

## Cones

Quadratic cone  $\mathcal{Q}^n$

$$x_1 \geq \sqrt{x_2^2 + \dots + x_n^2}$$

Rotated quadratic cone  $\mathcal{Q}_r^n$

$$2x_1x_2 \geq x_3^2 + \dots + x_n^2, \quad x_1, x_2 \geq 0$$

Power cone  $\mathcal{P}_3^{\alpha, 1-\alpha}$ ,  $\alpha \in (0, 1)$

$$x_1^\alpha x_2^{1-\alpha} \geq |x_3|, \quad x_1, x_2 \geq 0$$

Exponential cone  $K_{\text{exp}}$

$$x_1 \geq x_2 e^{x_3/x_2}, \quad x_2 \geq 0$$

## Simple bounds

$$t \geq x^2 \quad (0.5, t, x) \in \mathcal{Q}_r^3$$

$$|t| \leq \sqrt{x} \quad (0.5, x, t) \in \mathcal{Q}_r^3$$

$$t \geq |x| \quad (t, x) \in \mathcal{Q}^2$$

$$t \geq 1/x, \quad x > 0 \quad (x, t, \sqrt{2}) \in \mathcal{Q}_r^3$$

$$t \geq |x|^p, \quad p > 1 \quad (t, 1, x) \in \mathcal{P}_3^{1/p, 1-1/p}$$

$$t \geq 1/x^p, \quad x > 0, \quad p > 0 \quad (t, x, 1) \in \mathcal{P}_3^{1/(1+p), p/(1+p)}$$

$$|t| \leq x^p, \quad x > 0, \quad p \in (0, 1) \quad (x, 1, t) \in \mathcal{P}_3^{p, 1-p}$$

$$t \geq |x|^p/y^{p-1}, \quad y \geq 0 \quad (t, y, x) \in \mathcal{P}_3^{1/p, 1-1/p}$$

$$p > 1$$

$$t \geq x^T x / y, \quad y \geq 0 \quad (0.5t, y, x) \in \mathcal{Q}_r^{n+2}$$

$$t \geq e^x \quad (t, 1, x) \in K_{\text{exp}}$$

$$t \leq \log x \quad (x, 1, t) \in K_{\text{exp}}$$

$$t \geq 1/\log x, \quad x > 1 \quad (u, t, \sqrt{2}) \in \mathcal{Q}_r^3$$

$$(x, 1, u) \in K_{\text{exp}}$$

$$t \geq a_1^{x_1} \cdots a_n^{x_n}, \quad a_i > 0 \quad (t, 1, \sum x_i \log a_i) \in K_{\text{exp}}$$

$$t \geq x e^x, \quad x \geq 0 \quad (t, x, u) \in K_{\text{exp}}$$

$$(0.5, u, x) \in \mathcal{Q}_r^3$$

$$t \geq \log(1 + e^x) \quad u + v \leq 1$$

$$(u, 1, x - t) \in K_{\text{exp}}$$

$$(v, 1, -t) \in K_{\text{exp}}$$

$$t \geq |x|^{3/2} \quad (t, 1, x) \in \mathcal{P}_3^{2/3, 1/3}$$

$$t \geq x^{3/2}, \quad x \geq 0 \quad (s, t, x), (x, 1/8, s) \in \mathcal{Q}_r^3$$

$$t \geq 1/x^3, \quad x > 0 \quad (t, x, 1) \in \mathcal{P}_3^{3/4, 1/4}$$

$$0 \leq t \leq x^{2/5}, \quad x \geq 0 \quad (x, 1, t) \in \mathcal{P}_3^{2/5, 3/5}, \quad t \geq 0$$

## Means and averaging

$$\begin{array}{ll} \text{Log-sum-exp} & (z_i, 1, x_i - t) \in K_{\text{exp}} \\ t \geq \log(\sum e^{x_i}) & i = 1, \dots, n \end{array}$$

$$\sum z_i \leq 1$$

$$(z_i, x_i, t) \in \mathcal{Q}_r^3$$

$$i = 1, \dots, n$$

$$x_i > 0 \quad \sum z_i = nt/2$$

$$(z_i, x_i, z_{i+1}) \in \mathcal{P}_3^{1-1/i, 1/i}$$

$$i = 2, \dots, n$$

$$x_i > 0 \quad z_2 = x_1, \quad z_{n+1} = t$$

$$|t| \leq \sqrt{xy}, \quad x, y > 0 \quad (x, y, \sqrt{2}t) \in \mathcal{Q}_r^3$$

$$\text{Weighted geom. mean} \quad (z_i, x_i, z_{i+1}) \in \mathcal{P}_3^{1-\beta_i, \beta_i}$$

$$|t| \leq x_1^{\alpha_1} \cdots x_n^{\alpha_n}, \quad x_i > 0 \quad \beta_i = \alpha_i / (\alpha_1 + \dots + \alpha_i)$$

$$\alpha_i > 0, \quad \sum \alpha_i = 1 \quad i = 2, \dots, n$$

$$z_2 = x_1, \quad z_{n+1} = t$$

$$|t| \leq x^{1/4} y^{5/12} z^{1/3} \quad (s, z, t) \in \mathcal{P}_3^{2/3, 1/3}$$

