

# The Mathematics of How We See

Optometrics  $\Rightarrow$  Mathematics

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# Introduction

- **What is Optometry?**
  - Profession devoted to the examination of the eyes for visual defects, diagnosis of problems or impairments, and prescription of corrective lenses or provision of other types of treatment.
  - Much evolution since its inception
  - Optometry vs. Ophthalmology

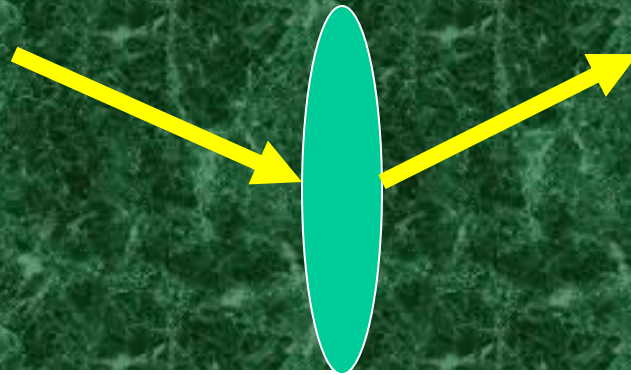
# Introduction

- Major Players
  - Isaac Newton
    - Axioms on Optics
  - Herman Snellen
    - Visual Acuity
  - Bausch & Lomb
  - Wesley-Jessen

# Newton's Axioms



- Axiom I
  - The angles of reflection and refraction lie in one and the same plane with the angle of incidence.

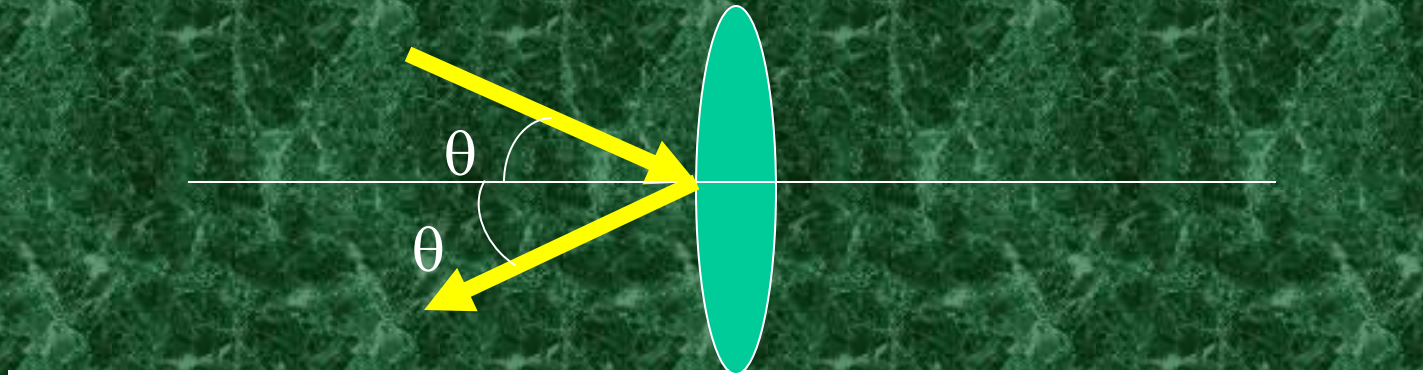




# Newton's Axioms



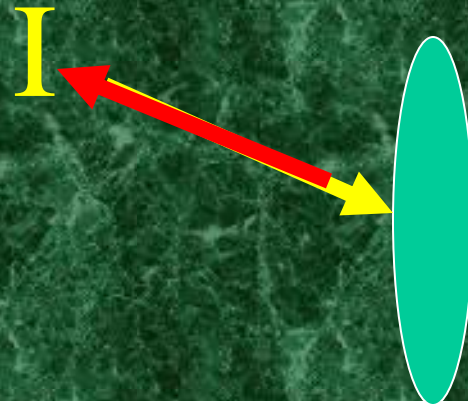
- Axiom II
  - The angle of reflection is equal to the angle of incidence.



