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# The Scientific Theories of Michael Faraday and James Clerk Maxwell

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The origins of modern electromagnetism and electrodynamics can be traced back to the influential work of two nineteenth century thinkers: Michael Faraday and James Clerk Maxwell. Michael Faraday, a British scientist with little formal training, was a great experimentalist. He devised a number of unique, inventive ways to test and explore electromagnetic phenomena, and he made several important discoveries regarding the nature of electromagnetism. These insights later provided a foundation for the work of James Clerk Maxwell, who used his mathematical skill to unify the then disparate science of electromagnetism into a coherent theory, complete with mathematical formalism.

It is tempting to hold the joint work of Faraday and Maxwell up as a paradigmatic example of the modern scientific process in practice, in which experimentation and observation yield facts that can then be organized into theory, the result being the mathematical formalism that allows for the prediction of future phenomena. And it is certainly true that the work that began with Faraday's experiments and ended with Oliver Heaviside's reformulation of Maxwell's mathematical relations did, in 1884, produce what are now known as Maxwell's Four Unifying Equations, which physicist Daniel Fleisch calls "four of the most influential equations in all of science."<sup>1</sup> But, in addition to this, the history of the work of Faraday and Maxwell can also serve as a case study for an interesting question regarding the place of mathematical formalism and systematic theories in physics. Although Faraday eschewed theoretical systems involving any mathematical formalism (or any mathematics at all beyond simple geometries), he did produce theoretical work, a type of theory that he felt more closely resembled the physical processes it was meant to describe. An analysis, then, of both the experiments and theoretical summations of Faraday can be usefully compared to the theoretical system and mathematical formalism produced by Maxwell in order to examine the differences between a physics theory

<sup>&</sup>lt;sup>1</sup> Daniel Fleisch, A Student's Guide to Maxwell's Equations (Cambridge: Cambridge University Press, 2008), vii.

without formalisms and a physics theory in which systematized mathematics have been incorporated—in other words, to examine why physicists feel the need to go beyond experimentation and relation of facts to the creation of complete and formal mathematized systems.

As will be demonstrated in this analysis, the expression of physics concepts and relationships in mathematical form can bring to light the fundamental features of a phenomenon that would otherwise remain opaque, as well as allow for a more precise exploration of the individual components of the phenomenon in question.

#### **The Historical Backdrop**

The continuity between the work of Faraday and Maxwell is not just thematic; it exists even in historical terms: "At age 64 in [the year] 1855," science writer Thomas K. Simpson notes, "Faraday...[was] finishing his productive scientific work at the moment when Maxwell, aged 24, [was] beginning his: the quest for the electromagnetic field [was] passed in the course of one year from Faraday's hands to Maxwell's. In one of history's magic moments, Faraday in 1855 handed his *Experimental Researches* directly to their ideal reader."<sup>2</sup>

There are, perhaps, two areas of historical backdrop relevant to the comparison of the work of Faraday and Maxwell: the individual educational background of each scientist, and their place in the overall scientific tenor of the time period.

James Clerk Maxwell was born in Scotland in 1831, the only child of an upper-middle class father. And he graduated from Trinity College at Cambridge University—a fact that is not insignificant in appraising Maxwell's mathematical training. As noted by Thomas K. Simpson,

<sup>&</sup>lt;sup>2</sup> Thomas K. Simpson, *Maxwell on the Electromagnetic Field: A Guided Study* (New Brunswick, NJ: Rutgers University Press, 1997), 11.

The centerpiece of liberal education as understood at Trinity College in the midnineteenth century [was] not so much the traditional humanities but rather mathematics...The 'mathematics' involved [was] itself broadly understood. Founded on the one hand in geometry and the geometrical style adopted by Isaac Newton in the *Principia*, it extend[ed] to mastery of powerful methods in differential and integral calculus which [were] newer and [owed] their origin rather to the French. Furthermore, 'mathematics' in the Cambridge curriculum...[included] without strict distinction both pure mathematics and a range of topics, such as mechanics, optics, and heat, which would fall today under the heading 'mathematical physics'.<sup>3</sup>

It seems likely that this broad interpretation of mathematics grounded Maxwell's view that science, at least in part, was mathematical in nature, for it prepared him to produce a physics theory that was more mathematically-oriented.

