AN ALTERNATIVE TO AGGREGATION IN INPUT-OUTPUT ANALYSIS AND NATIONAL ACCOUNTS

Wassily Leontief *

I

THE SCHEMATIC uniformity of standard input-output computations accounts for certain practical advantages of that approach as well as for some of its peculiar limitations. One of the principal advantages of such uniformity is the opportunity it offers for using the matrix of technical coefficients, A, as a central storage bin for the basic factual information used again and again in various computations.

A comparison of the structural properties of two economies — or of the structural characteristics of the same economy at two different points of time — is reduced in this context to a comparison of two A matrices. The only (and admittedly very serious) difficulty arising in any attempt to ascertain the differences and similarities between the magnitudes of individual technical coefficients — or of the whole rows, or entire columns of such coefficients — in two matrices is often caused by the incomparability of the sectoral breakdown in terms of which the two tables were originally compiled.

These differences might turn out to be of a merely terminological or classificatory kind. This means that, in principle, at least, with full access to all the basic facts and figures, new matrices could be constructed that would describe the two essentially comparable economic structures in appropriately comparable terms.

The lack of perfect correspondence between the sectoral headings of two input-output tables might, however, frequently reflect the presence in one of the two economies of some goods or services that are neither produced nor consumed in the other. In this instance, re-classification will not help. In the extreme, albeit most unlikely, case in which the two economies have no goods or services in common, the very thought of structural comparison would have to be given up.

More often, when all the justifiable preliminary realignments of the original classifications have been made, the two matrices will turn out to have some reasonably comparable sectors, while some of the other sectors contained in one of them will have no matching counterparts in the other. Even when such incomparability is known to be due only to differences in the commodity and industry classifications used, the figures entered in those rows and columns must be treated as describing structures of incomparable kinds.

In current statistical practice, the solution of the difficulties described above is sought in aggregation. The difference between copper and nickel vanishes as soon as both are treated as "non-ferrous metals" and both become indistinguishable from steel as soon as the qualifying specification "non-ferrous" has been dropped too. The fact that comparability through aggregation is secured at the cost of analytical sharpness in the description of the underlying structural relationships is too well known to require explanation.

The method of double inversion described below permits us to reduce to a common denominator two input-output matrices that contain some comparable and also some incomparable sectors. In contrast to conventional aggregation, such analytical reduction is achieved without distortion of any of the basic structural relationships. The comparability of input-output tables attained through double inversion is limited in the sense that their respective structures are described only in terms of input-output relationships between goods and services of directly comparable kinds. It is, nevertheless, an overall comparability to the extent that

* I want to express my thanks to the staff of the Harvard Economic Research Project and particularly to Mrs. Brookes Byrd for the indispensable assistance in the preparation of the material presented in this paper. Frankly, the responsibility for the minor errors that might have crept into it rests with them.

all the structural characteristics of each of the two systems, including the magnitudes of the technical coefficients located in the "incomparable" rows and columns, are taken into account fully without omission or distortion.

II

To facilitate the intuitive understanding of the transformation that leads to the construction of what might be called a reduced input-output matrix of a national economy, we will ask the reader to visualize a situation in which — for trading purposes — all industries of a country have been divided into two groups. The industries belonging to group I are identified as the "contracting," those in group II as the "subcontracting," industries.

Each contracting, i.e., group I industry covers its direct input requirements for the products of other group I industries by direct purchases, and each group II industry makes direct purchases from other group II industries. However, the products of group II industries delivered to group I industries are manufactured on the basis of special work contracts. Under such a contract, the group I industry placing an order with a group II industry provides the latter with the products of all group I industries (including its own), in amounts required to fill that particular order. To be able to do so, it purchases all these goods — from the group I industries that make them — on its own account. The relationship between a contracting (group I) and a subcontracting (group II) industry is thus analogous to the relationship between a customer who buys the cloth himself and the tailor who makes it up for him into a suit.

In determining the amounts of goods and services that he will have to purchase from his own and all the other group I industries, the procurement officer of each group I industry will have to add to the immediate input requirements of his own sector the amounts to be processed for it — under contract — by various group II industries. For all practical purposes, such augmented shopping lists now constitute the effective input vectors of all the group I industries.

