tau_{yz} = \tau_{zx} = 0$ -)

با حل معادله فوق، تنش‌های اصلی به دست می‌آیند. این معادله را می‌توان از طریق نرم‌افزار، ماشین حساب و یا روش زیر حل کرد:

(می‌دانیم یکی از تنش‌های اصلی $\sigma_z = 100\ Mpa$ است، چرا که در هیچ‌یک از صفحات شامل z تنش برشی نداریم.)

$$\sigma^3 - (80)\sigma^2 + (-4400)\sigma - (-240000) = (\sigma - 100)(\sigma^2 + 20\sigma - 2400) = 0$$

$$(\sigma^2 + 20\sigma - 2400) = 0 \quad \rightarrow \quad \boldsymbol{\sigma_1} \ \& \ \boldsymbol{\sigma_2} = \boldsymbol{-60} \ \& \ \boldsymbol{40\ Mpa}$$

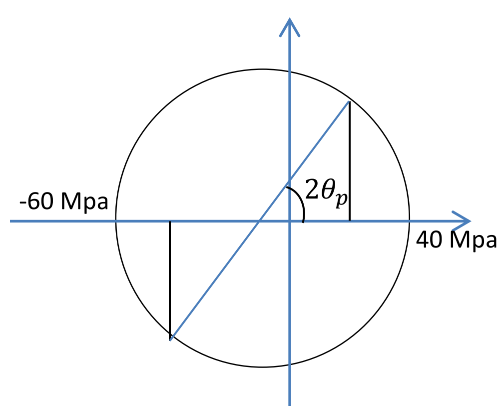
$$\boldsymbol{\sigma_3 = 100\ Mpa}$$

Method (2) - knowing $\sigma_3 = 100\ Mpa$, Mohr Circle can be used to determine σ_1 & σ_2.

$$\boldsymbol{\sigma_1} \ \& \ \boldsymbol{\sigma_2} = \boldsymbol{-60} \ \& \ \boldsymbol{40\ Mpa}$$

Using Mohr Circle to find Angle of Rotation for the principal stresses:

$$\tan(2\theta_p) = \frac{40}{20-(-40)/2} = \frac{4}{3} \quad \rightarrow \quad \boldsymbol{\theta_p = 26.56}\ °$$



2. The stresses at a point are given by the stress-matrix shown. Determine the overall maximum shear stress.

$$\mathbf{A}=\begin{bmatrix} \mathbf{90} & \mathbf{-200} & \mathbf{0} \\ \mathbf{-200} & \mathbf{175} & \mathbf{225} \\ \mathbf{0} & \mathbf{225} & \mathbf{150} \end{bmatrix}\mathbf{Mpa}$$

Ans.

$\sigma_x = 90\ Mpa$, $\sigma_y = 175\ Mpa$, $\sigma_z = 150\ Mpa$, $\tau_{xy} = -200\ Mpa$, $\tau_{yz} = 150\ Mpa$

Method (1) - Using Equation 3-15 $\rightarrow$ $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$I_1 = \sigma_x + \sigma_y + \sigma_z = 90 + 175 + 150 = 415$

$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = -35125$

$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_z\tau_{xy}^2 - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 = -8193750$

$\rightarrow \quad \sigma^3 - (415)\sigma^2 + (-35125)\sigma - (-8193750) = 0$

Method (2) – Solve equation below for λ

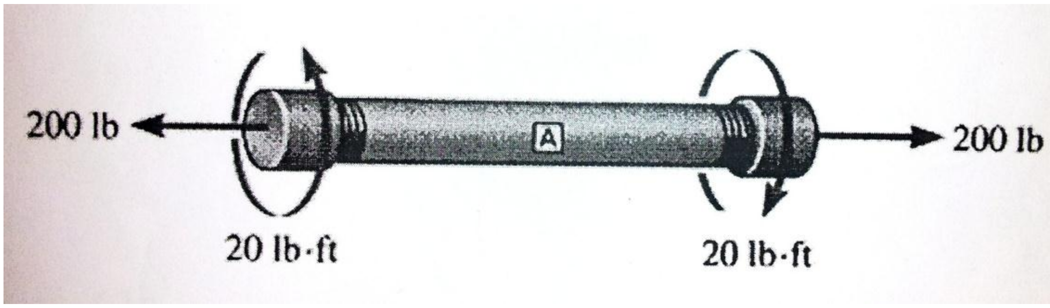
$|\lambda I - A| = 0 \qquad \rightarrow \qquad \lambda^3 - 415\lambda^2 - 35125\lambda + 8193750 = 0$

This equation can be solved using softwares or calculator.

