PUPT-1577
CTP-2487
hep-th/9511104

Screening vs. Confinement in 1+1 Dimensions

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Abstract

We show that, in 1+1 dimensional gauge theories, a heavy probe charge is screened by dynamical massless fermions both in the case when the source and the dynamical fermions belong to the same representation of the gauge group and, unexpectedly, in the case when the representation of the probe charge is smaller than the representation of the massless fermions . Thus, a fractionally charged heavy probe is screened by dynamical fermions of integer charge in the massless Schwinger model, and a colored probe in the fundamental representation is screened in QCD_2 with adjoint massless Majorana fermions. The screening disappears and confinement is restored as soon as the dynamical fermions are given a non-zero mass. For small masses, the string tension is given by the product of the light fermion mass and the fermion condensate with a known numerical coefficient.

Parallels with 3+1 dimensional QCD and supersymmetric gauge theories are discussed.

1 Introduction

The proof of confinement in QCD remains a major unsolved problem. The heuristic picture of confinement is well known. In pure Yang- Mills theory, the static potential between heavy probe charges in the fundamental color representation is believed to grow linearly at large distances

$$V_{Q\bar{Q}}(r) \sim \sigma r. \tag{1.1}$$

This corresponds to the famous area law behavior of the Wilson loop vacuum expectation value,

$$< W(C) >= \left\langle \frac{1}{N_c} \mathrm{Tr} P \exp\left\{ ig \int_C \hat{A}_\mu dx_\mu \right\} \right\rangle \ \sim \ \exp\{-\sigma \mathcal{A}_C\}, \tag{1.2}$$

for large smooth quasi-planar loops where $\mathcal{A}_C$ is the area of the minimal surface with boundary C.

It is also well-known that the area law (1.2) does *not* hold in QCD with dynamical quarks. The string may tear by creating a light quark-antiquark pair so that the color charge of heavy sources is screened by the dynamical quarks. The potential $V_{Q\bar{Q}}(r)$ should then approach a constant for large r, and the Wilson loop average should display a perimeter law. The same is true for the Wilson loop in the *adjoint* representation in pure Yang-Mills theory. Although the spectrum of QCD with dynamical quarks contains only colorless states, it is important to distinguish this *screening* picture from true *confinement* with an area law for the Wilson loop.

Recently, the first $4D$ field theory example where confinement is proved (at least at a physical level of rigorousness) has been constructed [1] . The theory is an $\mathcal{N} = 2$ supersymmetric $SU(2)$ Yang-Mills theory with an extra term that breaks $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ giving a small mass to one of the two adjoint Majorana fermions and its $\mathcal{N} = 1$ scalar superpartner. Confinement is absent when $m = 0$ but *does* appear for any non-zero

m. Due to the special nature of this model, the confinement affects only the $U(1)$ subgroup of $SU(2)$; the rest of the group is in the Higgs phase.

In this paper we show that a similar onset of confinement as a mass is introduced takes place in some simple 2D models: the Schwinger model for fractional probe charges, and $SU(N)$ gauge theories coupled to Majorana fermions in the adjoint representation for heavy probe charges which are in the fundamental representation of the color group. In the latter case, the entire $SU(N)$ is in a screening phase for vanishing fermion mass but becomes confining as the mass is turned on.

2 Higgs phase vs. confinement in the Schwinger model

First, we consider the well understood case of the Schwinger model, with Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \epsilon_{\mu\nu}F. \tag{2.2}$$

The coupling constant e has dimension of mass. After bosonizing the Dirac fermion we arrive at the following equivalent Lagrangian,

$$\mathcal{L} = \frac{1}{2}F^2 + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{e}{\sqrt{\pi}}F\phi + m\Sigma[\cos(2\sqrt{\pi}\phi) - 1], \tag{2.3}$$

where

$$\Sigma \;=\; e\,\frac{\exp(\gamma)}{2\pi^{3/2}} \tag{2.4}$$

is the absolute value of the fermion condensate in the Schwinger model (γ is the Euler constant). We have added -1 to the cosine so as to have zero classical vacuum energy.

For our purposes it is convenient to integrate out ϕ (or equivalently ψ) in order to derive the effective action for the gauge field. This is particularly easy in the case of $m = 0$, where the theory is quadratic in ϕ, and we obtain

$$\mathcal{L}_{eff} = \frac{1}{2}F^2 + \frac{e^2}{2\pi}F\frac{1}{\partial^2}F. \tag{2.5}$$

The non-local term acts essentially as a mass term for the gauge field. To show this, we pick $A_1 = 0$ gauge and restrict ourselves to static fields, so that $1/\partial^2$ may be replaced by $-1/\partial_1^2$. After an integration by parts the effective Lagrangian reduces to

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_1 A_0)^2 + \frac{e^2}{2\pi}A_0^2. \tag{2.6}$$

This may be interpreted as a peculiar two-dimensional version of the Higgs phenomenon: the Coulomb force is replaced by a force of finite range with a mass scale $\mu = e/\sqrt{\pi}$. The consequences of this may be probed by introducing a static external charge distribution $\rho(x^1)$. This adds $-\rho A_0$ to $\mathcal{L}_{eff}$, and the equation of motion becomes

$$\partial_1^2 A_0 - \mu^2 A_0 = -\rho(x^1). \tag{2.7}$$

Suppose, for instance, that we fix an external charge e' at $x^1 = 0$, and $-e'$ at $x^1 = a$. Solving (2.7) with

$$\rho(x^1) = e'(\delta(x^1) - \delta(x^1 - a)), \tag{2.8}$$

we get

$$A_0(x^1) = \frac{e'}{2\mu}(e^{-\mu|x^1|} - e^{-\mu|x^1-a|}). \tag{2.9}$$

Substituting this back into $\mathcal{L}_{eff}$ we find that the energy of the two test charges is

$$V(a) = \frac{e'^2}{2\mu}(1 - e^{-\mu a}). \tag{2.10}$$

While $V(a)$ increases linearly for small a, it saturates at $e'^2/(2\mu)$ for large separations. This indicates a remarkable phenomenon: *any fractional charge e' is screened by integer massless charges.* Does this also occur when the dynamical charges are massive ? One way to find the answer is to integrate out ϕ. The fact that the massive theory is non-polynomial in ϕ

leads to a non-polynomial effective action for F. The expansion of $\mathcal{L}_{eff}$ in powers of F may be constructed by integrating ϕ out order by order in eF,

$$\mathcal{L}_{eff} = \frac{1}{2}F^2 + \frac{e^2}{2\pi}F\frac{1}{\partial^2 + 4\pi m\Sigma}F + \frac{16me^4}{\pi}\left[\frac{1}{\partial^2 + 4\pi m\Sigma}F\right]^4 + \mathcal{O}(F^6). \tag{2.11}$$

For weak, slowly varying fields this may be approximated by

$$\mathcal{L}_{eff} = \frac{1}{2}F^2\left(1 + \frac{e^2}{4\pi^2 m\Sigma}\right). \tag{2.12}$$

Thus, the leading effect of integrating out a massive fermion is a finite renormalization of electric charge: the Higgs phenomenon has disappeared. The absence of a mass term for the gauge field means that we can no longer screen a fractional charge by integer charges. In other words, $V(a) \sim a$ as $a \to \infty$, and the theory is in the confining phase.