The square array of n_1 such column vectors — each containing n_1 elements (some of which may of course be zero) — represents the reduced table of input coefficients that we seek. It describes the same system as the original table, however, it describes it only in terms of goods and services produced by the selected contracting industries included in group I.

The relationship between the two tables is similar to the relationship of an abbreviated time table that lists only selected large stations, to the complete detailed time table that also shows all the intermediate stops. The subdivision of all the sectors of an economy into groups I and II must, of course, depend on the specific purpose that the consolidated system is intended to serve.

Using a reduced table for planning purposes, we can be sure that if the input-output flows among the group I industries shown in it are properly balanced, the balance between the outputs and inputs of all the group II industries omitted from it will be secured, too.

In the process of consolidation, the allocation of so-called primary inputs will change, as well. The new labor and capital coefficients of each group I industry must now reflect not only its own immediate labor and capital requirements, but also the labor and capital requirements of all the group II industries from which it draws some of its supplies. It is as if, under the imaginary contractual arrangements described above, each group I industry had to provide the group II industries working for it, not only with the goods and services produced by any of the group I sectors, but also with all the capital and labor required by these group II industries to fulfill these contracts. Thus, the output levels of all the group I industries, as projected on the basis of a reduced input-output table (multiplied with the appropriate consolidated capital and labor coefficients), will account not only for the capital and labor requirements of these group I industries, but also for those of all the group II industries without whose support these output levels could not have been attained.

III

Not unlike conventional aggregation, the analytical procedure described below is aimed at a reduction of the number of sectors in terms of which the particular economic structure was originally described. It is, however, a "clean" — not an index number — operation. It does not involve introduction of weights or any other arbitrary constants.

Equation (1) describes — in conventional matrix notation — the relationships between the total output vector, X, of all the sectors of a particular economy, and the corresponding final bill of goods, Y.

$$(I - A) X = Y. \tag{1}$$

In equation (2), both vectors are split into two parts: the column vectors X_1 and Y_1 represent the total outputs and the final deliveries of group I industries that produce the n_1 goods that will be retained in the reduced matrix, while X_2 and Y_2 represent the outputs and the final deliveries of all the other, i.e., the n_2, goods produced by the group II industries that have to be eliminated.

$$\left[\begin{array}{c|c} (I - A_{11}) & -A_{12} \\ \hline -A_{21} & (I - A_{22}) \end{array}\right]\left[\begin{array}{c} X_1 \\ \hline X_2 \end{array}\right] = \left[\begin{array}{c} Y_1 \\ \hline Y_2 \end{array}\right] \tag{2}$$

The matrix $(I - A)$ on the left-hand side is partitioned, in conformity with the output vector into which it is multiplied. A_{11} and A_{22} are square matrices whose elements are technical coefficients that govern the internal flows between the sectors of the first and of the second groups, respectively, while A_{12} and A_{21} are rectangular (not necessarily square) matrices describing the direct requirements of industries of the second group for outputs of the first group and vice versa.

Equation (3) is the solution of (2) for X in terms of Y.

$$\left[\begin{array}{c} X_1 \\ \hline X_2 \end{array}\right] = \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right]\left[\begin{array}{c} Y_1 \\ \hline Y_2 \end{array}\right]. \tag{3}$$

Matrix B is the *inverse* of $(I - A)$. It is partitioned in conformity with the partitioning of $(I - A)$ in equation (2). After the multiplication has been carried out on its right-hand side, equation (3) can be split in two:

$$X_1 = B_{11}Y_1 + B_{12}Y_2 \tag{4}$$

$$X_2 = B_{21}Y_1 + B_{22}Y_2. \tag{5}$$

Premultiplying both sides of (4) by B_{11}^{-1}, we have:

$$B_{11}^{-1}X_1 = Y_1 + B_{11}^{-1}B_{12}Y_2. \tag{6}$$

This equation can be interpreted as a reduced version of the original system (2). It describes the same structural relationships, however, it represents them only in terms of the goods and services produced by the n_1 industries assigned to group I. The variables contained in vector X_2 — that is, the outputs of the n_2 industries assigned to group II — have been eliminated by means of two successive matrix inversions that led from (2) to (6).