# Newton's Axioms



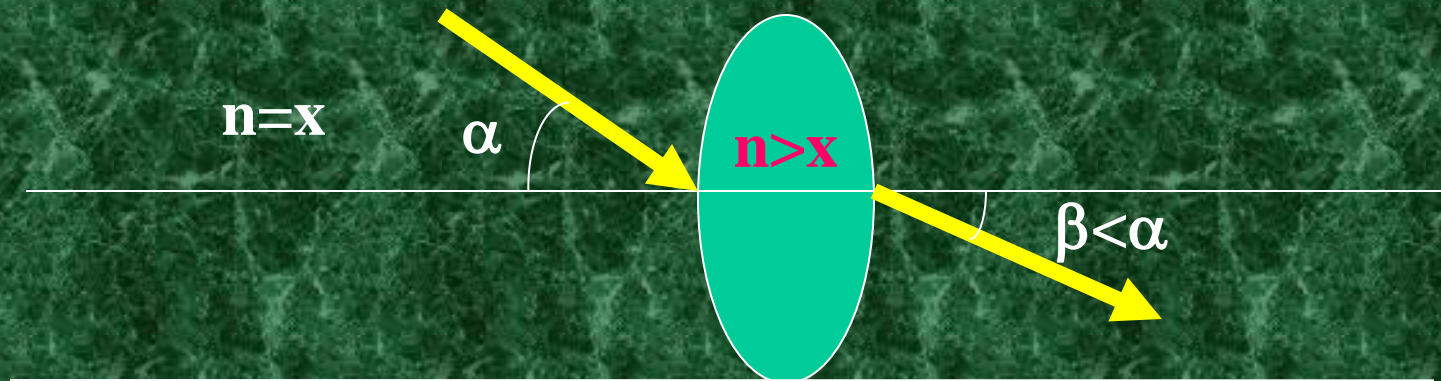
- Axiom III
  - If the refracted ray be returned directly back to the point of incidence, it shall be refracted into the line before described by the incident ray.



# Newton's Axioms



- Axiom IV
  - Refraction out of the rarer medium into the denser is made towards the perpendicular; that is, so that the angle of refraction be less than the angle of incidence.



# Optometric Terms

- What does “20/20” actually mean?
  - Herman Snellen
  - Normal Vision
  - 20/400 = very poor sight
  - 20/15 = better than normal



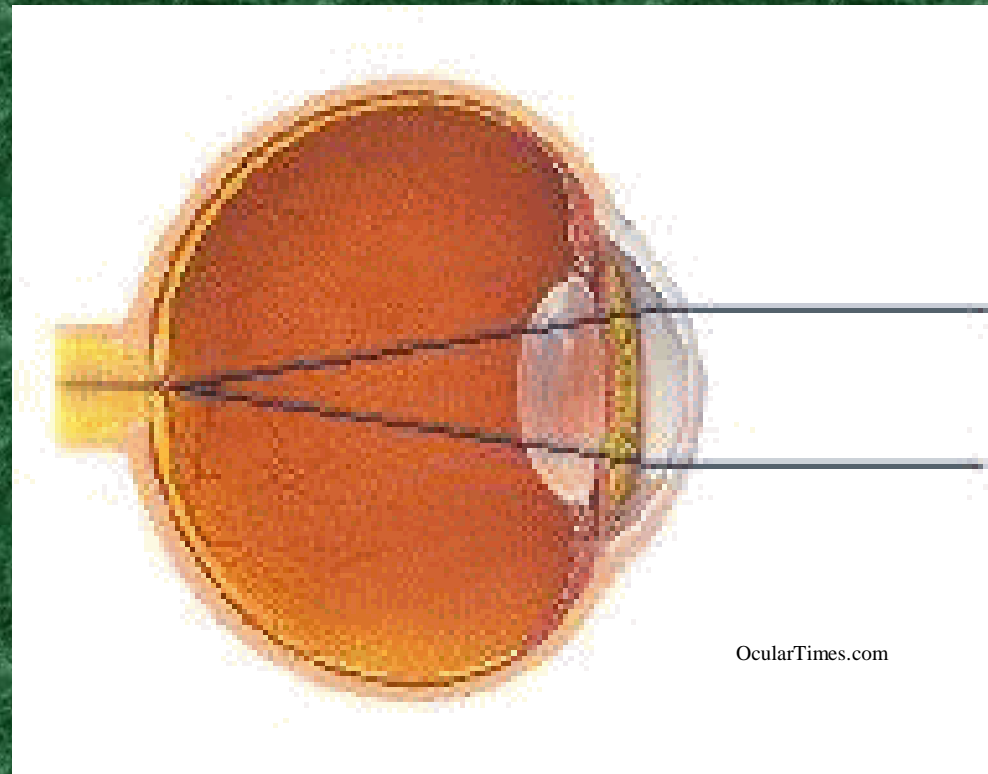
# Optometric Terms

In normal 20/20 vision, a person can see the 85 on the sign at a distance of 200 feet.

In contrast, a person with 20/200 vision must be 20 feet from the 85 to see it.