Solving the equations of motion which follow from the truncated Lagrangian (2.12) with the source (2.8) and calculating the energy, we get for small $m \ll e$

$$V(a) \;=\; \left(\frac{e'}{e}\right)^2 2\pi^2 m\Sigma a. \tag{2.13}$$

This is true, however, only as long as $e' \ll e$. Otherwise, the higher-order terms in the effective Lagrangian (2.11) cannot be neglected and the string tension is renormalized. In the following we determine the exact dependence of the string tension on e'/e and show that it vanishes for *integer* probe charges. For fractional probe charges, it vanishes only when $m = 0$, but does *not* vanish in the massive Schwinger model.

One of the ways to reach this conclusion is by studying classical solutions of the bosonized equations, as in [2]. Let us make the fermions of charge $e' = qe$ and large mass M dynamical and bosonize them in terms of a new scalar field χ. The complete Schwinger model Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}F^2 + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 + \frac{e}{\sqrt{\pi}}F(\phi + q\chi) + m\Sigma[\cos(2\sqrt{\pi}\phi) - 1] + cM^2[\cos(2\sqrt{\pi}\chi) - 1]\ , \tag{2.14}$$

where c is a numerical constant. After integrating out the gauge field, we arrive at the following Lagrangian

$$\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{e^2}{2\pi}(\phi + q\chi)^2 + m\Sigma[\cos(2\sqrt{\pi}\phi) - 1] + cM^2[\cos(2\sqrt{\pi}\chi) - 1]. \quad (2.15)$$

Following [2] we may look for static solutions, $\phi(x^1)$, $\chi(x^1)$, to the resulting equations of motion. The requirement of finite energy leads to the boundary conditions $\phi(-\infty) = \chi(-\infty) = 0$. ϕ and χ must also approach constant values as $x^1 \to \infty$. For $m = 0$ there exists a finite energy solution with

$$\chi(\infty) = \sqrt{\pi}\ , \qquad\qquad \phi(\infty) = -q\sqrt{\pi}\ . \quad (2.16)$$

The total charge,

$$Q = \frac{e}{\sqrt{\pi}}[\phi(\infty) - \phi(-\infty)] + \frac{e'}{\sqrt{\pi}}[\chi(\infty) - \chi(-\infty)]\ , \quad (2.17)$$

vanishes for such a solution, as it should. This solution describes a massive charge e' screened by a cloud of massless charges e. It provides us with a rather detailed understanding of the mechanism for this screening. The bosonized theory with a massless field ϕ possesses finite energy configurations containing any desired charge $-e'$ in a localized region of space. Upon gauging of the theory, these configurations bind to charge e' and neutralize it. Remarkably, such fractionally charged ϕ-solitons acquire infinite energy as soon as m is turned on, due to the $m\Sigma[\cos(2\sqrt{\pi}\phi) - 1]$ term in $\mathcal{L}$. For small m, the energy per unit length (i.e. the string tension) may be found from the first order perturbation theory and is given by

$$\sigma = m\Sigma[1 - \cos(2\pi q)]. \quad (2.18)$$

In section 4 this result will be rederived by analyzing the behavior of the Wilson loop in the path integral approach.

We see that the string tension indeed vanishes when e' is an integer multiple of e. This has an obvious physical interpretation: one can always screen an integer charge by binding to it a number of particles of charge $-e$.

3 Non-abelian Higgs phase in the massless adjoint fermion model

In the previous section we discussed the Schwinger model. As we have shown, it is in the Higgs phase for massless fermions and, for fractional probe charges, in the confining phase for massive fermions. In this section we show that essentially the same conclusions hold in certain non-Abelian 1+1 dimensional gauge theories.

While in 3+1 dimensions confinement is a rather miraculous phenomenon, which is not yet fully understood, in 1+1 dimensions it is hardly a mystery due to the confining nature of the Coulomb force. In a pure $SU(N)$ gauge theory, for example, there are no dynamical gluons, but there exists an exactly linear Coulomb potential between test charges. In other words, Wilson loops in any representation will exhibit an area law.

If, however, we couple dynamical fermions in the fundamental representation to the gauge field, then the situation changes. The Wilson loops in the fundamental (or any other) representation now exhibit the perimeter law because the dynamical fundamental charges screen the test charges.

A more interesting situation is expected to occur in theories where all the dynamical fields are in the adjoint representation of $SU(N)$. Such 1+1 dimensional models have received some recent attention because of their many similarities with 3+1 dimensional gauge theories [3, 4, 5]. The adjoint fields play a physical role similar to that of transverse gluons. In theories where all the dynamical fields are in the adjoint representation the adjoint Wilson loop exhibits the perimeter law, while the Wilson loop in the fundamental representation is usually expected to obey the area law corresponding to confinement. It is interesting that in 1+1 dimensional models with adjoint matter the criteria for confinement are the same as in the 3+1 dimensional gauge theories. The proof of confinement is expected to be much simpler in 1+1 dimensions. To our surprise, however, we will find non-Abelian models where the confining phase is replaced by the Higgs phase, much like in the Schwinger model. In this section we discuss the simplest such model: SU(N) gauge theory coupled to a massless Majorana fermion in the adjoint representation. We will show that test charges in the fundamental representation are screened by the massless adjoint fermions. This is the non-Abelian analogue of the screening of fractional charge by massless integer charges that we observed in the Schwinger model.

The gauged Lagrangian for a single flavor of massless Majorana fermions is

$$\mathcal{L} = \mathrm{tr}\left[i\bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}\right] , \tag{3.1}$$

where $\psi = \psi^a t^a$, $A_\mu = A_\mu^a t^a$, and t^a are the N^2-1 hermitian generators of $SU(N)$. The field strength and covariant derivative are defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] = \epsilon_{\mu\nu}F , \tag{3.2}$$

$$D_\mu\psi = \partial_\mu\psi + i[A_\mu, \psi] . \tag{3.3}$$

We will integrate out ψ to derive the effective action for A_μ which is known explicitly in 1+1 dimensions [6, 7],

$$S_{eff} = \mathrm{tr}\int d^2x\left[\frac{1}{2g^2}F^2 + \frac{N}{2\pi}(\partial_- A_+ - \partial_+ A_-)\frac{1}{\partial^2}(\partial_- A_+ - \partial_+ A_-)\right] + NS_{WZ}(A_+) - NS_{WZ}(A_-). \tag{3.4}$$

The non-local Wess-Zumino term is an integral over manifold B whose boundary is space-time,

$$S_{WZ}(A_+) = \frac{1}{12\pi}\int_B d^3x\epsilon^{ijk}\partial_i gg^{-1}\partial_j gg^{-1}\partial_k gg^{-1} , \tag{3.5}$$

$$S_{WZ}(A_-) = \frac{1}{12\pi}\int_B d^3x\epsilon^{ijk}\partial_i hh^{-1}\partial_j hh^{-1}\partial_k hh^{-1} , \tag{3.6}$$

where $A_+ = \partial_+ gg^{-1}$ and $A_- = \partial_- hh^{-1}$. The factor of N in the induced action is the central charge of the affine algebra of the $SU(N)$ gauge currents. In [8] it was noted that an identical current algebra results in the gauged model of N flavors of massless Dirac fermions in the fundamental representation of $SU(N)$ (theory II). It was further shown [8] that the massive spectrum of theory II is identical to that of theory I (gauge theory coupled to one massless adjoint multiplet). Theories I and II are not completely equivalent because I has no massless bound states while II does, but the massless sector in II is in some sense decoupled from the rest of the spectrum. The fact most important for us is that, since their gauge current algebras are identical, theories I and II have identical effective actions for A_μ. [1] As a result, the expectation value of any Wilson loop,

$$\langle W \rangle = \int [\mathcal{D}A]\; W \mathrm{e}^{-S_E(A)}, \tag{3.7}$$

is the same in theories I and II ($S_E(A)$ is the Euclidean continuation of $S_{eff}(A)$). It is physically clear that in the model with massless fundamental fermions (II) the Wilson loops in the fundamental (and all other) representations must obey the perimeter law: the fundamental fermions can screen test charge in any representation. This implies that in theory I the fundamental Wilson loop also obeys the perimeter law. Surprisingly, we have shown that the theory with a massless adjoint Majorana multiplet is not confining: it is rather in the Higgs phase. In the following we will confirm this unexpected conclusion in a number of ways.