Let a new structural matrix and a new final demand vector be defined by:

$$A_{11}^* = I - B_{11}^{-1} \tag{7}$$

$$Y_1^* = Y_1 + B_{11}^{-1}B_{12}Y_2. \tag{8}$$

In this notation (6) can be re-written as:

$$(I - A_{11}^*)X_1 = Y_1^*. \tag{9}$$

In perfect analogy with the original system (1) this equation describes the input-output relationships between the redefined vector of final deliveries, Y_1^*, and the corresponding vector of total outputs X_1.[1] Solved for X_1 in terms of Y_1^*, it yields:

$$X_1 = (I - A_{11}^*)^{-1}Y_1^*. \tag{10}$$

This equation is, of course, formally equivalent to (4). A_{11}^* is the structural matrix of the economy that was originally described by A. However, the same structure is now described in terms of the n_1 group I industries alone. The first column of A_{11}^* consists, for example, of n_1 technical coefficients, a_{11}^*, a_{21}^*, . . . a_{n1}, showing the number of units of each of these n_1 industries of group I required per unit of the total output, x_1, of the first. Although not referring to them explicitly, implicitly these coefficients reflect the input requirements also of the other n_2 industries eliminated in the reduction process.

Let, for example, industry 1 produce "steel" and industry 2, "electric energy," both assigned to group I. In the reduced matrix A_{11}^*, the coefficients a_{21}^* thus represents the number of kwh (or a dollar's worth) of electricity required to produce a ton (or a dollar's worth) of steel. This requirement is computed to cover not only the direct deliveries of electricity from generating stations to steel plants, but also the indirect deliveries channeled through industries assigned to group II. If "iron mining" were, for instance, considered as belonging to group

[1] The symbol X_1^* is not used because the reduced system has been derived in such a way that $X_1 \equiv X_1^*$.

II, the electricity used in extraction and preparation of the iron ore that went into the production of one ton (or a dollar's worth) of steel would also be included in the input coefficient a_{21}^*, and so would electric power absorbed by the steel industry via all other sectors assigned to group II.

In other words, the array of the input coefficients (with asterisks) that make up the first column of matrix A_{11}^* describes the combination of the products of industries included in group I with which the economy in question would be capable of turning out a ton (or a dollar's worth) of steel. Some of these inputs reach the steel industry indirectly through industries assigned to group II.

The reduced structural matrix A_{11}^* describes explicitly only the input structure of the group I industries and this only in terms of their own products. Implicitly, it reflects, nevertheless, the technological characteristics of all the other industries as well. The relationship between elements of the reduced and the original matrix is displayed clearly if A_{11}^* is expressed directly in terms of the elements of the partitioned matrix A:[2]

$$A_{11}^* = A_{11} + A_{12}(I - A_{22})^{-1}A_{21}. \tag{11}$$

The well-known sufficient conditions for the ability of the given input-output system to maintain — without drawing on outside help — a positive level of final consumption, i.e., to possess a positive inverse $(I - A)^{-1}$, requires that none of the column (or rows) totals of the technical coefficients in A_{11} exceed one, and at least one of these sum totals be less than one. This implies that the inverse $(I - A)^{-1}$ is non-negative. All the components of the second term on the right-hand side of (11) being either zero or positive, each element a_{ij}^* of the consolidated structural matrix has to be either equal to, or larger than, the corresponding originally given input coefficient, a_{ij}.

The final deliveries on the right-hand side of the reduced system (6) are composed of two parts. Vector Y_1 is the demand for the products of the group I industries as it appears in the original system (2). Vector $B_{11}^{-1}B_{12}Y_2$ $(\equiv A_{12}(I - A_{22})^{-1}Y_2)$ represents the final demand for the products of the second group of goods translated into the requirements for inputs of goods belonging to the first. In the special case in which the final users happen to demand directly only commodities and services of group I, while group II consists exclusively of intermediate goods, Y_2 vanishes and, save for the omission of its zero components, the final deliveries vector of the original system would enter without any change into the smaller, reduced system, too.

