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Understanding and using the minus sign in Faraday's law

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Abstract

The inclusion of minus signs in physics equations is often a barrier to student understanding, and perhaps never more so than in Faraday's Law of electromagnetic induction. This paper carefully explains the origin of the minus sign in Faraday's law and proposes a simple method by which students can use it to directly predict the direction of induced current in a coil.

Introduction

At advanced level, the Faraday–Neumann law for the induced electromotive force (emf) in a rigid coil of wire is often presented in the form

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \qquad (1)$$

where $\mathcal{E}$ is the induced emf, N the number of turns of wire in the coil and Φ the magnetic flux linking the coil. The latter is usually presented as

$$\Phi = BA \qquad (2)$$

where B is the magnetic flux density and A the area of the coil.

The minus sign in equation (1) inevitably becomes an issue with students and it is always tempting to explain it briefly as being a simple consequence of Lenz's law, safe to ignore in calculations but included in the equation to show that the induced emf *opposes* the change of flux. However, exactly how the minus sign shows this is by no means intuitive and requires some careful explanation. To illustrate this, consider the use of the minus sign in another equation, that which describes the dependence of the acceleration a of a simple harmonic oscillator (frequency f) on its displacement from equilibrium position, x:

$$a = -(2\pi f)^2 x. \qquad (3)$$

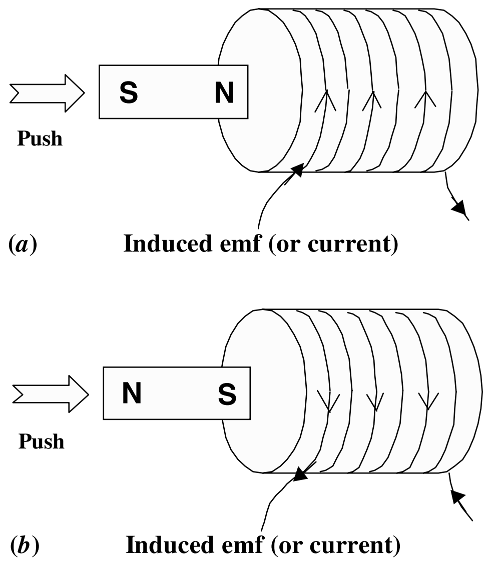

Figure 1. The problem of associating the minus sign with the direction of induced emf. (*a*) According to equation (1) $\mathcal{E}$ is negative; (*b*) according to equation (1) $\mathcal{E}$ is still negative!

In this equation, both a and x are vector quantities, and the minus sign is used simply to indicate that a points in the *opposite direction* to x.

The use of the minus sign in Faraday's law (equation (1)) is clearly different, since none of the

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Box 1. Using the equation $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t}$ to work out the direction of the induced current around a coil.

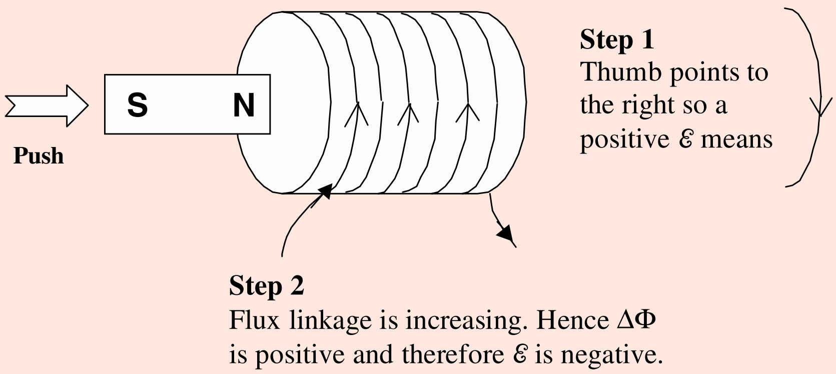
Step 1:

Point the thumb of your right hand in the direction of the applied magnetic field through the coil, and curl your fingers. The way your fingers point around the coil establishes what we'll call the positive (+) sense of the induced emf.

Step 2:

Look at the equation at the top of this box. Think of Φ as representing the **number of field lines going through the coil** ('flux linkage'). If this is getting *larger*, then $\Delta\Phi$ is positive and hence $\mathcal{E}$ is negative. If, on the other hand, the number of lines is getting smaller, then $\Delta\Phi$ is negative and hence $\mathcal{E}$ is positive.

Example:



quantities in it are vector quantities. This means that we cannot associate the minus sign explicitly with the *direction* of induced emf, direction of flux or direction of current. Figure 1 illustrates the potential pitfalls of such an association: a minus sign for $\mathcal{E}$ can mean current induced *either* way around a coil, depending on the circumstances.