There is a subtlety in the above argument that requires further explanation. Theory I has gauge group $SU(N)/Z_N$. As we explain in detail in section 4, there are N different topological classes for A_μ associated with the elements of $\Pi_1(SU(N)/Z_N) = Z_N$. Only one of them, the trivial class, is present in theory II. However, as explained in section 4, each of the topologically non-trivial classes in I has fermion zero modes and does not contribute to $\langle W \rangle$. We expect, therefore, that $S_E(A)$, the Euclidean effective action obtained by integrating the fermions out, diverges for the topologically non-trivial configurations. To show this, let us consider the Euclidean theory defined on S^2. In the topologically non-trivial sectors A_μ is not single-valued. It has singularities of the Dirac string type where the infinitesimal Wilson

[1] Cf. the abelian case: the Schwinger model with four massless fermions of charge $e/2$ has the same effective action as the theory with one massless fermion of charge e.

loop surrounding the north pole is a non-trivial element of Z_N. It is not hard to show that such a singularity creates a divergence in the effective action (3.4). For simplicity, we consider $N = 2$, but the argument generalizes to other N. Near the north pole (as $r \to 0$)

$$A_\mu \to i\Omega^\dagger \partial_\mu \Omega. \tag{3.8}$$

In the instanton configuration we may choose a gauge where

$$\Omega \to \exp(i\theta\sigma_3/2), \tag{3.9}$$

so that only A^3_μ is not single valued and

$$F^3 = \pi\delta^2(x) + \text{regular terms}. \tag{3.10}$$

Thus, the effective action in the instanton class diverges due to the term

$$-2\pi \int d^2x \delta^2(x) \frac{1}{\partial^2} \delta^2(x). \tag{3.11}$$

This is the well-known expression for the electrostatic self-energy of a two-dimensional charge, which is logarithmically divergent. The fact that $S_E(A)$ turns out to be infinitely large in the instanton sectors is directly related to the presence of the fermion zero modes before the fermions are integrated out. [2] The divergence of $S_E(A)$ suppresses the instantons in theory I and restores its equivalence to theory II. As a result, all Wilson loops in I and II are identical.

One interesting check of the screening phenomenon involves a calculation of the static quark–antiquark potential. The charge of the quark and the antiquark points in one of the $N^2 - 1$ directions of $SU(N)$, which we call direction 1 without any loss of generality,

$$\rho^1(x) \sim \delta(x) - \delta(x-a)\ , \qquad \rho^a(x) = 0\ ,\ a \neq 1. \tag{3.12}$$

[2] Strictly speaking, this reasoning is not quite rigourous. To be precise, one should treat the topologically distinct sectors separately and single out the contribution of zero modes explicitly (see Ref.[9] for a detailed analysis in the Schwinger model case). The more precise treatment of topologically non-trivial sectors in QCD_2 with adjoint fermions will be given in Sect.5, but it is rather remarkable that heuristic arguments based on the universal form of the effective action give essentially the same answer.

We will choose the $A_1 = 0$ gauge and look for a static classical solution for A_0 in the background of this charge density. The classical gauge field points in the same group direction as the charge density, $A_0^a = 0$ for $a \neq 1$. The Wess-Zumino terms may be neglected because they involve group commutators, while g, h and their derivatives commute. Thus, the equations satisfies by a static A_0^1 are the same as in the Abelian theory,

$$\partial_x^2 A_0^1 - \frac{g^2 N}{\pi} A_0^1 = g\sqrt{N}(\delta(x-a) - \delta(x)) \ . \tag{3.13}$$

Note that the screening mass-squared is $\mu^2 = g^2 N/\pi$, which is finite in the large N limit. Substituting the solution into the effective action, we find that the static quark–antiquark potential behaves as

$$V(a) \sim \mu(1 - e^{-\mu a}). \tag{3.14}$$

In the Schwinger model, where the effective action for A_μ was quadratic, this method of calculation was exact. In the non-Abelian case we may only hope to have found the dominant saddle point. The fluctuations around it probably change the simple formula (3.14) but do not alter its qualitative behavior, which is characteristic of the Higgs phase.

At this point it is interesting to ask how the transition from confinement to screening affects the spectrum of the theory. In the large N limit, the spectrum of single-"glueball" states is expected to be fully discrete for any non-vanishing fermion mass. For highly excited states, however, the gaps become astronomically small due to the exponentially growing density of states. This is the kind of structure one expects to find in physically interesting confining gauge theories. As we have argued above, for $m = 0$ confinement is replaced by screening. The disappearance of the string tension may lead to a continuous spectrum, at least for high enough excitation number. [3] In the numerical work of [5] the lowest couple of

[3] While in a theory with the adjoint matter alone the continuous spectrum is only a hypothesis, we can do better for a theory with both a massless adjoint and a fundamental fermion mutliplets. In addition to the glueball-like states this theory contains mesons (open strings) with the fundamental fermions at the end-points. The large N spectrum of such mesons should become continuous at the energy sufficient for a decay into a quark screened by a cloud of massless adjoint quanta and an antiquark screened by a cloud of massless adjoint quanta. Thus we expect a meson spectrum consisting perhaps of a few low-lying discrete states followed by a continuum.

states were found to have discrete gaps, but beyond that it was difficult to judge whether the spectrum is continuous or discrete with very small gaps. It is necessary to improve numerical techniques to the point where it is possible to judge whether the transition to continuous spectrum takes place. If it does, then it is clearly interesting to identify the precise energy where the spectrum becomes continuous.

4 Changing the group representation via bosonization.

To make the screening of fundamental charges by massless adjoint fermions less mysterious we should identify the operators in the free fermion theory which transform in the fundamental representation of $SU(N)$. Such operators are analogous to the fractionally charged solitons of the massless bosonized field which, as we showed in the previous section, screen elementary fractional charges in the Schwinger model. Below we sketch a similar construction in the simplest adjoint fermion model corresponding to $SU(2)$.