IV

A primary input, such as labor, a natural resource, or — in a static system — a stock of some kind of capital goods, can be treated in the process of reduction as if it were a product of a separate industry included in group I.

The row assigned to each primary factor in the original matrix A will contain the appropriate technical input coefficients: labor coefficients, capital coefficients, and so on. The columns corresponding to these rows will consist of zeros, since, in contrast to other goods and services, the output of a primary factor is not considered to be formally dependent on inputs originating in other industries.[3]

The labor, capital, and other primary factor coefficients appearing in the appropriate rows of matrix A^* will never be smaller — and in most instances they will be larger — than the corresponding elements of the original matrix A. As all the other input coefficients in the reduced system, they cover not only the immediate requirements of each group I industry, but also the labor and capital employed by group II industries (eliminated in the process of analytical reduction) from which that industry receives all its group II supplies.

V

Any static input-output system implies the existence of linear relationships between the prices of all products and the "value added" in

[2] Since $B = (I - A)_{-1}$,
$$B(I - A) = I.$$
In particular:
$$B_{11}(I - A_{11}) - B_{12}A_{21} = I$$
$$-B_{11}A_{21} + B_{12}(I - A_{22}) = 0.$$
Eliminating B_{12} and rearranging yields:
$$A_{11}^* = (I - B_{11})_-^1 = A_{11} + A_{12}(I - A_{22})_-^1A_{21}.$$

[3] The matrix $(I - A)$ is nevertheless not singular: its main diagonal contains positive elements throughout.

all the sectors per unit of their respective outputs.[4] While a reduction of a structural matrix eliminates some of the prices from the picture, it leaves the relationship between the remaining prices and the values added essentially intact.

Let P be the price vector of the original system and V the vector of values added per unit of output in its n different sectors. The basic relationships between the two vectors,

$$(I - A')P = V \tag{12}$$

can be solved for the unknown prices in terms of given values added:

$$\left[\begin{array}{c} P_1 \\ \hline P_2 \end{array}\right] = \left[\begin{array}{c|c} B_{11}' & B_{21}' \\ \hline B_{12}' & B_{22}' \end{array}\right]\left[\begin{array}{c} V_1 \\ \hline V_2 \end{array}\right]. \tag{13}$$

The "primes" above the B's indicate transposition, i.e., permutation of rows and columns. The partitioning of the two vectors and of the structural matrix corresponds to a similar partitioning in (3) above. Solving for P_1 we have:

$$P_1 = B_{11}'V_1^*, \text{ where} \tag{14}$$

$$V_1^* = V_1 + (B_{11}')^{-1}B_{21}'V_2. \tag{15}$$

The last equation shows that, analogous to the reduced final bill of goods, Y_1^*, in (8), V_1^* represents the augmented values added vector of the group I industries. Each element of that augmented vector contains not only the value added — shown for each one of them in the original table — but also the value added in group II industries imputed through all the goods and services which the particular group I sector receives from them. In view of (7), (14) can be rewritten as:

$$P_1 = (I - A^{*\prime})^{-1}V_1^*. \tag{16}$$

Inserting on the right-hand side the augmented values added in group I industries, we obtain on the left-hand side a set of prices identical with those that would have been derived for group I outputs from the original (unreduced) set of price equations (13–15).

VI

A recently completed study of metalworking industries called for analysis of interdependence among the several branches of production belonging to this group, and for an assessment of its position within the United States national economy as a whole. Of the 73 producing sectors in the 1958 input-output table,[5] 23 are making or transforming metals, five of them supply intermediate ferrous or nonferrous products, while the other 18 are engaged in the manufacture of basic materials and finished metal goods.

The immediate technical interdependence among the 23 metalworking sectors is reflected in the magnitude of the input coefficients located on the intersections of the 23 rows and the corresponding 23 columns in the large 73-sector table mentioned above.

The production of the nonmetal inputs absorbed by metalworking industries often requires the use of various metal products in its turn. The dependence of each metalworking sector upon all the others (taking into account such indirect requirements) is described by the augmented input coefficients entered in the 23 rows and columns of the reduced matrix that was obtained through analytical elimination of all the 50 nonmetalworking sectors from the original table. The full interdependence between the 18 metalworking industries engaged in the manufacture of raw and finished metal products, can be brought out through further reduction that eliminates from the large table also the five intermediate metalworking industries.