Two issues arise from the discussion above. Firstly, what does a negative sign for $\mathcal{E}$ actually mean? Secondly, is there a way in which we can teach students to *use* the minus sign in order to predict the direction of the induced current? This article deals with the second point first, since this is of more immediate practical relevance to students. The section on the origin of the minus sign and its link to Lenz's law includes vector and calculus notation which may be unfamiliar to students but is included to help teachers clarify how an equation that appears to include only scalar quantities can give information about the *direction* of induced current.

Getting students to use the minus sign

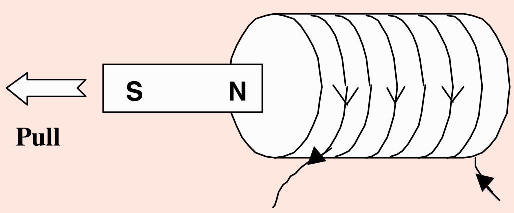
The two-step method described in Box 1 enables students to work out the direction of induced current in a coil directly from equation (1), that is *without the need for an independent application of Lenz's law*. A variety of practical examples, which illustrate the use of the method, are described in Box 2.

Discussion of the method

A small amount of practice with this procedure enables students to predict the direction of the induced emf (and hence the induced current)

Box 2.

A. Withdrawing a bar magnet from a coil.

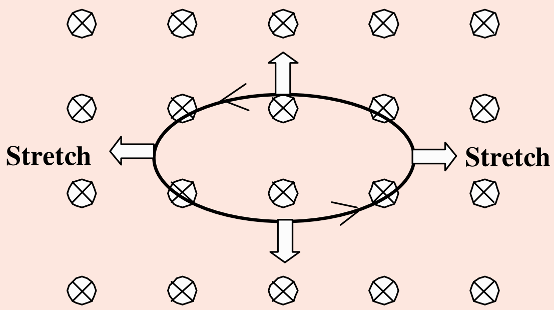


Step 1
Positive sense.

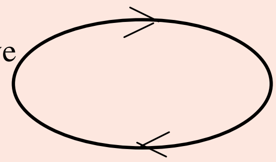
Step 2
Flux linkage *decreasing*, so $\Delta\Phi$ is negative and therefore $\mathcal{E}$ is positive.

B. Stretching a loop of conducting rubber.

(Magnetic field into page)



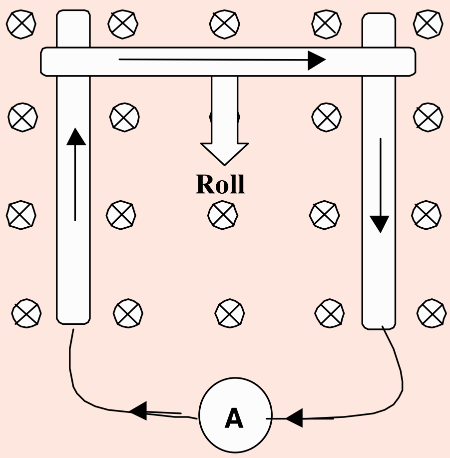
Step 1
Positive sense.



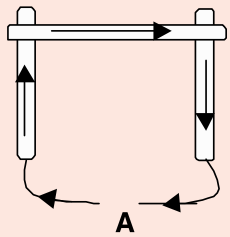
Step 2
Flux linkage *increasing*, so $\Delta\Phi$ is positive and therefore $\mathcal{E}$ is negative.

C. Rolling a conducting bar along conducting rollers.

(Magnetic field into page)



Step 1
Positive sense.



Step 2
Flux linkage *decreasing*, so $\Delta\Phi$ is negative and therefore $\mathcal{E}$ is positive.

relatively *quickly* in all cases involving electromagnetic induction in coils or single loops.

Once students have become familiar with the use of the method, it is of course necessary to justify its use by demonstrating that the method gives results that are *in agreement with experiment*. That is, if the minus sign *is left out of the equation*, the method predicts current the opposite way around to that actually observed! This would be a convenient point to introduce Lenz's law in its usual form.

I believe that there are advantages in using the method discussed in this paper to predict the direction of the induced current. Firstly, students gain an appreciation that the minus sign is of practical importance in equation (1), and is not something included almost as an afterthought for 'completeness'. Secondly, by practising the method, students become familiar with the delta notation and the various symbols involved in Faraday's equation. Thirdly, the method is fully consistent with the right-handed convention used in more mathematical treatments; a student pursuing physics at an undergraduate level would not have to 'unlearn' the method; indeed practice with the method could help to clarify the use of negative signs in vector algebra. These points are discussed further in the next section.

The origin of the minus sign

Although the quantities $\mathcal{E}$ and Φ presented in equation (1) are not themselves vector quantities, they are each related to the *dot product* of two other vector quantities. Hence they each contain information about the *relative* directions of these two quantities.