We will consider the left-moving (holomorphic) sector of the free fermion theory (the antiholomorphic sector behaves analogously). The fermion fields $\psi^a(z)$, $a = 1, 2, 3$, transform in the adjoint (triplet) representation under the $SU(2)$ currents

$$J^a(z) = \frac{i}{2}\epsilon^{abc}\psi^b\psi^c. \tag{4.1}$$

It is convenient to combine ψ^1 and ψ^2 into a Dirac fermion, which may be bosonized in terms of the holomorphic part of a boson field,

$$\psi^1 + i\psi^2 = \sqrt{2}e^{i\phi(z)} \ , \qquad\qquad \psi^1 - i\psi^2 = \sqrt{2}e^{-i\phi(z)}. \tag{4.2}$$

The currents assume the form

$$J^+ = J^1 + iJ^2 = \sqrt{2}\psi^3 e^{i\phi} \ , \qquad\qquad J^3 = -i\partial_z\phi \ , \tag{4.3}$$

$$J^- = J^1 - iJ^2 = \sqrt{2}\psi^3 e^{-i\phi} \ . \tag{4.4}$$

Let us recall that the $c = 1/2$ theory corresponding to ψ^3 contains order and disorder operators with the following OPE,

$$\psi^3(z)\sigma(0) = -\frac{1}{\sqrt{2z}}\mu(0) \ , \qquad \psi^3(z)\mu(0) = -\frac{1}{\sqrt{2z}}\sigma(0) \ . \tag{4.5}$$

Now it is not hard to see that the operators

$$\Psi_+ = \sigma e^{i\phi/2} \ , \qquad \Psi_- = \mu e^{-i\phi/2} \tag{4.6}$$

are local with respect to the $SU(2)$ currents and, in fact, transform in the fundamental (doublet) representation,

$$J^3(z)\Psi_+(0) = -\tfrac{1}{2z}\Psi_+(0) \ , \qquad J^3(z)\Psi_-(0) = \tfrac{1}{2z}\Psi_-(0) \ , \tag{4.7}$$

$$J^+(z)\Psi_-(0) = -\tfrac{1}{z}\Psi_+(0) \ , \qquad J^-(z)\Psi_+(0) = -\tfrac{1}{z}\Psi_-(0) \ , \tag{4.8}$$

where we have exhibited only the singular terms in the OPE. The doublet fields have a rather exotic holomorphic dimension, 3/16 (this is the sum of the holomorphic dimension of μ or σ, 1/16, and that of $e^{\pm i\phi/2}$, which is 1/8). This is not too surprising because in the bosonized theory of a single massless Dirac fermion the fractionally charged objects, $e^{iq\phi}$, also have fractional dimensions, $q^2/2$. Nevertheless, it is these objects that screen external fractional static charges in the Schwinger model.

It is thus plausible that the composite doublets we found in the free adjoint theory are capable of screening the external test doublets in the gauged theory. We believe, although have not checked in detail, that similar constructions of fundamentals from adjoints are possible for all $SU(N)$ gauge groups. In fact, somewhat simpler constructions of a similar type demonstrate the screening of external spinor charges in $SO(2n)$ gauge theory with massless fermions in the vector representation.

Consider, for instance, the $SO(8)$ gauge theory with fermion fields $\psi^a(z)$, $a = 1, 2, \ldots, 8$, transforming as a vector. We may combine the 8 Majorana fermions into 4 Dirac fermions and bosonize them

$$\psi^1 + i\psi^2 = \sqrt{2}C_1 e^{i\phi_1(z)}\ , \qquad\qquad \psi^3 + i\psi^4 = \sqrt{2}C_2 e^{i\phi_2(z)}\ , \tag{4.9}$$

$$\psi^5 + i\psi^6 = \sqrt{2}C_3 e^{i\phi_3(z)}\ , \qquad\qquad \psi^7 + i\psi^8 = \sqrt{2}C_4 e^{i\phi_4(z)}\ , \tag{4.10}$$

where C_i are the cocycle operators necessary for maintaining the proper anticommutation relations between different fermion fields. As is well known in string theory [10], the fields that transform as spinors of $SO(8)$ may be easily constructed out of the bosonic fields as

$$\tilde{C} \exp i \left(\pm \frac{\phi_1}{2} \pm \frac{\phi_2}{2} \pm \frac{\phi_3}{2} \pm \frac{\phi_4}{2} \right) \tag{4.11}$$

where $\tilde{C}$ are the necessary cocycles. The chirality of the spinor is the product of the signs that appear in the exponent. The special property of the $SO(8)$ is that the objects that transform as spinors have dimension 1/2 which means that they are fermions, just like the original fields that transform as a vector. This is not surprising because the two spinors (of positive and negative chirality) and the vector are interchanged by the triality of $SO(8)$. We conclude that there exists an exact transformation that maps the gauge theory with massless fermions that transform as a vector of $SO(8)$ and into the theory of massless fermions that transform as a spinor of definite chirality (we are free to chose whether it is positive or negative). This tranformation preserves the number of fermion fields. To show that the external static spinor charges are screened rather than confined we simply perform the transformation on the lagrangian. Thus, it is certain that the "composite" spinor fermions screen the external static spinor charges. While for other groups the "composite" objects have more exotic dimensions, it is still very plausible that they screen external static charges that transform in different representation from those appearing in the lagrangian.

Our ability to carry out constructions such as those shown above depends crucially on special properties of conformal field theories. Once the mass is turned on, the fractionally charged solitons of the type we used no longer exist. Thus, we expect that external fundamental charges can no longer be screened and we have confinement. Examination of the quadratic terms in $S_{eff}(A)$ for $m \neq 0$ also indicates that the mass term for the gauge field is no longer present. For small m we expect the theory to be confining, with a small string tension. As we show in the next section, this is indeed what happens.

5 Wilson loops and the topological structure.

5.1 The Schwinger model

Consider first the Wilson loop with unit probe charge. In the massless Schwinger model the functional integral is Gaussian, and the higher-order correlators factorize into products of pair correlators. Therefore, we find

$$\left\langle \mathrm{e}^{ie\int_C A_\mu dx_\mu} \right\rangle = \left\langle \mathrm{e}^{ie\int_D F(x)d^2x} \right\rangle =$$
$$\exp\left\{ -\frac{1}{2}e^2 \int_D \int_D d^2x d^2y < F(x)F(y) > \right\}. \tag{5.1}$$

The correlator $< F(x)F(y) >$ has the form (see e.g. [9])

$$< F(x)F(y) >= \delta(x-y) - \frac{\mu^2}{2\pi} K_0(\mu|x-y|), \tag{5.2}$$

where $\mu^2 = e^2/\pi$. It satisfies the property

$$\int d^2x < F(x)F(0) > = \ 0. \tag{5.3}$$

The property (5.3) is natural, of course. In the Schwinger model, $F(x)$ is the local density of the topological charge:

$$\nu = \frac{e}{2\pi} \int F(x) d^2x. \tag{5.4}$$

The integral on the LHS of Eq. (5.3) is proportional to the topological susceptibility

$$\chi = \frac{1}{V} < \nu^2 >= \left(\frac{e}{2\pi}\right)^2 \int d^2x < F(x)F(0) >, \tag{5.5}$$

which is zero in the theory with massless fermions: the topologically non-trivial sectors with $\nu \neq 0$ involve fermion zero modes which make the corresponding contributions to the partition function vanish.

Note that in the quenched Schwinger model (without dynamical fermions) the correlator $< F(x)F(y) >$ is just $\delta(x-y)$ and the topological susceptibility (5.5) is not zero (cf. the well-known situation in 4D Yang-Mills theory: the topological susceptibility is zero in QCD_4 with massless quarks, but has a non-zero value $\chi_{YM} \sim \Lambda_{YM}^4$ in the pure Yang-Mills theory).

The property (5.3) leads to the vanishing of the coefficient of the area in $\ln < W(C) >$, and the Wilson loop has the perimeter law

$$< W(C) > \sim \exp\left\{-e^2 P/(4\mu)\right\} \tag{5.6}$$

for large contours.[4] Static heavy charged sources are screened by the massless dynamical fermions. In the quenched Schwinger model, the susceptibility (5.5) is non-zero, and the Wilson loop has the area law corresponding to the linearly rising static Coulomb potential.