A row of labor coefficients, and another of (total) capital coefficients, was added at the outset to the original 73-sector matrix. After reduction, appropriately augmented labor and capital coefficients appeared in the last two rows of both reduced matrices as well.

In table 1, the technical coefficients describing the inputs of various metal products required by the "motor vehicles and equipment" industry, as they appear in the original 73-sector matrix, are shown in column (1). The second column contains the corresponding augmented coefficients, as they appear in the reduced matrix composed of the 23 metalworking sectors. The third column shows the 18

[4] The "value added" in any industry can, in its turn, be described as a sum of the input coefficients of all factors multiplied by their respective prices augmented by the amount of positive or negative net surplus earned per unit of its output.

[5] U.S. Department of Commerce, *Survey of Current Business*, 44, No. 11 (Nov. 1964); and Anne P. Carter, "Changes in the Structure of the American Economy, 1947 to 1958 and 1962," this REVIEW, XLIX (May 1967).

TABLE 1. — INPUT COEFFICIENTS DESCRIBING THE REQUIREMENTS OF THE MOTOR VEHICLES AND EQUIPMENT INDUSTRY FOR PRODUCTS FROM OTHER UNITED STATES METALWORKING INDUSTRIES [a] IN 1958

Sector Number the 73-Sector Matrix	Industry	Input Coefficients in the: Original 73-Sector Matrix [b]	Reduced 23-Sector Matrix [c]	Reduced 18-Sector Matrix
59	Motor Vehicles and Equipment	0.29757	0.29817	0.29991
37	Primary Iron and Steel Manufacturing	0.08780	0.08874	0.10714
42	Other Fabricated Metal Products	0.03603	0.03713	
41	Screw Machine Products, Bolts, Nuts, etc., Metal Stamping	0.03103	0.03137	
47	General Industrial and Metalworking Machinery, and Equipment	0.02364	0.02456	
58	Miscellaneous Electrical Machinery Equipment and Supplies	0.01543	0.01557	0.01564
38	Primary Non-ferrous Metals Manufacturing	0.01144	0.01205	0.01871
56	Radio, Television, and Communication Equipment	0.00523	0.00557	0.00576
62	Professional, Scientific, and Control Instruments and Supplies	0.00438	0.00460	0.00498
55	Electric Lighting and Wiring Equipment	0.00420	0.00441	0.00475
43	Engines and Turbines	0.00379	0.00402	0.00437
53	Electrical Industrial Equipment	0.00217	0.00236	
52	Service Industrial Machinery, Household Appliances	0.00129	0.00157	0.00208
44	Farm Machinery and Equipment	0.00105	0.00129	0.00144
40	Heating, Plumbing, and Structural Metal Products	0.00102	0.00147	
64	Miscellaneous Manufacturing	0.00092	0.00201	0.00245
61	Transportation Equipment, Miscellaneous	0.00089	0.00123	0.00143
57	Electronic Components and Accessories	0.00079	0.00090	0.00111
45	Construction, Mining, Oil Field Machinery and Equipment	0.00044	0.00062	0.00094
60	Aircraft and Parts	0.00039	0.00086	0.00123
46	Materials Handling Machinery and Equipment	0.00022	0.00027	0.00046
63	Optical, Ophthalmic, Photographic Equipment	0.00005	0.00045	0.00053
51	Office, Computing and Accounting Machines	0.00000	0.00069	0.00079
	Labor	0.02645	0.04729	0.05614
	Capital Stock	0.24313	0.47495	0.55890