For the case of the induced emf the two vector quantities concerned are $\boldsymbol{E}$, the induced electric field and $\mathbf{d}\boldsymbol{l}$, an increment of path around a loop C:

$$\mathcal{E} = \oint_{\mathrm{C}} \boldsymbol{E} \cdot \mathbf{d}\boldsymbol{l}. \tag{4}$$

Here $\boldsymbol{E} \cdot \mathbf{d}\boldsymbol{l}$ is the dot product of the quantities $\boldsymbol{E}$ and $\mathbf{d}\boldsymbol{l}$ (the dot product is positive if $\boldsymbol{E}$ and $\mathbf{d}\boldsymbol{l}$ are in the same direction).

Similarly, the magnetic flux *across* a surface S is

$$\Phi = \int_{\mathrm{S}} \boldsymbol{B} \cdot \mathbf{d}\boldsymbol{S} \tag{5}$$

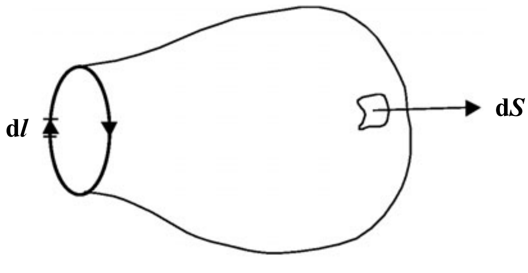


Figure 2. The definitions of $\mathbf{d}\boldsymbol{l}$ and $\mathbf{d}\boldsymbol{S}$.

where $\boldsymbol{B}$ is the magnetic flux density and $\mathbf{d}\boldsymbol{S}$ is an increment of area, part of any surface bounded by C; see figure 2.

The directions of $\boldsymbol{E}$ and $\boldsymbol{B}$ follow the usual conventions for electric fields and magnetic fields respectively. The direction of $\mathbf{d}\boldsymbol{S}$ may be chosen *arbitrarily*, either outwards from the surface (as shown in figure 2) or inwards. This arbitrary choice means that we might as well choose $\mathbf{d}\boldsymbol{S}$ to be in the direction of the magnetic field, i.e. in the direction of vector quantity $\boldsymbol{B}$ (see for example [1, 2]). This is convenient since the dot product $\boldsymbol{B} \cdot \mathbf{d}\boldsymbol{S}$ is then always positive, and hence the *magnetic flux linkage* Φ *may always be taken as positive*. This justifies the use of Φ as a positive number indicative of the flux linkage (see step 2 of the method).

Once a direction has been chosen for the vector quantity $\mathbf{d}\boldsymbol{S}$, it is necessary to use a mathematical convention to relate the direction of $\mathbf{d}\boldsymbol{l}$ to $\mathbf{d}\boldsymbol{S}$, and it is this convention that is ultimately responsible for the necessity of the minus sign in equation (1). The convention is simply the one used in step 1: the right-hand convention. That is, the positive sense for $\mathbf{d}\boldsymbol{l}$ is set by the right-hand grip rule with the thumb pointing in the direction of $\mathbf{d}\boldsymbol{S}$ (see figure 2).

A positive $\mathcal{E}$ is therefore one in which the induced electric field $\boldsymbol{E}$ (and hence the induced current) is in the *same direction* as $\mathbf{d}\boldsymbol{l}$, making the dot product $\boldsymbol{E} \cdot \mathbf{d}\boldsymbol{l}$ (and hence the right-hand side of equation (4)) *positive*. A negative $\mathcal{E}$ is one in which the induced electric field $\boldsymbol{E}$ (and hence the induced current) is in the *opposite* direction to $\mathbf{d}\boldsymbol{l}$, making the dot product $\boldsymbol{E} \cdot \mathbf{d}\boldsymbol{l}$ (and hence the right-hand side of equation (4)) *negative*. Of course, the positive sense for $\mathbf{d}\boldsymbol{l}$ must first have been determined using the convention described above; this is what is involved in step 1 of the method presented in this paper.

Conclusion

Explaining the minus sign in Faraday's law by saying it is a simple consequence of Lenz's law is not telling the whole story! It is in fact a consequence of two things: (i) the decision to use the right-handed convention, and (ii) the necessity that the resulting equation predicts a direction for the induced current which agrees with experiment (i.e. Lenz's law). In other words, if we had chosen to use a *left-handed* convention, then a minus sign would not be necessary but the equation would still agree with Lenz's law!

This article has presented a method by which Advanced level students (16–18 years) can use the minus sign in Faraday's law to predict the direction of an induced current in a coil or loop of wire. The method is simple to use for all students and also completely consistent with the right-handed convention, which is useful for students going on to study undergraduate physics.

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References

[1] Duffin W J 1990 *Electricity and Magnetism* 3rd edn (New York: McGraw-Hill) ch 9
[2] Lorrain P, Corson D P and Lorrain F 1988 *Electromagnetic Fields and Waves* 3rd edn (San Francisco: Freeman) ch 23

Chris Jones has taught AS-level and A-level physics at Hereford Sixth Form College for five years, after gaining a PhD in semiconductor physics from the University of Exeter.