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Group-Theoretic Algorithms and Graph Isomorphism

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PREFACE

This monograph develops the recent algebraic approach to Graph Isomorphism and some of its implications for Computational Complexity. Graph Isomorphism can be rephrased as a purely algebraic problem that exposes a surprising structural similarity with a number of problems in Group Theory. These problems are easily shown to be in **NP** but are not likely **NP**-complete. Moreover, there is a good possibility that they are harder than Graph Isomorphism, with respect to polynomial time reduction. Because of this possibility, the algebraic approach detailed in this book could prove to be very important for Computational Complexity.

The roots of this approach predate Babai's Colored Graph Automorphism Problem and my investigation of cone graphs. Nevertheless, these two papers appear to have been the stimulus leading to the break-through subexponential isomorphism test for trivalent graphs by Furst, Hopcroft and Luks. That paper already contained many of the techniques applied later by Luks in his polynomial time isomorphism test for graphs of fixed valence, most notably the inductive approach to determining automorphisms. Luks' contributions have been primarily a novel way for exploiting the imprimitivity structure of certain permutation groups and his analysis of the structure of the automorphism groups of graphs of fixed valence.

I give my thanks to Juris Hartmanis for suggesting that this material be brought together into a systematic survey of the area as it is at present. John Hopcroft's dedication to Computer Science has been exemplary. I wish to thank him for his willingness to introduce me to Graph Isomorphism. Charles Sims has been my tutor in the mathematical aspects of this work and has been one of those rare individuals willing to carefully read the manuscript and make suggestions for improvement. Paul Young has been exceptionally willing to listen to my ideas and patient enough to criticize them. Francine Berman contributed by partially relieving my teaching load. Merrick Furst and Michael O'Donnell have thoroughly read the manuscript and improved it. I wish to thank them all.

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