[a] Units of measurement: for labor coefficients, man-years per $1,000 of output; for all other coefficients, 1958 dollars per dollar of output.
[b] This matrix is based on the 1958 input-output table published by the Office of Business Economics, Department of Commerce. See Anne Carter, "Changes in the Structure of the American Economy, 1947–1958, 1962," this REVIEW, XLIX (May 1967). The labor coefficients are based on, Jack Alterman, "Interindustry Employment Requirements," *Monthly Labor Review*, 88, No. 7 (July 1965). The capital coefficients for manufacturing sectors were obtained from Waddell, Ritz, Norton, De Witt, and Marshall K. Wood, "Capital Expansion Planning Factors, Manufacturing Industries," *National Planning Association* (Washington, D.C., April 1966). For nonmanufacturing sectors, the capital coefficients were compiled at the Harvard Economic Research Project.
[c] The sectors eliminated through the reduction procedure are those included in the 73-sector input-output table, but not represented in this column of augmented coefficients.

still more augmented coefficients as they appear in the motor vehicles and equipment column of a reduced matrix, from which the five basic metalworking industries were eliminated too. Appropriate labor and capital coefficients are entered at the bottom of all three columns.

VII

Table 2 is an example of a reduced national input-output table. This complete, but compact, flow chart was derived from the official 1958 United States table [6] in two successive steps.

[6] U.S. Department of Commerce, *Survey of Current Business*, 45, No. 9 (Sept. 1965).

First, 34 of the 83 productive sectors of the original table were combined into eight groups. The resulting smaller 57-sector table contained these eight aggregated industries, the 49 sectors carried over from the original 83-order table, a corresponding column of final demand and a value added row.

This 57-sector table was reduced, in a second step, through elimination of all the 49 non-aggregated industries, to a compact eight-sector table. It should be noted that the figures shown in table 2 are total flows, not input coefficients. They were obtained through multiplication of all elements of each column of the corresponding reduced coefficient matrix by the given total output figure of the industry,

TABLE 2. — INPUT-OUTPUT TABLE OF THE UNITED STATES ECONOMY FOR THE YEAR 1958 REDUCED TO EIGHT FROM 57 PRODUCING SECTORS [a]

Column Row	Industry	Food and Drugs (1)	Housewares (2)	Machinery (3)	Transportation Equipment and Consumer Appliances (4)	Construction (5)	Metals (6)	Energy (7)	Chemicals (8)	Final Demand	Gross Domestic Output
1	Food and Drugs	15,202	547	161	353	513	165	218	386	58,728	76,272
		(12,468)	(96)	(11)	(49)	(17)	(53)	(62)	(288)	(55,320)	
2	Textiles, Clothing and Furnishings	347	12,815	92	821	761	171	63	61	21,369	36,500
		(155)	(12,692)	(37)	(636)	(524)	(47)	(8)	(38)	(20,033)	
3	Machinery	430	215	2,321	2,061	1,397	819	406	200	13,385	21,233
		(28)	(105)	(2,186)	(1,644)	(748)	(545)	(141)	(150)	(11,293)	
4	Transportation Equipment and Consumer Appliances	363	158	816	11,791	1,372	485	183	53	38,691	53,912
		(29)	(55)	(691)	(11,196)	(753)	(101)	(29)	(5)	(32,670)	
5	Construction	1,158	218	115	308	48	284	1,541	70	65,117	69,291
		(235)	(18)	(26)	(109)	(8)	(131)	(579)	(6)	(56,836)	
6	Metals	1,033	475	3,073	6,038	6,468	7,959	388	479	2,244	28,158
		(46)	(277)	(2,631)	(4,618)	(3,650)	(7,335)	(110)	(389)	(−45)	
7	Energy	2,158	652	371	805	2,774	1,704	6,888	1,127	23,851	40,330
		(783)	(293)	(226)	(404)	(1,536)	(1,391)	(6,236)	(1,007)	(17,702)	
8	Chemicals	1,956	1,030	201	475	1,218	459	713	2,500	3,218	11,770
		(1,056)	(218)	(117)	(115)	(437)	(283)	(576)	(2,351)	(1,510)	
	Value Added	53,625	20,390	14,083	31,260	54,308	16,112	29,930	6,894	178,912	405,515
		(22,252)	(12,844)	(10,254)	(20,677)	(28,937)	(10,509)	(15,127)	(4,674)		
TOTAL		76,272	36,500	21,233	53,912	69,291	28,158	40,330	11,770	405,515	
	Labor	8,182	3,929	1,820	3,891	8,581	1,867	1,775	671	26,430	57,146
		(2,202)	(2,808)	(1,307)	(2,467)	(4,847)	(1,155)	(1,003)	(403)		

