

Sample Problem Sets

KSEA National Mathematics Competition 2007

These sets cover : 4th Grade – 11th Grade

Notes

- These problems do not necessarily cover every aspect of the actual test. These are problems chosen from the pool of old problem sets with some modification. This is to give you an idea about the style and scope of the Test. (Warning: No one can guarantee that in the actual test there will not be an abrupt change from these samples.)
- Each set of problems in this sample consists of 15 problems, which is one-half of the number of problems that will be given in the competition. (No answer keys are provided here.)
- Each set is composed of multiple choice problems and write-in problems. Regardless of the difficulty, all problems will be equally weighed, worth 1 point. No partial credit will be provided for any problems.
- No scrap paper, calculator, dictionary, ruler, or note will be allowed in this sample as well as in the competition. (You will need to use the provided space and back of the problem sheets to solve all the problems in the competition. No communication devices will be allowed during the test either.)
- In the competition, you will be asked to write all the answers into the “Answers” part of the answer sheet! For the multiple-choice problems, you are asked to clearly write your answer choice(a, b, c, d, or e), and for the write-in problems, write a simplified answer (fractions must be reduced completely, etc). A sample of answer sheet is provided.
- In the competition, when you will be done with the test, you will be asked to return both the problem sheets and the answer sheet to the exam proctor.
- Any comments, corrections, and suggestions should be addressed directly to the sample provider Dr. Sung Y. Song at sysong@iastate.edu.

KSEA National Mathematics Competition 2007

(Sample Answer Sheet)

Identification

Registration #	English Name	Korean Name
School Grade	School Name	KSEA Chapter

Answers

1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

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N1		N2		N3	
G1		G2		G3	
		R2		R3	

Grade 4

- (1) $125 + 367 + 12 = ?$
(a) 414 (b) 404 (c) 489 (d) 504 (e) 514
- (2) Which number is divisible by 5?
(a) 138 (b) 128 (c) 125 (d) 118 (e) 64
- (3) John has \$126.20 and the price of a 10-speed bike is \$144.99. How much more does he need to buy this bike?
(a) \$10 (b) \$14.50 (c) \$20.99 (d) \$18.79 (e) \$19.99
- (4) Which one of these is equivalent to $\frac{56}{48}$?
(a) $\frac{4}{3}$ (b) $\frac{5}{4}$ (c) $\frac{6}{5}$ (d) $\frac{7}{6}$ (e) $\frac{8}{7}$
- (5) What is the greatest common divisor (factor) of 24 and 64?
(a) 7 (b) 3 (c) 8 (d) 24 (e) 64
- (6) What is the 15% of \$60.00?
(a) \$6 (b) \$7 (c) \$8 (d) \$9 (e) \$10
- (7) Which of the following is not equal to the others?
(a) $\frac{3}{8}$ (b) $\frac{6}{16}$ (c) $\frac{15}{40}$ (d) $\frac{35}{80}$ (e) $\frac{27}{72}$
- (8) A positive integer is called a prime number if it cannot be divided exactly except by itself and the number 1. Which of the following is a prime number?
(a) 17 (b) 21 (c) 27 (d) 35 (e) 49
- (9) A box of apple is \$30 and there are exactly 60 apples in one box. What is the price for one apple?
- (10) Grace's two test scores are 90 and 86. What is her average test score?
- (11) James can travel 5 miles in 30 minutes by his bike. How many miles can he travel by his bike in 2 hours?
- (12) A can of tennis balls costs \$3.50. How many cans of tennis balls can you buy with \$28.
- (13) In a tennis tournament, one out of ten players are over 50 in age. 250 people are going to participate the Fourth of July-tennis tournament. How many participants are of age over 50?
- (14) Hyungmin has seven quarters. A piece of gum costs 35 cents. How many pieces of gum can he buy if he uses all his quarters?
- (15) The volume of a cube is 125 cubic inches. How many square inches are there in the surface area of the cube? (There are six faces)