Let us consider now the Wilson loop for a fractional probe charge $e' = qe$

$$< W_q(C) > = \left\langle \exp\left\{ieq \int_C A_\mu dx_\mu\right\}\right\rangle \tag{5.7}$$

The derivation presented above can be easily generalized to this case, and we find that $< W_q(C) >$ displays the perimeter law, i.e. the dynamical fermions with integer charges somehow manage to screen a heavy probe of arbitrary charge. This fact has been noted by many people and is a common lore. The mechanism of this strange screening deserves some further explanation, however.

Let us note that the perimeter law holds for the integer q Wilson loops even after the fermions are endowed with a mass. However, for non-integer q, $< W_q(C) >$ exhibits the area law behavior corresponding to confinement for any non-zero m, however small it is. This was already shown in section 3 using bosonization, but here we give an independent derivation of this remarkable phenomenon.

Note first of all that the topological susceptibility (5.5) is no longer zero when $m \neq 0$. For $m \ll e$ it can be calculated exactly. The quickest way to find it is by introducing the

[4] The coefficient of P may be calculated by doing the integral in (5.1) with the account of boundary effects [11] or, alternatively, from (2.10) after taking the limit $a \to \infty$.

vacuum angle θ (this adds the term $i\nu\theta$ to the Euclidean Lagrangian). Then the relation

$$\chi = \frac{\partial^2 \epsilon_{vac}(\theta)}{\partial \theta^2} |_{\theta=0} \tag{5.8}$$

holds (ϵ_{vac} is the vacuum energy density). Consider the function $\epsilon_{vac}(m, \theta)$ where m can be complex in general. The point is that ϵ_{vac} is not an arbitrary function, but rather a function of a single complex variable $z = me^{i\theta}$ (and its complex conjugate). This follows from the Ward identities and the topological structure of the theory, and can be derived in the same way as in QCD_4. When m is small and real, we can expand the real function $\epsilon_{vac}(z, \bar{z})$ in a Taylor series,

$$\epsilon_{vac} = \epsilon_{vac}(0) - \frac{1}{2}\Sigma(z + \bar{z}) + \mathcal{O}(z^2) = \epsilon_{vac}(0) - \Sigma m \cos\theta + \mathcal{O}(m^2). \tag{5.9}$$

The quantity $-\Sigma$ is simply the fermion condensate at $\theta = 0$ as given in Eq.(2.4):

$$< \bar{\psi}^a \psi^a >_{\theta=0} = \frac{\partial}{\partial m} \epsilon_{vac}(m) |_{m=\theta=0} = -\Sigma. \tag{5.10}$$

Substituting (5.9) into (5.8), we immediately get the relation

$$\chi = \Sigma m. \tag{5.11}$$

Again, this relation is analogous to the well-known relation $\chi = \Sigma m / N_f$ in QCD_4 derived in [13] (see also [12]).

If (5.11) is substituted into (5.1), then the string tension is found to be non-vanishing (and proportional to m) for any probe charge q. This is wrong, however. The point is that the *massive* Schwinger model is no longer a Gaussian, exactly soluble, theory. The higher-order correlators no longer factorize into products of pair correlators but involve non-trivial connected pieces. For the $q = 1$ Wilson loop one can write

$$< W_1(C) >= \exp\left\{ -\frac{1}{2} e^2 \int_D \int_D d^2x d^2y < F(x)F(y) > + \right.$$
$$\left. \frac{e^4}{24} \int_D \int_D \int_D \int_D d^2x d^2y d^2z d^2u < F(x)F(y)F(z)F(u) >_c - \ \ldots \right\}. \tag{5.12}$$

Since we are interested only in the coefficient of the area in ln $< W_1(C) >$ for large contours, Eq.(5.12) can be rewritten as

$$< W_1(C) >= \exp\left\{ \mathcal{A}_D \sum_{n=1}^{\infty} (-1)^n \frac{(2\pi)^{2n}}{(2n)!} \chi_{2n} \right\}, \tag{5.13}$$

where

$$\chi_{2n} = (-1)^{n+1} \frac{\partial^{2n} \epsilon_{vac}(\theta)}{(\partial\theta)^{2n}} = (\frac{e}{2\pi})^{2n} \prod_{i=1}^{2n-1} \int d^2x_i < F(0)F(x_1)\cdots F(x_{2n-1}) >_c, \tag{5.14}$$

are the generalized susceptibilities. For $m \ll e$ they can be easily found from (5.9), and we get for the string tension

$$\sigma = -\frac{1}{\mathcal{A}_D} \ln < W_1(C) >= \Sigma m(1 - \cos 2\pi) = 0. \tag{5.15}$$

This can be easily understood by noting that, if one is interested only in the coefficient of the area, the integral in $< \exp\{ie \int d^2xF(x)\} >$ can be extended over the whole two- dimensional manifold where the theory is defined (the manifold may be very large but compact to provide for infrared regularization of the path integral). This is implicit in (5.14). The flux of the electric field through the area has the meaning of the net topological charge (5.4) on the whole manifold. We thus have that

$$< W_1^{asympt}(C) >= \left\langle e^{2\pi i\nu} \right\rangle, \tag{5.16}$$

and, since ν is quantized to be an integer, this is manifestly equal to 1 (the perimeter corrections are due to the boundary effects in the flux integral and are disregarded in this reasoning).

Now consider the Wilson loop of arbitrary test charge q, $W_q(C)$. Eqs. (5.12-5.15) are easily generalized, and one finds that the string tension is

$$\sigma = -\frac{1}{\mathcal{A}_D} \ln < W_q(C) >= \Sigma m(1 - \cos(2\pi q)). \tag{5.17}$$

This result was obtained earlier via bosonization, and it can also be understood as follows. In the limit where the boundary effects are neglected, we get

$$< W_q^{asympt}(C) >= \left\langle e^{2\pi i q\nu} \right\rangle . \tag{5.18}$$

The value of $e^{2\pi i q\nu}$ depends on ν and the average is not 1 anymore. Actually, (5.18) can be written as

$$< W_q^{asympt}(C) >= \frac{Z(\theta = 2\pi q)}{Z(\theta = 0)}, \tag{5.19}$$

where

$$Z(\theta) \equiv \sum_\nu Z_\nu e^{i\nu\theta} \ = \ \exp\{-\epsilon_{vac}(\theta)\mathcal{A}\}, \tag{5.20}$$

and $\mathcal{A}$ is the total area. Substituting (5.9), we immediately find (5.17). The string tension goes to zero and confinement disappears in the limit $m \to 0$. Again, this can be easily understood from the representation (5.19) and the Fourier decomposition for $Z(\theta)$. For massless fermions, only the trivial topological sector with $\nu = 0$ contributes to the partition function. The contribution of the non-trivial sectors is killed by the fermion zero modes which appear due to the index theorem. Thus, Z is θ-independent and $< W_q^{asympt}(C) > |_{m=0} \ = \ 1$.

5.2 QCD_2 with adjoint fermions.

The behavior of the Wilson loop may be related to the topological structure of the theory also in QCD_2 with adjoint fermions. It was observed in [14] and shown in detail in [15], and later using the Hamiltonian formalism in [16], that the adjoint QCD_2 has N distinct topological classes for Euclidean gauge field configurations. This is because the true gauge group in this theory is $SU(N)/Z_N$ rather than $SU(N)$ (the adjoint fields are not transformed under the action of the center), and $\pi_1[SU(N)/Z_N] = Z_N$ is non-trivial. If we define the theory on a Euclidean plane, for instance, then the admissible boundary conditions are

$$\lim_{r\to\infty} A_\mu = i\Omega^\dagger \partial_\mu \Omega, \tag{5.21}$$

where $\Omega \in SU(N)/Z_N$. There are N topologically distinct ways to map the circle at infinity into $SU(N)/Z_N$. Therefore, there are N distinct topological classes for A_μ.