Faraday's academic background contrasted greatly with Maxwell's. As was typical of someone with his impoverished economic background, Faraday had little formal education. "Beyond the rudiments of reading, writing, and calculation, [Faraday] was self-educated, learning from books," Simpson writes. "He learned his electricity from an article in the *Encyclopedia Britannica*."<sup>4</sup> Though one might at first glance attribute his exclusion of mathematics in physics theories to this lack of mathematical training, it must be remembered that, as a self-educated scientist, Faraday certainly could have trained himself in mathematics had he simply added the subject to his studies in chemistry and electricity. Additionally, Faraday's own comments on the relationship between mathematics and science must be taken into account:

I do not remember that Math. have *predicted* much. Perhaps in Ampère's theory one or at most two independent facts. I am doubtful of two. Facts have preceded the math. or where they have not the facts have remained unsuspected though the calculations were ready...; and sometimes when the fact was present...the calculations were insufficient to illustrate its true nature until other facts came into help.<sup>5</sup>

<sup>&</sup>lt;sup>3</sup> Simpson, Maxwell on the Electromagnetic Field, 1-3.

<sup>&</sup>lt;sup>4</sup> Simpson, *Maxwell on the Electromagnetic Field*, 11.

<sup>&</sup>lt;sup>5</sup> Olivier Darrigol, *Electrodynamics from Ampère to Einstein* (Oxford: Oxford University Press, 2000), 38.

However, Faraday's antipathy towards mathematics may have been justified given the state of the study of electricity and magnetism in the early nineteenth century. When Faraday entered the field, most theoretical work involved Newtonian fluid mechanics, which were heavily mathematical, allowing for a great deal of quantitative analysis, but often producing little in the way of useful qualitative descriptions of the actual phenomenon. The champion of this particular form of theorizing was French scientist Ampère, who is appropriately recognized as the founder of electrodynamics. This "hegemony of Newtonian fluid theories" was prevalent in most European scientific communities, but, as Darrigol goes on to note, "a few British physicists escaped this general evolution...First and foremost was Faraday," joined by William Thomson (Lord Kelvin), and, of course, by Maxwell.<sup>6</sup>

#### **Faraday: Methods and Theory**

Faraday consistently broke with traditional methods in his approach to electromagnetic research. In order to properly identify the difference between a mathematically-based theory, such as Maxwell's, and physics research done without recourse to such formalisms, it is vital that one understand Faraday's view of the relationship between experimentation, theory, and scientific discovery.

Faraday held what might be termed a purist's view of physics theories—he wanted any theoretical explanation to provide a direct correlation between the account of a phenomenon and the phenomenon itself, without intervening conceptual abstractions or mathematical formulations. Faraday described his method as one of "feel[ing] my way by facts closely placed together."<sup>7</sup> While it is tempting to distill this view into one where science is either purely

<sup>&</sup>lt;sup>6</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 77.

<sup>&</sup>lt;sup>7</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 21.

experimental or purely descriptive, such a perspective does not appear to have been adopted— Faraday's works do, in fact, include speculation and even formulations. However, for Faraday, these are always an attempt to actually come to *know*, in the sense of coming to meet, the physical phenomena qua phenomena.<sup>8</sup> Faraday thought that what the scientist ought to desire was "not a variety of different methods of representing forces, but the one true physical signification of that which is rendered apparent to us by the phaenomena."<sup>9</sup> One can conclude, then, that Faraday's objection to mathematics was that it was something that came *between* the scientist and the observed phenomenon.

Simpson writes that a paradigmatic example of how Faraday viewed the relationship between theoretical concepts and physical phenomena can be found in the scientist's own speculations regarding the physical existence of magnetic lines of force.<sup>10</sup> Originally, these lines of force held only explanatory power, but as he continually noted the way these lines could be integrated into other aspects of electromagnetic phenomena, Faraday began to believe that these lines must have some kind of physical existence—the explanatory concept, if experimentally relevant, must possess a physical reality.