Grade 5

- (1) Which of following is not equal to the other four?
(a) 8.31 (b) $8\frac{31}{100}$ (c) $8 + \frac{31}{100}$ (d) $\frac{831}{100}$ (e) $8 + \frac{31}{1000}$
- (2) Which of the following cannot be the measure of an angle in a triangle?
(a) 185° (b) 60° (c) 45° (d) 160° (e) 90°
- (3) For $\frac{28}{6} = \frac{x}{3}$, what is x ? (a) 25 (b) 14 (c) 22 (d) 19 (e) 17
- (4) If $10 < a$ and $a < b$, which of the following statements is always true?
(a) $a + 1 = b$ (b) $a = b$ (c) $b < 12$ (d) $10 < b$ (e) $b > 20$
- (5) Which of the following must be the next number in the sequence : 2, 5, 11, 20, ___ ?
(a) 29 (b) 31 (c) 32 (d) 36 (e) 30
- (6) What is the least common multiple of 30 and 45 ?
(a) 30 (b) 45 (c) 60 (d) 90 (e) 120
- (7) Joe's recipe calls for 6 tablespoons of butter and 5 oz of flour to make a piecrust. If Joe wants to make slightly more piecrust by using 8 oz of flour, how many tablespoons of butter should he use to keep the proportion of butter to flour the same?
(a) 6 (b) 6.7 (c) 7 (d) 9 (e) 9.6
- (8) $21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30 = ?$
(a) 250 (b) 255 (c) 510 (d) 350 (e) 300
- (9) The scale on a map is given as 1 inch = 2 feet. If a wall is 14 feet long, how long would it be in inches in the map?
- (10) What is the greatest common factor (divisor) of 42 and 70?
- (11) $\frac{1}{\frac{1}{2} - \frac{1}{3}} = ?$
- (12) If $2^3 + 2^3 + 2^3 + 2^3 = 2^n$, what is n ?
- (13) If $\frac{5}{8} = \frac{1.25}{x}$, what is x ?
- (14) Bill took 3 exams. The average of 3 exams is 92. What does he need to score on his fourth exam to have a new average of 93 ?
- (15) Hyunjoo can finish a job in 3 hours. Minkyung can complete the same job in 2 hours. If both of them work together on the same job, how long does it take them to complete the job?

Grade 6

- (1) Grace has 20% of \$100. Chulsoo has 10% of Grace's. What percent of \$100 does Chulsoo have?

(a) 1% (b) 2% (c) 5% (d) 10% (e) 20%

- (2) A line segment \overline{AB} is divided into two parts, \overline{AC} and \overline{CB} , according to the golden ratio where $\overline{AC}/\overline{CB} = 8/5$. If $\overline{CB} = 15$, what is \overline{AC} ?



(a) 18 (b) 16 (c) 40 (d) 32 (e) 24

- (3) Jinho realized that he did not record the data measured at time= 5 in the table below. Fortunately, he remembered that the mean value of the total data set was 23. What is the missing value in his table?

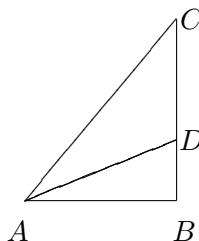
Time	1	2	3	4	5	6	7
Data	8	13	18	23	?	33	38

(a) 24 (b) 26 (c) 28 (d) 30 (e) none of the above

- (4) The ratio of Graces weight to Todd's weight is 2 : 3. If the ratio of Todd's weight to Sam's weight is 4 : 5, what is the ratio of Grace's to Sam's?

(a) 2 : 5 (b) 3 : 4 (c) 7 : 13 (d) 8 : 15 (e) 9 : 16

- (5) If $\overline{BC}/\overline{BD} = 3$ and the area of the right triangle $\triangle ABC$ is 12, what is the area of the triangle $\triangle ADC$?



(a) 4 (b) 6 (c) 8 (d) 10 (e) none of the above

- (6) A rectangular box has length L , height H , and width W . A square box has length L , height L , and width L . If $L = H$ and $W = 2H$, what is the ratio of the surface area of the rectangular box to the surface area of the square box?

(a) 5/3 (b) 2 (c) 4/3 (d) 3 (e) 3/2

- (7) Which digit will appear in the 201th place after the decimal point in the decimal representation of A ?

$$A = \frac{1}{3} + \frac{5}{11}.$$

(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

- (8) The average of two numbers a and b is C , and the product of a and b is D . What is then $a^2 + b^2$?