Consider first the well-understood case $N = 2$. The gauge group is $SU(2)/Z_2 = SO(3)$. There are just two topological classes — the trivial class and the class containing one instanton. One can be convinced [14] that for all the topologically trivial fields

$$W(C) = \frac{1}{2}\mathrm{Tr}\ \mathrm{P}\exp\left\{ig\int_C A_\mu^a t^a dx_\mu\right\} \ = \ 1, \tag{5.22}$$

and for the non-trivial fields $W(C) = -1$. The contour C runs around infinity on the Euclidean plane. Alternatively we may compactify the Euclidean space to S^2 by, say, the stereographic projection. Then the contour C surrounds the north pole of the sphere where the field $A_\mu(x)$ is pure gauge $i\Omega^\dagger(x)\partial_\mu\Omega(x)$ with a trivial or non-trivial mapping $S^1 \to SO(3)$.

The average of (5.22) is the order parameter for the screening or the confinement phase. The loop C in that case should be large but not necessarily surrounding the whole two-dimensional Euclidean manifold. However, as we have seen when discussing the Schwinger model, since we are interested only in the string tension, it is sufficient to study $< W(C) >$ for loops at infinity.

In the hamiltonian language, there are 2 classical vacua related by a topologically non-trivial *large* gauge transformation, and a superselection rule which is quite analogous to the standard θ-angle superselection rule in QCD [17] may be imposed. The only difference is that here there are only two possible values of θ: $\theta = 0$ and $\theta = \pi$. The partition function in these two sectors has the form

$$Z_\pm = Z_{triv} \pm Z_{inst}. \tag{5.23}$$

The crucial observation is that any gauge field in the instanton sector involves 2 fermion zero modes [15], which implies that the expectation value of $W(C)$ is equal to its value in the topologically trivial sector, $\langle W(C)\rangle = 1$. Therefore, the string tension is zero and we are in the screening or Higgs phase, in accordance with what we argued in section 3.

The appearance of the fermion zero modes in the non-abelian 2D instanton background is not as straightforward as in the abelian case because we do not have an index theorem of the Atiyah-Singer kind: the topological charge cannot be presented here as an integral of a local charge density. However, the presence of the zero modes can be seen in a number of ways. In [15], they have been constructed explicitly in a particular instanton gauge field configuration $A_\mu^{(0)}$ on a torus, and it was shown that they are also there in a perturbed background $A_\mu^{(0)} + a_\mu$ to all orders in a_μ. In [16], the theory was studied in the Hamiltonian approach and the level crossing phenomenon showed the existence of the zero modes.

Consider QCD_2 with adjoint fermions defined on a finite (not necessarily small) spatial circle of length L. Impose the gauge $A_0^a = 0$. It is possible to show that the trivial perturbative vacuum $A_1^a = 0$ has a gauge copy

$$A_1^a = \frac{2\pi}{gL} n^a, \qquad (n^a)^2 = 1. \tag{5.24}$$

The field (5.24) is related to $A_1^a = 0$ by a *large* gauge transformation not reducible to zero by infinitesimal deformations (the configurations (5.24) with different n^a are related to each other by topologically trivial gauge transformations). Therefore, the *energy* spectrum of the Dirac operator in the background (5.24) is exactly the same as for the free operator. Studying the spectrum in the constant A_1^a background smoothly interpolating between (5.24) and the trivial vacuum, one can be convinced that one left-handed mode and one right-handed mode cross zero and the spectrum is rearranged. Therefore, the level crossing should occur on *any* interpolating path which implies the presence of 2 zero modes of the Euclidean Dirac operator in any instanton background interpolating between the inequivalent vacua.

Let us now give the fermion a small mass $m \ll g$. The zero modes in the instanton sector generate a bilinear fermion condensate [15]:

$$- < \bar{\psi}^a \psi^a > \equiv \Sigma \sim g. \tag{5.25}$$

There is a gap in the physical spectrum, hence the partition functions $Z_\pm$ enjoy the extensive property $Z_\pm = \exp\{-\epsilon_\pm(m)\mathcal{A}\}$. The fermion condensate is just the first Taylor coefficient in the expansion of $\epsilon_\pm$ in m, and we have for small masses

$$Z_\pm \sim \exp\{\pm\Sigma m\mathcal{A}\}. \tag{5.26}$$

Let us calculate $\langle W(C)\rangle$ in the sector $|+>$. We have

$$\langle W_+(C)\rangle = \frac{Z_{triv} - Z_{inst}}{Z_{triv} + Z_{inst}} = \frac{Z_-}{Z_+} \sim e^{-2\Sigma m\mathcal{A}}. \tag{5.27}$$

Hence in the theory with non-zero Majorana fermion mass, confinement is restored and the string tension is

$$\sigma = 2\Sigma m. \tag{5.28}$$

In calculating the string tension for theories with $N \geq 3$ we encounter a peculiar difficulty. These theories are in a sense paradoxical and the paradox is still unresolved. There are N distinct topological sectors, and one finds that each non-trivial sector involves $2(N-1)$ fermion zero modes. This number is too large for a bilinear fermion condensate to be generated. On the other hand, bosonization arguments suggest that the fermion condensate *is* generated[5]. The paradox is akin to a similar controversy which arises in supersymmetric Yang-Mills theories with higher orthogonal or exceptional gauge groups [19]. These issues are discussed in detail in [15].

For $m = 0$, however, $\langle W(C)\rangle$ is not sensitive to the exact number of zero modes in the topologically non-trivial sectors. The important fact is that the zero modes *are* present and suppress the contribution of all topologically non-trivial sectors. Thus, $\langle W(C)\rangle$ is simply equal to its value in the trivial sector, $\langle W(C)\rangle = 1$, and we find a vanishing string tension. For non-zero mass, the string tension is not zero anymore, but its dependence on m has not been sorted out.

[5]Independent arguments show that it is generated in the infinite N_c limit [18].

5.3 Loop equations in QCD$_2$ with adjoint fermions.

The screening of the fundamental Wilson loop by massless adjoint fermions follows also from the loop equations. The idea is to regard the expectation value of a Wilson loop $W[C]$ as a functional of the contour C. Observing how $W[C]$ changes as we make an infinitesimal variation of C one obtains a functional differential equation which constrains the Wilson loop [21].

To derive this equation, consider the path integral defining the Wilson loop,

$$W[C] = \frac{1}{N} \int [\mathcal{D}A_\mu][\mathcal{D}\psi]\, \mathrm{e}^{-S[A_\mu,\psi]}\, \mathrm{tr} \left[\mathrm{P} \exp \left(i \oint_C A_\mu(x)\, dx^\mu \right) \right], \tag{5.29}$$

with the action

$$S[A_\mu, \psi] = \mathrm{tr} \int d^2x \left[i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \right]. \tag{5.30}$$

Make an infinitesimal change of the integration variables $A_\mu(x) \to A_\mu(x) + \delta A_\mu(x)$ and $\psi(x) \to \psi(x) + \delta\psi(x)$. Obviously, this does not change the total path integral. On the other hand, taken separately, the action and the path ordered exponential do change. Requiring that these changes balance each other we get a set of Schwinger–Dyson equations on the Wilson loops—the loop equations.