Though his theory was limited—it could offer description, speculation, and, at its highest points, direct relationships that could be used as laws, but never fundamental theoretical frameworks or definitions of the essence of the phenomena—Faraday's experimental methods were inversely thorough and often striking in their subtlety. Unconcerned with making broad statements, Faraday reserved his genius for application to his experimental researches. Faraday's method was to identify a salient feature of electromagnetism that bore inquiry, devise a method for exploration, and then repeat the experiment, gradually altering various aspects to see what he

<sup>&</sup>lt;sup>8</sup> Simpson, *Maxwell on the Electromagnetic Field*, 15.

<sup>&</sup>lt;sup>9</sup> Simpson, Maxwell on the Electromagnetic Field, 15.

<sup>&</sup>lt;sup>10</sup> Simpson, Maxwell on the Electromagnetic Field, 12.

could tease out. It is this character of Faraday's work that Darrigol picks out when he refers to Faraday's methods as the making of "chains or trees of experiments."<sup>11</sup>

#### **Faraday: Experiments and Findings**

Given Faraday's preference for experiment and observation above theoretical formalisms, it should not be surprising that Faraday proliferated far more varied and significant experimental results than any other electromagnetic physicist of his day. While this paper is principally concerned with Faraday's conceptual physics, his ideas must be placed within their framework of Faraday's extensive observations, explorations, and discoveries. It is therefore appropriate to discuss some of his more influential experimental work.

The phenomena either discovered or furthered by Faraday include rotations of the magnetic field around an electric current, exploration of all possible methods of induction, researches in diamagnetism (the effect on various materials to an applied magnetic field), electrochemical equivalence, dielectrics (or inductive capacity), and magneto-optical rotation. Though this list is by no means exhaustive, these are the areas that were most fruitful for Faraday, and which had the most effect on his thinking.

#### **Faraday: Important Concepts and the Field**

By the time he had completed some of his most important experiments, Faraday had begun to focus on certain speculations that would become central to the science of electromagnetism. Of these, lines of force were, for Faraday, paramount. His thoughts on this concept influenced his interpretation of the remaining phenomena, including contiguity, charge,

<sup>&</sup>lt;sup>11</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 38.

and induction; it also provided a conceptual foundation for a field theory of electromagnetic phenomena.

Lines of force rose to prominence for Faraday as the result of his focus on what he called "powers". In order to avoid the common use of fluid theories by his contemporaries, which he saw as ad hoc at best, Faraday instead focused on powers, which "referred to portions of space from which specific actions emanated. Powers could equally belong to a magnet or to the sides of a wire, and they could attract or repel each other."<sup>12</sup>

Further, rather than address the specific charged object and its state, as implied in any use of Coulomb's force law, Faraday focused on the "distribution of powers" that occurred during induction; attempts to visualize these distributions led Faraday on to the notion of the lines of force.<sup>13</sup>

The explanatory power of the lines of force was great—and would later prove to be even more so when Maxwell used the concept to fully formalize his field theory. Faraday speculated that the lines could, in fact, be physical in nature. The fact that induction occurred along these lines, even when curved, led Faraday to conclude that the forces were communicated by contiguous particles in the medium between charges; the line traced out the passage of action from one contiguous particle to another. In this way, charge was communicated through space.

While Faraday's notion of contiguity did not rid the science of electromagnetic attraction and repulsion of a problematic action-at-a-distance explanation, it did confine the action to extremely small distances. Additionally, this contiguity was invoked in the explanation of electric charge. For Faraday, charge was not an inherent characteristic of a particular body, but was rather a state of affairs in which every charge was one part of a polarized pair. In this way,

 <sup>&</sup>lt;sup>12</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 19.
<sup>13</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 40, 91.

charge was always the result of induction, because for every charged body there existed somewhere else in the contiguous, intervening medium that charge's polarized opposite.