(a) $C^2 - 2D$ (b) $2C^2 - 2D$ (c) $4C^2 - D$ (d) $2C^2 - D$ (e) $4C^2 - 2D$

- (9) The length of one side of a triangle is 3. The other sides of the triangle are selected at random from the set S . What is the probability that the selected sides form an isosceles?

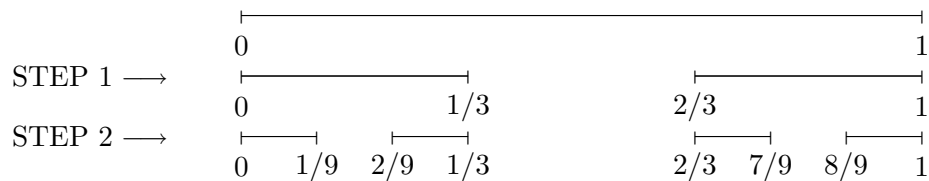
$$S = \{(3, 4), (4, 4), (4, 5), (5, 3), (1, 2), (2, 4)\}$$

- (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) 1

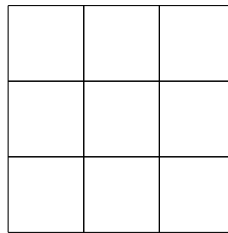
- (10) If the diameter of a circle is 1.5 times the diameter of a smaller circle, what is the ratio of the area of the bigger circle to that of the smaller circle?

- (a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{16}{9}$ (d) $\frac{25}{9}$ (e) $\frac{9}{4}$

- (11) Sejin has a line segment which has length 1. He removed its middle third (see the each of figure below, STEP 1). Then he removed the middle third from the remaining two segments (STEP 2). He then removed the middle third from each of the remaining four segments. He repeated this procedure until he got 32 segments. What is the total length of all 32 segments? Give your answer as a fraction.



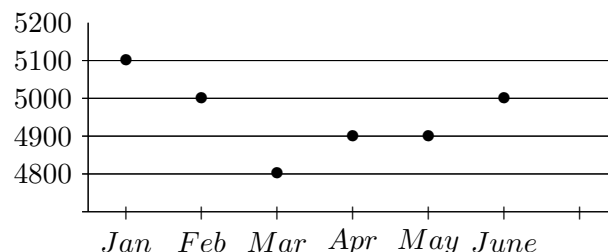
- (12) What is the sum of the areas of all possible squares of any size on the 3×3 board? The area of the smallest square is 1.



- (13) What is ab if the positive integers a and b satisfy the following equation?

$$(a + b)^2 + (a - b)^2 = 50.$$

- (14) The following graph shows how many books were sold at a bookstore each month. For example, the bookstore sold 5100 books in January in 2005. On Average, how many books were sold per month for the first half of 2005?

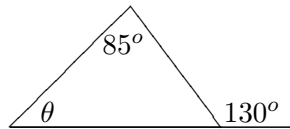


- (15) The local YMCA has two empty rectangular swimming pools, one for adults and the other for children. The volume of the children's pool is 60% of the adult's pool. The staff started supplying water to both pools with the same flow rate. If 80% of the children's pool was filled after T hours, how much more time do we need in terms of a percent of T until 90% of the adult's pool is filled?

Grade 7

- (1) What is the value of $\frac{5}{2x-5} - \frac{2x}{2x-5}$ if $x \neq \frac{5}{2}$?
- (a) 0 (b) 1 (c) -1 (d) $\frac{1}{2x-5}$ (e) None of the above

- (2) What is the degree measure of the angle θ in the following figure?

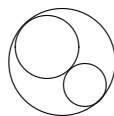


- (a) 35° (b) 30° (c) 45° (d) 95° (e) None of the above
- (3) If $12a + 10b = 120$, then what is the value of $\frac{a}{5} + \frac{b}{6}$?
- (a) 1 (b) 2 (c) 240 (d) 60 (e) None of the above
- (4) A 3×3 square is called a magic square if the sum of every row, column, and main diagonal is the same when arranging the numbers 1 through 9 into the square. Benjamin has placed three numbers to begin with, and then he has checked whether he could successfully complete it. Suppose he has succeeded to make it a magic square, what is the number he has filled in the middle of the square?

