THE VIBRATION OF THE AIR FILAMENT IN QUILL TUBES CAPPED AT BOTH ENDS*

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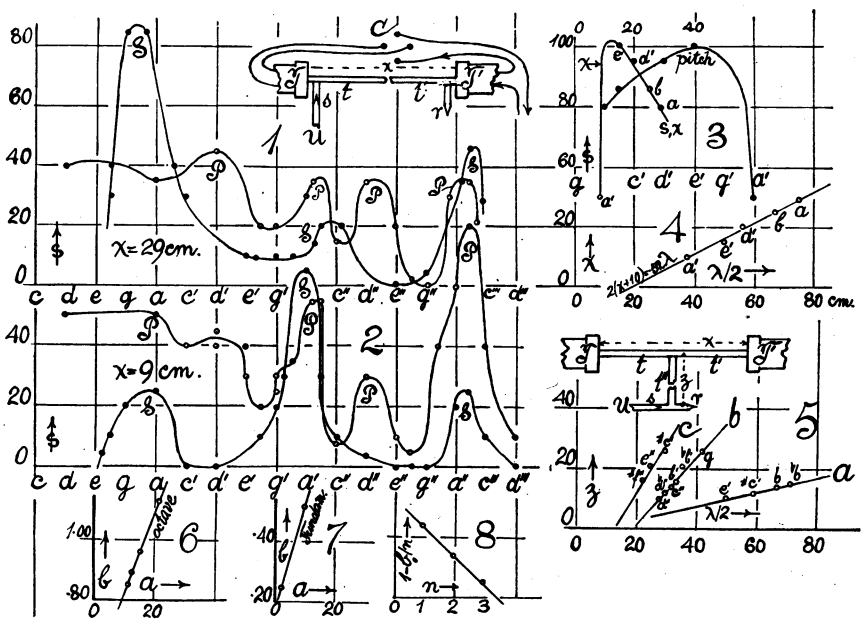
1. *Straight tubes.*—The plan of the experiments is similar to the preceding group for wide tubes. In figure 1 (inset) t, t' is the quill tube 0.35 cm. in diameter, actuated by the paired telephones T and T' at the ends and with salient (s) and reëntrant (r) pin-hole probes near the telephone plates. One shank of the interferometer U-gauge communicates with s, while r may be left free. The reversing switch C makes it possible to compel the plates of the telephones to vibrate in phase (P), or in opposed phases, i.e., in parallel or sequence (S). The length of quill tube is denoted



by x (if transverse by z) and the fringe displacement proportional to acoustic pressure by s. The telephones and the variable break circuit of a small inductor constitute an electric siren.

The graphs, figures 1 and 2, for tube lengths x = 9 cm. (the smallest available) and x = 29 cm. (the largest), for the like phase adjustment P at the two ends, are of the same character with a difference in intensity distribution. In the four foot octave, s is nearly constant or the resonance

is continuous. In view of the end position of the pin holes, it is difficult to detect a maximum moving with x. In the S adjustment, however, besides the fixed nodes, the principal maximum S drops about an octave (a' to g) as x increases from 9 to 29 cm. This is the fundamental quill tube note.

In figure 3 the intermediate cases are given, the resonance displacement being summarized in relation to tube length x and also to pitch. The usual optimum, here between 10 and 15 cm., is pronounced even in this case of straight tubes.

Figure 4 finally shows the relation of tube length x and free wave-length λ and may be reproduced by $2(x + a) = b\lambda$, where the accessory tube length $a = 10$ cm. and the coefficient $b = 0.51$. As in the earlier paper, no special effect of frequency, n, is apparent within the octave examined, though the frictional effect is very marked. If we write the equation of Helmholtz and Rayleigh in terms of wave-length (λ free, λ' imprisoned) it may be put (viscosity η, density ρ, frequency n, radius R)

$$\frac{\lambda-\lambda'}{\lambda} = \frac{\lambda/2 - (x + a)}{\lambda/2} = \sqrt{\eta/R^2 2n\rho}$$

so that in the absence of a special frequency effect $R(1-b) = \text{const.}$ Thus for the two diameters examined, $2R = 2$ cm. in the former paper and $2R = 0.35$ cm. here, the coefficients are $1-b = 0.08$ and 0.49. While $2R$ has been decreased 5.7 times, the coefficient $1-b$ has increased about 6 times. But the small differential coefficient $1-b = 0.08$ is of course not trustworthy to one or two units in the second decimal, so that all the relations could quite well be reconstructed in accordance with $(1-b)R =$ const., throughout.

2. *Transverse tubes.*—The case of transverse tubes of total length z (figure 5, inset), of which I will only give the graphs for 3 harmonics in figure 5, presents much greater complications. The length of salient (s) and reëntrant (r) pin-hole probe (s joined to the shank U of the interferometer U-gauge) are not negligible here but must be included in z. The telephone plates vibrating in phase (P) evoke the successive resonances, but the curves corresponding to figures 1 and 2 must here be omitted.

The graph b was first worked out. In connection with it, the graph c was detected. As this is a scant fifth above graph b, the fundamental graph a was suspected and subsequently found. Its intensities (s, not shown) were in fact the largest, b intermediate and c small. So far as the data go, the following equations suffice to reproduce them, using $2(z + a) = b\lambda$:

frequency ratio:	$n = 1$	$z + 2.5 = .25(\lambda/2)$	$\lambda' = 0.25\lambda$
	2	$z + 22 = 1.13(\lambda/2)$	$\lambda' = 0.56\lambda$
	3	$z + 22 = 1.65(\lambda/2)$	$\lambda' = 0.55\lambda$

where $(z + a)/n = \lambda'$, the semi wave-length under friction. The last two coefficients might be taken to be of the same order as $b = .52$ in the preceding section for straight tubes; but the coefficient of the fundamental $b = .25$ is out of accord with this. It is tempting to refer the discrepancy to the constriction in t'', of the vibration in t and t', the sectional area being halved. In such a case, however, the same result should appear for $n = 2$ and $n = 3$. The theoretical equation $(1-b/n)\sqrt{n} = \text{const.}$ fails equally to give a consistent explanation.

The most puzzling feature is the interdependent variation of a and b in successive experiments and due to incidental causes not detected. Thus in case of the octave

$a =$	12	13	16	22
$b =$	.85	.89	.96	1.13

were found at different times. Similarly for the fundamental $a = 2.5$, $b = .25$; $a = 10$, $b = .51$ are given above. Figures 6 and 7 show that these relations are grouped consistently and do not differ much for the fundamental and the octave. Hence the value of $b = b_0$ for $a = 0$ is determinable and the reduction for the fifth may be made with the same coefficient as for the octave; i.e., $b = b_0 + \beta a$.

Thus $x + a = (b_0 + \beta a)\lambda/2$, for $a = 0$ becomes $x = b_0\lambda/2$, while $x = n\lambda'/2$ so that $\lambda' = (b_0/n)\lambda$ and finally

$$(\lambda - \lambda')/\lambda = 1 - b_0/n$$

Numerically this amounts to $1-b_0/n = .94-n/10$ or $b_0 = .06n + n^2/10$; or again,

$n =$ 1	$b_0 =$	.16 (observed)	$b_0 =$	.16 (computed)
2		.52		.52
3		1.05		1.08

Figure 8 indicates the proportionality of $1-b_0/n$ with n. Insofar as this general method of contrasting harmonics as a whole and of eliminating the a,b dependence is valid, a special frequency effect in b_0 cannot be rejected; but it does not follow the theoretical equation.

* Advance note from a request to the Carnegie Institution, Washington, D. C.