[a] Derived from the 83-sector table published in "Transaction Table of the 1958 Input-Output Study and Revised Direct Requirements Data," *Survey of Current Business*, 45, No. 9 (Sept. 1965). Each of the eight sectors of the intermediate 57-sector table retained in this reduced table represents an aggregate of the following industries identified by the numbers they carry in the original 83-sector table:
1) Food and Drugs: 14, 15, 29; 2) Textiles, Clothing, Furnishings: 16, 17, 18, 19, 34, 22, 23; 3) Machinery (only final): 51, 44, 45, 46, 47, 48, 49, 50, 63; 4) Transportation Equipment and Consumer Appliances: 52, 54, 56, 59, 60, 61, 62; 5) Construction: 11, 12; 6) Metals: 37, 38; 7) 31, 68; 8) Chemicals: 27.
Corresponding entries in the unreduced 57-sector table appear in parentheses. The unit are man-years in the labor row and millions of dollars in all other rows.

the input structure of which that particular column describes.

Table 2 thus depicts the structure of the American economy in terms of flows of commodities and services among eight industrial sectors, a value added row, and a column of final demand, both reduced in conformity with the rest of the table (see, equation 8). Wages and salaries paid out by various sectors are, of course, included in the value added row. In addition, a separate row of labor inputs, measured in man-years, was carried along through all computations. This row is reproduced separately at the bottom of the table.

In each cell of the table, below the number describing the appropriately augmented intersectoral transaction is entered, enclosed in parentheses, another figure. This number represents the magnitude of the input — from the sector named on the left to the sector identified at the head of the column — as it appeared in the unreduced 57-sector table obtained at the end of the first step, i.e., before the 49 un-aggregated sectors were eliminated from the table in the second step.

In the final demand column, the larger entries represent the augmented deliveries to households, government, and other final users, while the entries in parentheses show the corresponding figures, as they appeared in the 57-sector table. The first entry exceeds, in each instance, the figure in parentheses below by the amount of the particular type of goods that was absorbed in the production of those final deliveries which were eliminated from the original table. Value added in general — and labor inputs in particular — that were absorbed in this way, appear now in the final demand column.

VIII

The idea that, in the description of an economic system, some processes and outputs can be reduced, that is, expressed in terms of others, goes quite far back into the history of economic thought. Adam Smith discussed at length the question of whether corn should be measured in labor units required to grow it, or, on the contrary, labor measured in terms of corn that a worker needs to live. Quesnay insisted that various branches of manufacturing should be represented in his tableau only by the amounts of rough materials that they transformed into finished products.

The notion of unproductive — as contrasted with productive — labor, whose product does not deserve to be included in the grand total of national product, was still propounded by Stuart Mill. The Marxian doctrine caused, up until recently, the Soviet official statistician to exclude transportation of persons and products of many service industries from national accounts, and, in the West, the output of governmental and other public services are still often treated in the same way.

In the latter case, the elimination of the output — as contrasted with the input — of the public sector from national accounts, is justified, not so much by the distinction between productive and unproductive activities, but rather by the difficulty of measuring the output of "public administration," of "education," or of "national defense."

The number of goods and services that more and more detailed observation of various processes of production and consumption would permit us to distinguish is much greater than even an input-output matrix containing many thousands of rows and columns can possibly hold. For many purposes, that number might also be larger than we would need to carry from the first stage of the analytical procedure to the last. Aggregation, i.e., summation of essentially heterogeneous quantities, is one of the two devices that the economist uses to limit the number of variables and functional relationships in terms of which he describes what he observes. The other is reduction, that is, elimination of certain goods and processes. In this paper, a systematic procedure has been presented that permits us to reduce the size of an input-output table through analytical elimination of any of its rows and columns. A less systematic, intuitive elimination of a much larger number of variables — considered to be secondary or intermediate — occurs, however, already during the collection of the primary statistical information. Thus, even a most detailed input-output table, as well as the national accounts constructed around it, can be said to present the actual economic system, not only in an aggregated, but also in a reduced form.