Ultimately, these concepts, so different than any other contemporary theory, were the seeds of a field theory. Darrigol defines a field theory as "the introduction of physical or mathematical entities in the space intervening between electric and magnetic sources."<sup>14</sup> The fundamental characteristic of a field theory is that in such a theory, the phenomena being explained is not limited to the specific bodies within the system, but is rather given a holistic place within the whole system qua system, replacing its emphasis on isolated body analysis with descriptions of interactions. Faraday's lines of force and emphasis on the intervening space is clearly just such a notion; his was the "first precise and quantitative concept of a field."<sup>15</sup> Though his ideas were poorly understood by most of his peers, Faraday's field approach would ultimately prove to be the key to forming a successful electromagnetic explanation.

#### **Maxwell's Equations**

Much of Faraday's work had already been published when James Clerk Maxwell first begin studying issues related to electricity and magnetism in the 1850s. Although the former's studies were widely praised for the wealth of experimental data they contained, his contentions were, nonetheless, more often than not ignored. Maxwell, however, determined to make a precise evaluation of the science, dedicated himself to a study of Faraday's entire body of work.

Darrigol has described what followed as a "stepwise" development of Maxwell's field theory of electromagnetics: Maxwell completed his theory in discrete stages, documenting the

<sup>&</sup>lt;sup>14</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 78.

<sup>&</sup>lt;sup>15</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 78.

progression of his thought in a series of three "memoirs" which began with Faraday's lines of force and ended with a unified electromagnetic field theory.<sup>16</sup>

Maxwell's theory built upon Faraday's work. For this reason, it was both unique and inventive. It provided a theoretical framework for the entire science that was more accurate than anything that had yet been devised. The specific concepts that Maxwell adopted from Faraday included "the field-based definitions of electric charge and current, the concept of conduction as the competition between polarization build-up and decay, and the reduction of all electric and magnetic actions to stresses in the field."<sup>17</sup>

Maxwell's notion of what made an effective theory was explicitly stated in the introduction to his first paper on the subject of electromagnetism, titled, "On Faraday's Lines of Force". Disturbed by the chaotic state of the science, Maxwell wrote,

The first process therefore in the effectual study of the science, must be one of simplification and reduction of the results of previous investigation to a form in which the mind can grasp them. The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis. In the first case we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject. If, on the other hand, we adopt a physical hypothesis, we see the phenomena only through a medium, and are liable to that blindness to facts and rashness in assumption which a partial explanation encourages. We must therefore discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on the physical science from which that conception is borrowed, so that it is neither drawn aside from the subject in pursuit of analytical subtleties, nor carried beyond the truth by a favourite hypothesis.<sup>18</sup>

It seems that Maxwell's clear and holistic view of what a good physics theory ought to

contain-mathematical analysis, which could validate but not usurp the physical description-

<sup>&</sup>lt;sup>16</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 172.

<sup>&</sup>lt;sup>17</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 168.

<sup>&</sup>lt;sup>18</sup> Simpson, *Maxwell on the Electromagnetic Field*, 58.

paid off, for after a great deal of research and a number of publications on the subject, Maxwell accomplished precisely what he had set out to do: he provided a clear physical description of the electromagnetic field, complete with mathematical equations that could be used to relate the fundamental concepts and make measurement and prediction possible.

The result of Maxwell's work was summed up in twenty field equations; but, toward the end of the nineteenth century, Oliver Heaviside managed to reduce them to four. Because all four were derived from Maxwell's original twenty, they are still referred to as "Maxwell's Field Equations," and can be described explicitly in terms of Maxwell's theory.

#### **The Four Equations**

The modern expression of Maxwell's unified electromagnetic field theory is most famously represented in the four Maxwell Equations. Each equation is expressed in both an integral form and a partial differential form in order to highlight different aspects of the same phenomenon.<sup>19</sup>

**Gauss' Law for Electric Fields.** Gauss's Law is the only of the four equations that deals solely with the electrostatic field, the field produced by an individual charge (as opposed to an induced electromagnetic field produced by an oscillating magnetic field). The integral form relates the electric flux (best understood as the number of field lines in a given space) to an enclosed electric charge. The differential form relates the divergence of the field, or the tendency of charge to "flow away" from a charge, to the density of that charge.