	7	
		9
8		

- (a) 3 (b) 4 (c) 5 (d) 6 (e) None of the above
- (5) Tom is asked his age, and he responds, "My age is nine less than half of my math teacher's age." The teacher is asked her age, and she responds, "My age is three times Tom's age." What is Tom's age?
- (a) 18 (b) 54 (c) 27 (d) 16 (e) None of the above
- (6) What is the product of the least common multiple and the greatest common factor of 22 and 48?
- (a) 2 (b) 528 (c) 1056 (d) 96 (e) None of the above
- (7) The point $(5, 3)$ is reflected about the x -axis. The image point is then reflected about the y -axis. If the resulting point is (x, y) , What is the value of $x + y$?
- (a) -2 (b) 2 (c) 8 (d) -8 (e) None of the above

- (8) From a bag of coins, $\frac{1}{3}$ were given to Mary, $\frac{1}{5}$ to Norma, $\frac{1}{6}$ to Anna, and $\frac{1}{4}$ to Sam. The six left were given to Troy. How many coins were originally in the bag?
- (a) 20 (b) 120 (c) 60 (d) 240 (e) None of the above
- (9) A 3-digit counting number is created using the digits from the set $\{0, 1, 2, 3, 4\}$ (allowing repetition). How many odd numbers can be constructed?
- (10) An advertisement reads, "Take 10% off any item which is already discounted 30%." What is the percentage value of the combined discounts?
- (11) In linear measurements, 7 palms equal 1 cubit, and 28 digits equal 1 cubit. What is the total number of cubits in 8 palms, and 6 digits? Express your answer as a mixed fraction.
- (12) In a small town, three children deliver all the newspapers. Abby delivers three times as many papers as Bob, and Connie delivers 13 more than Abby. If the three children deliver a total of 496 papers, how many papers does Connie deliver?
- (13) Two small circles with radii $2cm$ and $3cm$ are externally tangent. A third circle is circumscribed about the first two as shown. What is the ratio of the area of the smallest circle to the area of the third circle subtracted by the area of the first two circles?



- (14) $a + b = 8$, $b + c = -3$, and $a + c = -5$. What is the value of the product abc ?
- (15) Five coins look the same, but one is a counterfeit coin with a different weight than each of the four genuine coins. Using a balance scale, what is the least number of weighings needed to ensure that, in every case, the counterfeit coin is found and is shown to be heavier or lighter?

Grade 8

- (1) July 4, 1903, was a Thursday. On what day of the week was July 4, 1904?

(a) Monday (b) Tuesday (c) Wednesday (d) Thursday (e) Saturday

- (2) The mean of eight numbers is 101. If four numbers are each increased by 10, and four numbers are each decreased by 10, what is the new mean of the eight numbers?

(a) 91 (b) 101 (c) 111 (d) 121 (e) None of the above

- (3) The following table relates weight on the moon and weight on Earth measured in the same unit. What is Julie's weight on the moon if her weight on Earth is 120 lb?

weight on Earth (lb)	1	2	3	4	5	100
weight on Moon (lb)	0.16	0.32	.48	0.64	0.80	16

(a) 18 (b) 20.8 (c) 32 (d) 19.2 (e) None of the above

- (4) The perimeter of a rectangle is 48 inches, and its length is twice its width. What is the number of square inches in the area of the rectangle?

(a) 96 (b) 128 (c) 72 (d) 64 (e) None of the above

- (5) Evaluate: $(2 + 3)^{-1} \times (2^{-1} + 3^{-1})$

(a) $-\frac{2}{5}$ (b) $\frac{25}{36}$ (c) $\frac{3}{5}$ (d) $\frac{1}{6}$ (e) None of the above

- (6) Gene, Tina, and Brenda have ordered a large pizza, Gene will eat three times as much pizza as Tina. Tina will eat twice as much pizza as Brenda. If the pizza is cut into 36 slices, how many slices will Tina eat?

(a) 8 (b) 6 (c) 12 (d) 9 (e) None of the above

- (7) If $x + y = 12$ and $x - y = 8$, what is the value of $3x - xy$?

(a) 0 (b) 1 (c) 10 (d) -1 (e) None of the above

- (8) A ball is thrown into the air at 128 feet per second. The ball will be h feet above the ground at t seconds, and $h = 128t - 16t^2$. How many seconds later does the ball hit the ground?