$\rightarrow \quad \boldsymbol{\sigma_1 = 452.6\ Mpa}$, $\boldsymbol{\sigma_2 = 117.05\ Mpa}$, $\boldsymbol{\sigma_3 = -154.46\ Mpa}$

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 303.6\ Mpa$

3. The thin-walled pipe has an inner diameter of 0.5in. and a wall thickness of 0.025in.. If it is subjected to an internal pressure of 500psi and to the axial tension and torsional loadings shown, determine the state of stresses on element A. Use x-axis as horizontal and y-axis as vertical upwards.



Ans.

$$\text{Thin-walled} \rightarrow \begin{cases} \sigma_t = \frac{pd_i}{2t} \\ \sigma_l = \frac{pd_i}{4t} \\ \xrightarrow{r=r_0} \ \sigma_r = 0 \end{cases}$$

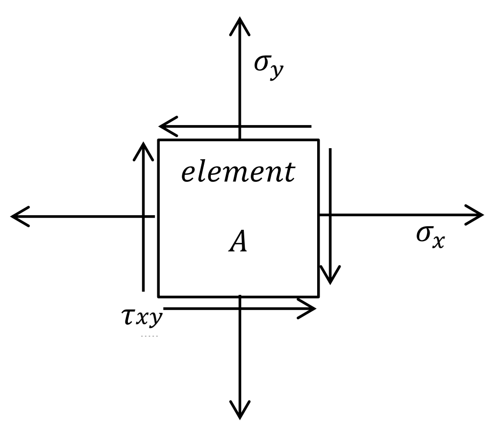
$$\frac{F_x}{A} = \frac{F_x}{\pi(r_o{}^2 - r_i{}^2)} = \frac{F_x}{2\pi r_{ave} t}$$

$$\sigma_x = \sigma_l + \frac{F_x}{A} = \frac{500\times0.5}{4\times0.025} + \frac{200}{2\times\pi\times0.2625\times0.025} = 2500 + 4850.4 = 7350.4\,psi = 7.35\,ksi$$

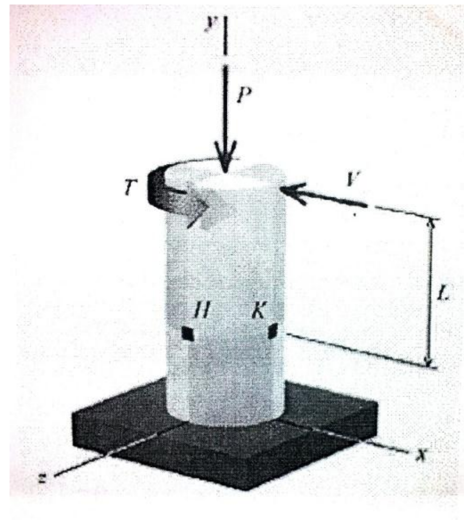
$$\sigma_y = \sigma_t = \frac{pd_i}{2t} = \frac{500\times0.5}{2\times0.025} = 5000\,psi = 5ksi$$

$$\sigma_z = \sigma_r = 0$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{Td_i/2}{\pi/_{32}(d_o{}^4 - d_i{}^4)} = \frac{(20\times12)0.55/2}{\pi/_{32}(0.55^4 - 0.5^4)} = 23176\,psi = 23.2\,ksi$$



4. **A 2.5-in.- diameter solid aluminium post is subjected to a horizontal force of V=3 kips, a vertical force of P=7 kips, and a concentrated torque of T=11 kip-in., acting in the direction shown. Determine the normal and shear stresses acting at point H (in ksi). Assume L=3.5 in.**



Ans.

At point H:

$$\sigma_x = \frac{My}{I} \overset{y=0}{\Longrightarrow} = 0$$

$$\sigma_y = \frac{P}{A} = \frac{-7}{\pi r^2} = 1.43\ ksi$$

$$\tau_{xy} = \frac{Tc}{J} + \frac{-VQ}{It} = \frac{16T}{\pi d^3} + \frac{-4V}{3\pi r^2} = \frac{16\times 11}{\pi 2.5^3} + \frac{-16\times 3}{3\pi 2.5^2} = 2.77\ ksi$$

$$\sigma_z = \tau_{xz} = \tau_{zy} = 0$$