Both of these forms clearly examine a state of the field; the integral in looking at field line density, and the differential in analyzing the flow direction of the field. As such, both forms

<sup>&</sup>lt;sup>19</sup> For the entirety of this section, I have relied heavily on Daniel Fleisch's excellent *Guide to Maxwell's Equation* (Cambridge: Cambridge University Press, 2008).

are clearly formulations of Faraday concepts. The usefulness of the mathematical form in which they were placed by Maxwell, however, is twofold: For one, Gauss' Law enables one to measure the field and derive therefrom knowledge about the associated charge, or vice versa. The second use is the application of knowledge about the field to describe either a surface (integral form) or a specific point in space (differential form). Ultimately, the formalization of the Gauss Law depicts a specific relationship between the electrostatic field and the amount of charge producing that field.

**Gauss' Law for Magnetic Fields.** Gauss' Law, when applied to magnetic fields, employs the same kind of questions about flux and field divergence. However, given the dipole nature of magnetism, which always occurs in a pair of opposite poles, both forms draw conclusions different from the electric law. In the case of the integral form, the total flux through any surface containing a magnetic dipole will be zero, because each end of the pole will necessarily create lines of force that are equal in magnitude but opposite in direction and will therefore cancel, or sum to zero. The same sort of occurrence is present in the differential form as well: the total divergence of the magnetic field will be zero, for the same reason.

This fact, brought to light through the mathematics of the surface integral or partial differential, is extremely useful; when it is known that a total situation will sum to zero, all sorts of information can be deduced about individual unknowns, particularly in the case of complex surfaces.

When Guass' Law for Electric Fields is contrasted with the way Gauss' Law works for magnetic fields, one can see an evaluation of one of Faraday's fundamental concepts, namely his idea that all electric induction always implied some kind of polarization of pairs of opposites. It is easy to see why Faraday might have thought this when one considers the nature of magnetic dipoles, which do occur in such pairs. However, as these laws demonstrate, the dipole nature of magnetism manifests itself in the mathematics of Gauss' Law for magnetic fields, where both the flux and the field divergence equate to zero due to the nature of a dipole. In the case of an electrostatic field, the flux and divergence can have non-zero sum values, indicating a field produced by a "monocharge" rather than distinct pairs of charges.

**Faraday's Law.** Faraday's Law is based on his (Faraday's) discovery that an oscillating magnetic field will induce an electric current. The integral form of the law describes the movement of a charged particle that has been motivated by an electromotive force through a magnetic field. The differential form relates the curl (the tendency of the field lines to circulate around a point) of the electric field created by the moving charge to the rate of change of the magnetic field.

This law allows one to grasp the cause of magnetic induction in an electric current: a magnetic flux creates a field that has the capacity to cause the directional motion of a charged particle. As noted, Faraday conceived of the induction of a current in terms of the intervening medium and field lines; the Maxwell equation named after Faraday provides a precise mathematical description of the field and force lines that Faraday emphasized.

The Ampère-Maxwell Law. The Ampère-Maxwell equation is based on Ampère's original formulation describing Oersted's discovery that a steady electric current will create a magnetic force. In his reformulation, Maxwell extended this relation to include a varying electric flux as well, as a steady current, in the creation of a circulating magnetic field. The integral form depicts the magnetic field circulating around any path bounded by the surface through which the electric current or flux moves. The differential form ties together this magnetic curl to the electric field density and the time rate of change of the electric field.

Again, though the concepts of electromagnetism that Maxwell expressed in this law are clearly based on Faraday's notions, the time-rate variation in this equation is what stands out as an element that simply could not have been described by Faraday without the use of mathematics. Additionally, the description of the path around which the magnetic field curls is made precise only with the use of integrals.