(a) 10 (b) 8 (c) 12 (d) 6 (e) None of the above

- (9) What is the sum of all integer solutions to $|x + 2| = 5$?

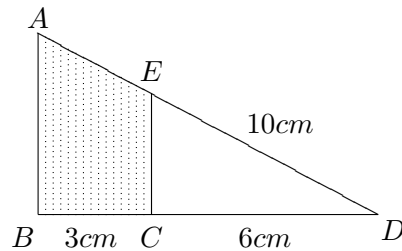
- (10) Mary is going to have a party for 95 guests. Hot dogs are sold in packages of eight and buns are sold in packages of ten. If she purchases the minimum number of packages of each to guarantee at least one hot dog and one bun for each guest, how many more buns than hot dogs does she have to buy?

- (11) In a survey of 110 college freshmen investigating their high school backgrounds, the following information was gathered:

25 took physics; 45 took biology; 48 took mathematics; 6 took physics and biology;
 10 took physics and mathematics; 8 took biology and mathematics;
 5 took all 3 subjects.

How many students took none of the three courses?

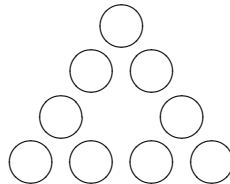
- (12) What is the area of the shaded region in the following figure if the length of ED is 10cm , $\angle B = \angle C = 90^\circ$? (Note the figure is not in correct scale.)



- (13) In a bag there are only red and white marbles. The probability of choosing a red marble is $3/5$. The probability of choosing two red marbles without replacement is $1/3$. How many white marbles are in the bag?

- (14) If α and β are the solutions of $x^2 - 4x - 3 = 0$, what is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

- (15) The digits $1, 2, 3, \dots, 9$ are arranged in the circles in the triangle shown so that the sum of the numbers on each side of the triangle is a constant. If the sum of the numbers in the three corner circles is 12, what is the constant sum of each side of the triangle?



Grade 9

- 15.** The value

$$2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \dots}}}}}$$

approaches to a number. What is this number?

Grade 10

1. Suppose that $3^{2x+1} = 12$. Which of the following value is $(\frac{1}{81})^x$?
- (A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{16}$ (D) $\frac{1}{32}$ (E) None of These

2. Consider the following system of two linear equations.

$$\begin{cases} 3x - 4y &= a, \\ bx - 12y &= 9. \end{cases}$$

Suppose the system has more than one solution. Which one of the following is equal to $\frac{b}{a}$?

- (A) 1 (B) 3 (C) 9 (D) 27 (E) None of These
3. How many real solutions does the equation $|-x^2 + 3x - 3| = 1$ have?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. Let $a = \log 2$ and $b = \log 3$. Which one of the following is the same as $\log 75$. Here \log means the common logarithm.
- (A) $-2a + b + 2$ (B) $-a + b + 1$ (C) $-a + b + 2$ (D) $2a + b + 2$ (E) None of These

5. Suppose z is the real number solution to $\sqrt{3 + \sqrt{z-1}} = 5$. What is the sum of the digits of z ?
- (A) 16 (B) 17 (C) 13 (D) 14 (E) None of These

6. Let a , b and c are integers. Suppose $x = 2$ and $x = 1 - \sqrt{2}$ are solutions of the equation $x^3 + ax^2 + bx + c = 0$. Find $a \cdot b \cdot c$?

7. Find the coefficient of x^8 in the expansion of $(\frac{1}{2} + 2x + x^{10})^{10}$.

8. Given the following system of equations

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{1}{3} \\ \frac{1}{x} + \frac{1}{z} &= \frac{1}{5} \\ \frac{1}{y} + \frac{1}{z} &= \frac{1}{7} \end{aligned}$$

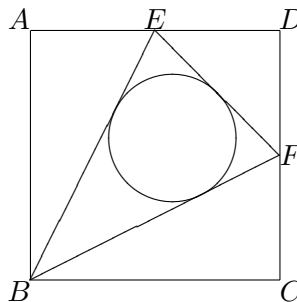
What is the value of the ratio $\frac{z}{y}$?