Together with the path ordered exponentials of the gauge field, such equations would involve correlators of fermions. However, in two dimensions, it is possible to eliminate all fermionic correlators thereby obtaining a closed equation for the Wilson loop (5.29). To this end, let the change of fields under path integral be of a special type[6],

$$\begin{cases} \delta A_+(x) = D_+ \chi_-(x) = \partial_+ \chi_-(x) + i[A_+(x), \chi_-(x)] \\ \delta A_-(x) = D_- \chi_+(x) = \partial_- \chi_+(x) + i[A_-(x), \chi_+(x)] \end{cases} \tag{5.31}$$

$$\begin{cases} \delta\psi_+(x) = i[\chi_+(x), \psi_+(x)] \\ \delta\psi_-(x) = i[\chi_-(x), \psi_-(x)]. \end{cases} \tag{5.32}$$

[6]We work in the Eucledian light cone coordinates $x^\pm = x^1 \pm ix^2$, denote $\psi = \binom{\psi_-}{\psi_+}$ and use the set of two dimensional Dirac matrices $\gamma^1 = \sigma_1$, $\gamma^2 = \sigma_2$.

with two arbitrary matrix valued parameters $\chi_+(x)$ and $\chi_-(x)$. Note that two parameters are exactly what one needs to parametrize an arbitrary change of vector potential.

Under this transformation the fermionic kinetic term does not change while the field strength term does, so that

$$\begin{aligned} \delta S[A_\mu,\psi] &= -\frac{1}{g^2}\mathrm{tr}\int d^2x\,[D_+\delta A_(x) - -D_-\delta A_+(x)]F_{01}(x) \\ &= -\frac{1}{g^2}\mathrm{tr}\int d^2x\,[\chi_+(x)-\chi_-(x)]D_+D_-F_{01}(x). \end{aligned} \tag{5.33}$$

However, the transformation (5.31, 5.32) also affects the path integral measure $\mathcal{D}\psi$, due to the chiral anomaly. Using standard methods [22] it is possible to show that under (5.32) the fermion measure transforms as

$$\mathcal{D}\psi_+\mathcal{D}\psi_- \to \mathcal{D}\psi_+\mathcal{D}\psi_- \exp\left[-\frac{gN}{4\pi}\mathrm{tr}\int d^2x\,[\chi_+(x)-\chi_-(x)]F_{01}(x)\right]. \tag{5.34}$$

Finally, the variation of the path ordered exponential in (5.29) yields a contribution

$$\begin{aligned} &\delta\left\{\mathrm{tr}\,\mathrm{P}\exp\left[-i\oint A_\mu(y)dy^\mu\right]\right\} \\ &= \mathrm{tr}\,\mathrm{P}\left\{\exp\left[-i\oint A_\mu(y)dy^\mu\right]\left(\oint dx^- D_-\chi_+(x) + \oint dx^+ D_+\chi_-(x)\right)\right\}. \end{aligned} \tag{5.35}$$

If $\chi_+ = \chi_-$ then (5.31) is a gauge transformation and the variation (5.35) vanishes. That is to say, similarly to (5.33, 5.34), the right hand side of (5.35) depends only on the difference $\chi_+ - \chi_-$ rather than on χ_+ and χ_- by themselves.

Demanding that $\delta W[C]/\delta\chi_+(x) = \delta W[C]/\delta\chi_-(x) = 0$, and using the Mandelstam formula [23]

$$\frac{1}{N}\left\langle \mathrm{tr}\left[F_{\mu\nu}(x)\mathrm{P}\exp\left(-i\oint A_\mu(y)dy^\mu\right)\right]\right\rangle = \frac{\delta W[C]}{\delta\sigma_{\mu\nu}(x)}, \tag{5.36}$$

we obtain the loop equation

$$
\begin{aligned}
&\left(\partial_+\partial_- - \frac{g^2N}{4\pi}\right) \frac{\delta W[C]}{\delta\sigma(x)}\bigg|_{x=x(\tau)} \\
&\qquad = +g^2 \oint dx^-(\tau')\frac{\partial}{\partial x^-(\tau)}\delta^{(2)}(x(\tau)-x(\tau'))\left\langle W_{x(\tau)x(\tau')}W_{x(\tau')x(\tau)}\right\rangle \\
&\qquad = -g^2 \oint dx^+(\tau')\frac{\partial}{\partial x^+(\tau)}\delta^{(2)}(x(\tau)-x(\tau'))\left\langle W_{x(\tau)x(\tau')}W_{x(\tau')x(\tau)}\right\rangle .
\end{aligned}
\tag{5.37}
$$

The right hand side of this equation involves the correlator of Wilson loops for the two subcontours of C which are obtained by cutting it at the points $x(\tau)$ and $x(\tau')$. Due to the presence of a delta function it is different from zero only if $x(\tau) = x(\tau')$, so that these subcontours are closed and $W_{x(\tau)x(\tau')}, W_{x(\tau')x(\tau)}$ are gauge invariant. For the same reason as in pure Yang–Mills theory [24] the contour integrals in the right hand side of (5.37) should be understood in the principal value sense—a small interval of $\tau' \in]\tau-\epsilon, \tau+\epsilon[$ should be excluded from the integration region. Then these integrals produce a nonzero contribution only for contours with self-intersections. A simple, nonselfintersecting Wilson loop obeys, therefore, the Klein–Gordon equation

$$
\left(\Delta - \frac{g^2N}{4\pi}\right)\frac{\delta W[C]}{\delta\sigma(x)} = 0. \tag{5.38}
$$

This equation is valid both for finite N and in the large N limit.

An immediate consequence of (5.38) is that, in contrast to pure Yang–Mills theory, $W[C]$ can not be merely a function of the total loop area. If this were the case, $\delta W/\delta\sigma(x)$ would be an x-independent constant which is not a solution of (5.38). Instead, as we shall see, (5.38) has a different solution which for large contours exhibits a perimeter, rather than the area, law.

To find this solution notice that the expectation value of a Wilson loop in Yang–Mills theory without fermions can be represented as

$$
W[C] = \exp\left(-\frac{g^2N}{2}A\right) = \exp\left[\frac{g^2N}{2}\oint dx^\mu dy_\mu G(x-y)\right], \tag{5.39}
$$

where $G(x-y)$ is the masless propagator defined by $\Delta G(x-y) = \delta^{(2)}(x-y)$. Indeed, converting the contour integrals into the area integrals by Stokes' theorem, we get

$$\oint dx^\mu dy_\mu G(x-y) = \int d^2x d^2y \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\mu} G(x-y) = -\int d^2x d^2y \delta^{(2)}(x-y) = -A.$$

Similarly, equation (5.38) will be satisfied if we consider

$$W[C] = \exp\left[\frac{g^2N}{2}\oint dx^\mu dy_\mu G_m(x-y)\right], \tag{5.40}$$

where $G_m(x-y)$ is now the massive propagator with $m^2 = g^2/4\pi$, satisfying the Klein–Gordon equation $(\Delta - m^2)G(x-y) = \delta^{(2)}(x-y)$. This fact is easy to check by direct substitution.

For large contours the solution (5.40) decays like $\propto \exp(-mP)$ where P is the perimeter of the loop. Indeed, the massive propagator $G_m(x-y)$ vanishes very fast for $|x-y| \gg 1/m$. Thus the contour integral in (5.40) is dominated by those x, y which are at most $1/m$ away, giving rise to the perimeter dependence of $W[C]$ for the loops of large size.