$$x, y, z \geq 0 \quad (x, y, s) \in \mathcal{P}_3^{3/8, 5/8}$$

## Norms, $x \in \mathbb{R}^n$

$$\|\cdot\|_1, \quad t \geq \sum |x_i| \quad (z_i, x_i) \in \mathcal{Q}^2, \quad t = \sum z_i$$

$$\|\cdot\|_2, \quad t \geq (\sum x_i^2)^{1/2} \quad (t, x) \in \mathcal{Q}^{n+1}$$

$$\|\cdot\|_p, \quad p > 1 \quad (z_i, t, x_i) \in \mathcal{P}_3^{1/p, 1-1/p}$$

$$t \geq (\sum |x_i|^p)^{1/p} \quad i = 1, \dots, n$$

$$\sum z_i = t$$

## Geometry

$$\text{Bounding ball} \quad \min r$$

$$\min_x \max_i \|x - x_i\|_2 \quad (r, x - x_i) \in \mathcal{Q}^{n+1}$$

$$\text{Geometric median} \quad \min \sum t_i$$

$$\min_x \sum \|x - x_i\|_2 \quad (t_i, x - x_i) \in \mathcal{Q}^{n+1}$$

$$\text{Analytic center} \quad \max \sum t_i$$

$$\max_x \sum \log(b_i - a_i^T x) \quad (b_i - a_i^T x, 1, t_i) \in K_{\text{exp}}$$

## Regression and fitting

$$\text{Regularized least squares} \quad \min t + \lambda r$$

$$\min_w \|Xw - y\|_2^2 + \lambda \|w\|_2^2 \quad (0.5, t, Xw - y) \in \mathcal{Q}_r^{m+2}$$

$$(0.5, r, w) \in \mathcal{Q}_r^{n+2}$$

$$\text{Max likelihood} \quad \max \sum a_i t_i$$

$$\max_p p_1^{a_1} \cdots p_n^{a_n} \quad (p_i, 1, t_i) \in K_{\text{exp}}$$

$$\text{Logistic cost function} \quad u + v \leq 1$$

$$t \geq -\log(1/(1 + e^{-\theta^T x})) \quad (u, 1, -\theta^T x - t) \in K_{\text{exp}}$$

$$(v, 1, -t) \in K_{\text{exp}}$$

## Risk-return

$$\Sigma \in \mathbb{R}^{n \times n} - \text{covariance}, \quad \Sigma = LL^T, \quad L \in \mathbb{R}^{n \times k}$$

$$\max_x \alpha^T x \quad \max_x \alpha^T x$$

$$\text{s.t. } x^T \Sigma x \leq \gamma \quad (\sqrt{\gamma}, L^T x) \in \mathcal{Q}^{k+1}$$

$$\max_x \alpha^T x - \delta x^T \Sigma x \quad \max_x \alpha^T x - \delta r$$

$$(0.5, r, L^T x) \in \mathcal{Q}^{k+2}$$

$$\text{Risk plus } x^{1.5} \text{ impact cost} \quad t \geq \delta r + \beta \sum u_i$$

$$t \geq \delta x^T \Sigma x + \beta \sum |x_i|^{3/2} \quad (0.5, r, L^T x) \in \mathcal{Q}^{k+2}$$

$$(u_i, 1, x_i) \in \mathcal{P}_3^{2/3, 1/3}$$

$$\text{Risk in factor model} \quad \gamma \geq x^T (D + FSF^T) x$$

$$(0.5, t, \sqrt{D}x) \in \mathcal{Q}_r^{n+2}$$

$$D - \text{specific risk (diag.)} \quad (0.5, s, U^T F^T x) \in \mathcal{Q}_r^{k+2}$$

$$F \in \mathbb{R}^{n \times k} - \text{factor loads}$$

$$S = UU^T - \text{factor cov.}$$

## Convex quadratic problems

Let  $\Sigma \in \mathbb{R}^{n \times n}$ , symmetric, p.s.d.

Find  $\Sigma = LL^T$ ,  $L \in \mathbb{R}^{n \times k}$  (Cholesky factor).

Then  $x^T \Sigma x = \|L^T x\|_2^2$ .

$$t \geq \frac{1}{2} x^T \Sigma x \quad (1, t, L^T x) \in \mathcal{Q}_r^{k+2}$$

$$t \geq \sqrt{x^T \Sigma x} \quad (t, L^T x) \in \mathcal{Q}^{k+1}$$

$$\frac{1}{2} x^T \Sigma x + p^T x + q \leq 0 \quad (1, -p^T x - q, L^T x) \in \mathcal{Q}_r^{k+2}$$

$$\max_x c^T x - \frac{1}{2} x^T \Sigma x \quad \max c^T x - r$$

$$(1, r, L^T x) \in \mathcal{Q}_r^{k+2}$$

$$c^T x + d \geq \|Ax + b\|_2 \quad (c^T x + d, Ax + b) \in \mathcal{Q}^{m+1}$$