9. When $\frac{125 + 311i}{125 - 311i} = x + iy$, find the value for $x^2 + y^2$.

10. Find the distance between the two foci of the hyperbola

$$\frac{x^2}{5} - \frac{y^2}{7} = 1.$$

11. There are three different French books and two different Spanish books. How many ways are there to arrange the books in a row on a shelf with all books of the same language grouped together?
12. Find the sum: $(1) + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + \cdots + 100)$.
13. A rectangular field is to be fenced off next to a straight river, with fencing on three sides, and the river's edge making the fourth side. Exactly 100 feet of fencing is to be used. What is the maximum area, in square feet, you can make?
14. Let $\square ABCD$ be a square of side length 2, and E and F be the midpoints of \overline{AD} and \overline{CD} respectively. If a circle is embedded in the triangle $\triangle BEF$ as shown in the figure, what is the radius of the circle?



Note : Figure is not drawn in scale

15. Three points are randomly located on a circle. What is the probability that the shortest distance between each point is less than or equal to the radius of the circle?

Grade 11

- Given point $P(1, 2)$ in the xy plane, let Q be the point obtained by rotating P by $\frac{\pi}{2}$ radians counter-clock-wise about the origin and then reflecting about the line $y = x$. What is the distance between P and Q ?
(A) $\sqrt{5}$ (B) 3 (C) $2\sqrt{5}$ (D) 4 (E) None of These
- Consider a round table with six identical chairs. If four students and two teachers randomly take seats, what is the probability that two teachers will not sit next to each other?
(A) 0.9 (B) 0.6 (C) 0.5 (D) 0.3 (E) None of These
- What is the sum of all roots of the following equation?
$$\log_2(x + 1) + \log_2 x = 1$$

(A) 3 (B) -3 (C) 1 (D) -1 (E) None of these
- Find the y -intercept of the parabola satisfying the following properties: (i) the vertex is $(3, 0)$, (ii) the axis of symmetry is parallel to the y -axis, and (iii) it passes through the point $(2, 2)$.
(A) 8 (B) 12 (C) 14 (D) 18 (E) None of These
- If $z^2 = 1 + \sqrt{3}i$, what is the value of z^{12} ?
(A) $-8i$ (B) -16 (C) $32i$ (D) 64 (E) None of These
- Given two real-valued functions $f(x) = \log_2 x$ and $g(x) = 2 - \sqrt{9 - x^2}$, what is the domain of the composite $f \circ g$?
(A) $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ (B) $[-3, 3]$ (C) $[-3, -\sqrt{5}) \cup (\sqrt{5}, 3]$ (D) $(-\sqrt{5}, \sqrt{5})$ (E) None
- Let α be a solution of $x^2 + x + 1 = 0$. Then what is the value of
$$1 + \alpha + \alpha^2 + \cdots + \alpha^{12}$$
?
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0.
- If $\sin \theta + \cos \theta = 0.8$, what is the value of $\tan \theta + \cot \theta$?
- Suppose a circle C is inscribed in a square S , and an equilateral triangle T is inscribed in circle C . What is the ratio of the areas of S , C , and T ?
- Find the sum of all solutions of the equation $\log_x(6x^2 - 11x + 6) = 3$.

11. Solve the equation $8^x - 7 \cdot 4^x + 7 \cdot 2^{x+1} - 8 = 0$.

12. Consider triangles ABC and DEF which having the following properties: (i) the vertices D , E , and F lie on the sides CA , AB and BC , respectively, (ii) the triangles EAD and EBF are isosceles triangles with $\overline{EA} = \overline{ED}$ and $\overline{EB} = \overline{EF}$. What is angle measure in degrees of $\angle DEF$ when $\angle ACB = 55^\circ$?

13. Consider a cone with the height of 40 cm and the radius 30 cm. If the top one third (in height) of the cone is removed, what is the surface area of the side of the bottom section?

14. Compute

$$\lim_{x \rightarrow 0} \frac{\tan(3x^3 + 5x^2 - 2x)}{2x^3 - 3x^2 + x}.$$

15. Consider the circle $x^2 + y^2 - 2\sqrt{3}x - 2y + 3 = 0$ and two lines $y = \frac{\sqrt{3}}{3}x$ and $y = \sqrt{3}x$ drawn on the same Cartesian coordinate system. What is the area of the region that is bounded by the two lines and the circle, and that is lying in the exterior of the circle?