**From Maxwell's Equations to the Wave Equation.** The Maxwell Equations possess one further feature that has yet to be discussed: consideration of all four equations, in conjunction with one another, leads directly to the wave equation, which relates electromagnetism to the spectrum of electromagnetic radiation. In other words, further manipulation of the mathematics of the Maxwell equations directly indicates the electromagnetic nature of light.

It is unlikely that this particular connection would ever have resulted from a continuation of Faraday's research—the electromagnetic nature of light was a mathematical discovery rather than an experimental one. But such a description of physical phenomena is precisely the sort of theory Faraday approved of. Just like his lines of force, wave-like electromagnetic phenomena is the sort of useful geometrical description of powers that Faraday was trying to get at with his lines of force.

A Unified Field Theory. Though the Maxwell equations are the crowning achievement of Maxwell's work, the emphasis placed on them in this paper may give the impression that they are the main feature of Maxwell's theory. For this reason, it is important to note that the truly fundamental aspect of Maxwell's theory was the specific way in which Maxwell unified all of the classical electromagnetism through his field theoretical approach. It was this physical framework that facilitated the unification of relevant mathematics, geometrical approaches, and observed phenomena.

#### Conclusions

Maxwell's conceptions of the ideal physical theory, and his application of it to the study of electromagnetism, have proven to be lasting contributions to science. Darrigol sums these contributions up, writing, "Maxwell defined a new kind of theoretical physics in which the classification of mathematical quantities, vector symbolism, and Lagrangian dynamics became major construction tools. He also revealed a tension between field macrophysics and the atomic structure of matter, and inaugurated ways of dealing with this tension. His physics was an unended quest. He provided methods that kept the theory open and alive."<sup>20</sup>

Though Maxwell's conception of physics theories differed greatly from Faraday's, even Faraday himself could appreciate at least some aspects of what Maxwell was trying to do. After its publication, Maxwell sent a copy of his "On Faraday's Lines of Force" to Faraday, who was then in the final decade of his life. Faraday wrote to Maxwell several times about this work, saying, "I was at first almost frightened when I saw so much mathematical force made to bear upon the subject, and then wondered to see that the subject stood it so well."<sup>21</sup> In another letter, he continued, "I have always found that you could convey to me a perfectly clear idea of your conclusions…neither above nor below the truth, and so clear in character that I can think and work from them."<sup>22</sup>

<sup>&</sup>lt;sup>20</sup> Darrigol, *Electrodynamics from Ampère to Einstein*, 176.

<sup>&</sup>lt;sup>21</sup> Simpson, *Maxwell on the Electromagnetic Field*, 407.

<sup>&</sup>lt;sup>22</sup> Simpson, Maxwell on the Electromagnetic Field: A Guided Study, 407.

Though Faraday never fully accepted that the use of mathematics could be integral to the study of science, he did recognize that it had been possible for Maxwell to employ mathematics without leaving behind the qualitative aspects of the phenomena about which he was theorizing. As Simpson states, "When Maxwell joins his efforts to those of Faraday, he seems to be drawn not only to a promising mode of explanation but to a concept of science itself, opting, despite his own powers in analytic mathematics, for a science centered not on such formalities but on an intimate understanding of the ways of nature in some simpler sense."<sup>23</sup>

Ultimately, Maxwell demonstrated that the use of mathematics allows for a deeper pursuit of scientific explanation when properly incorporated into a theoretical framework that is firmly based upon experiment and observation, just as Maxwell's ideas were built on the novel, innovative research done by Michael Faraday. In Maxwell's theoretical work, we see that it was—and is—possible to employ mathematics without abandoning a physical closeness to the phenomenon. Further, it is clear that the addition of mathematical formalism can assist in clarifying relationships, quantifying physical effects, and, in the strongest case, can even lead to the furtherance of a greater understanding of the phenomenon itself.

<sup>&</sup>lt;sup>23</sup> Simpson, *Maxwell on the Electromagnetic Field*, 15.

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