Although (5.40) satisfies the loop equation exactly, it does not give the exact expectation value of the Wilson loop in the adjoint fermion model. The reason is that, unless supplemented by certain boundary conditions [21], loop equations may have more than one solution. However, even without these boundary conditions it is clear that the area law $W[C] \propto \exp(-\sigma A)$ is inconsistent with (5.38), confirming that the Wilson loop is screened in the massless adjoint model.

6 Discussion.

The surprising result of this paper is that certain 1+1 dimensional gauge theories with massless adjoint fermions exhibit the screening of fundamental test charges rather than confinement. Our discussion was focussed on the simplest model, the $SU(N)/Z_N$ gauge theory with one massless adjoint multiplet. It is clear, however, that our methods carry over to more complicated theories, such as those with several massless adjoint multiplets, which are also in the screening phase.

A somewhat different example, which seems particularly interesting, is the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory,

$$\mathcal{L} = \mathrm{tr}\left[i\bar{\psi}\gamma^\mu D_\mu \psi + ig\bar{\psi}\gamma_5[\phi, \psi] + \frac{1}{2} D_\mu \phi D^\mu \phi - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \right] , \tag{6.41}$$

where ϕ is an adjoint scalar and ψ is an adjoint Majorana fermion. This theory, which may be obtained by dimensionally reducing the 2+1 dimensional $\mathcal{N} = 1$ SYM theory, was recently discussed in [20]. We find that the presence of fermion zero modes in the topologically non-trivial sectors of $SU(N)/Z_N$ once again guarantees that $\langle W(C) \rangle = 1$ for the contour at infinity. For $N = 2$ it is also possible to show that the model exhibits bilinear gluino condensation. These results imply that this theory is in the screening phase. This raises a tantalizing question: is it possible that $2 + 1$ and $3 + 1$ $\mathcal{N} = 1$ supersymmetric Yang-Mills theories are also in the screening, rather than the confining phase? We will return to this later.

There are two distinguishing features of 1+1 dimensional gauge theories which made our analysis possible:

1. The absence of dynamical degrees of freedom for gauge fields which leads to trivial Coulomb confinement in pure photodynamics or pure gluodynamics.

2. The rigid relation of the Wilson loop average to the topological structure of the theory.

Neither is true in 3+1 dimensions. This makes even more surprising the analogy of the phenomenon we observe with the situation in $\mathcal{N} = 2$ supersymmetric Yang-Mills theory [1]. In this theory the confinement of the unbroken $U(1)$ subgroup sets in as soon as a certain mass term, which breaks $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$, is added to the Lagrangian.

What possible lessons could we draw with respect to the more conventional 4D theories, in particular to QCD ? The physical case of QCD with dynamical quarks is well known to display screening, and we have nothing new to say about it. The pure 4D Yang-Mills theory is expected to be confining. In view of what we learned from 1+1 dimensional examples we may wonder, however, whether instead it could be in the screening phase: certain collective gluonic excitations might be capable of screening fundamental test charges. This possibility seems to be experimentally ruled out, however, since no states of fractional baryon number have been observed.

A more realistic scenario is that the pure gluodynamics is confining, while its $\mathcal{N}=1$ supersymmetric extension is not, due to the presence of the massless adjoint fermions. Our 1+1 dimensional examples show that a cloud of gluinos (with some help from the gluons) can screen a heavy fundamental charge, and we may be bold enough to conjecture that this is also possible in 3+1 dimensions. The screening disappears and the confinement is restored as soon as the gluinos are given a small mass (and the supersymmetry is broken). This scenario is sufficiently intriguing that, in our opinion, it deserves further investigation.

Acknowledgements

We are grateful to A. Hashimoto, D. Kutasov, J. Polchinski, M. Shifman, L. Susskind and A. Vainshtein for interesting discussions. This work was supported in part by DOE grant DE-FG02-91ER40671, NSF grant PHY90-21984, the NSF Presidential Young Investigator Award PHY-9157482, the James S. McDonnell Foundation grant No. 91-48 and funds provided by the U.S. Department of Energy under cooperative research agreement DE-FC02-94ER40818.

References

[1] N. Seiberg and E. Witten, *Nucl. Phys.* **B426**, 19 (1994).

[2] J. Ellis, Y. Frishman, A. Hanany, and M. Karliner, *Nucl. Phys.* **B382**, 189 (1992).

[3] S. Dalley and I. R. Klebanov, *Phys. Rev.* **D47**, 2517 (1993).

[4] D. Kutasov, *Nucl. Phys.* **B414**, 33 (1994).

[5] G. Bhanot, K. Demeterfi, and I. R. Klebanov, *Phys. Rev.* **D48**, 4980 (1993); K. Demeterfi, G. Bhanot, and I. R. Klebanov, *Nucl. Phys.* **B418**, 15 (1994); F. Antonuccio and S. Dalley, preprint OUTP-95-18-P, hep-lat/9505009.

[6] A. M. Polyakov and P. B. Wiegmann, *Phys. Lett.* **141B**, 223 (1984).

[7] A. M. Polyakov, Les Houches Lecture Notes, 1988, E. Brezin and J. Zinn-Justin, eds.

[8] D. Kutasov and A. Schwimmer, *Nucl. Phys.* **B442**, 447 (1995).

[9] C. Jaewardena, Helv. Phys. Acta **61**, 636 (1988); I. Sachs and A. Wipf, Helv. Phys. Acta **65**, 652 (1992).

[10] M. Green, J. Schwarz and E. Witten, *Superstring Theory*, vol. 1, Cambridge University Press 1987.

[11] A. Smilga, *Phys. Rev.* **D46**, 5598 (1992).

[12] H. Leutwyler and A. Smilga, *Phys. Rev.* **D46**, 5607 (1992).

[13] R. Crewther, *Phys. Lett.* **70B**, 349 (1977).

[14] E. Witten, *Nuovo Cimento* **A51** (1979) 325.

[15] A. Smilga, *Phys. Rev.* **D49**, 6836 (1994).

[16] F. Lenz, M. Shifman, and M. Thies, *Phys. Rev.* **D51**, 7060 (1995).

[17] C. Callan, R. Dashen, and D. Gross, *Phys. Lett.* **63B**, 334 (1976); R. Jackiw and C. Rebbi, *Phys. Rev. Lett.* **37**, 172 (1976).

[18] I. Kogan and A. Zhitnitsky, Preprint OUTP-95-25 P, hep-ph/9509322.

[19] M. Shifman and A. Vainshtein, *Nucl. Phys.* **B296**, 445 (1988); A. Morozov, M. Olshanetsky, and M. Shifman, *ibid* **B304**, 291 (1988).

[20] M. Li, hep-th/9503033; Y. Matsumura, N. Sakai, and T. Sakai, preprint TIT/HEP-290, hep-th/9504150.

[21] A. Migdal, *Phys. Rep.* **102**, 199 (1983).

[22] S. Treiman, R. Jackiw and D. Gross, Lectures on Current Algebra and its Applications, Princeton. Univ. Press, 1972; K.Fujikawa, *Phys. Rev. Lett.* **42**, 1195 (1979).

[23] S. Mandeltsam, *Phys. Rev.* **175**, 1580 (1968).

[24] V. Kazakov and I. Kostov, *Nucl. Phys.* **B176**, 199 (1980).