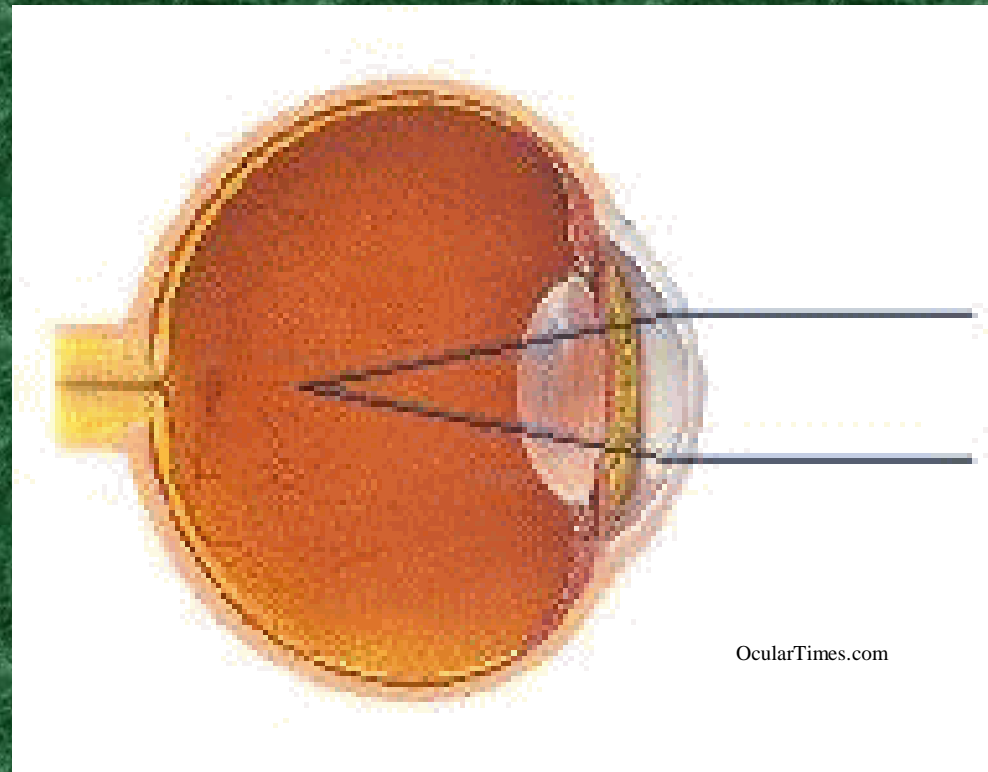
# Optometric Terms

- **Emmetropia**
  - **Normal Vision**
  - parallel rays of light meet directly on the macula



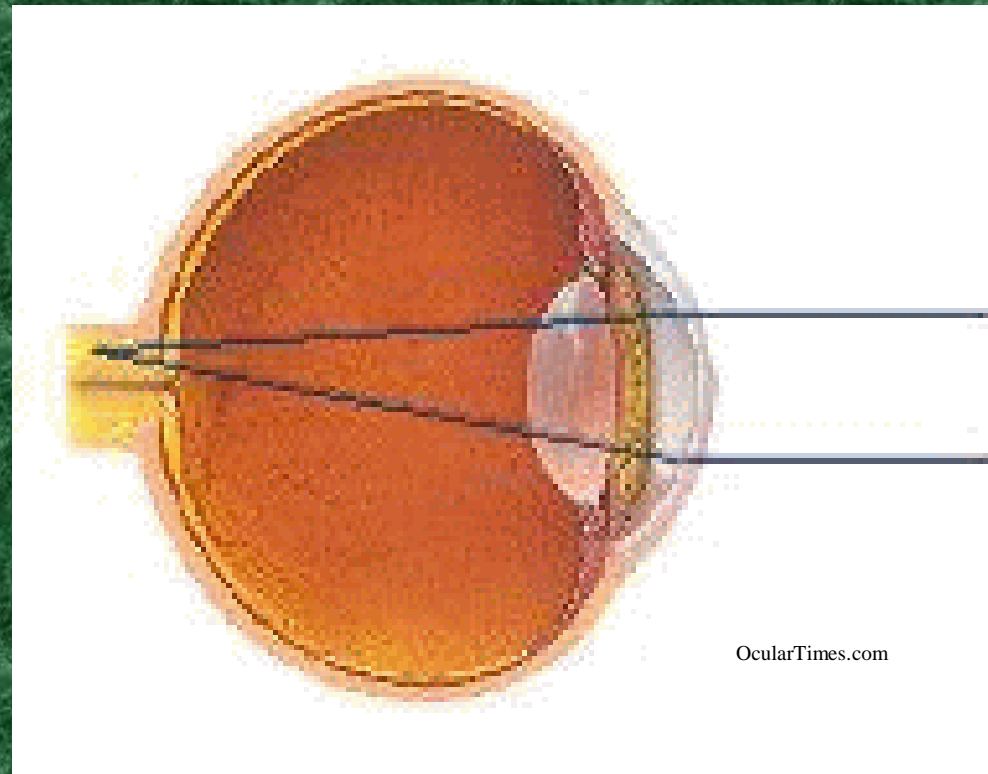
# Optometric Terms

- **Myopia**
  - **Nearsighted**
  - inability to see objects at a distance
  - negative powers



# Optometric Terms

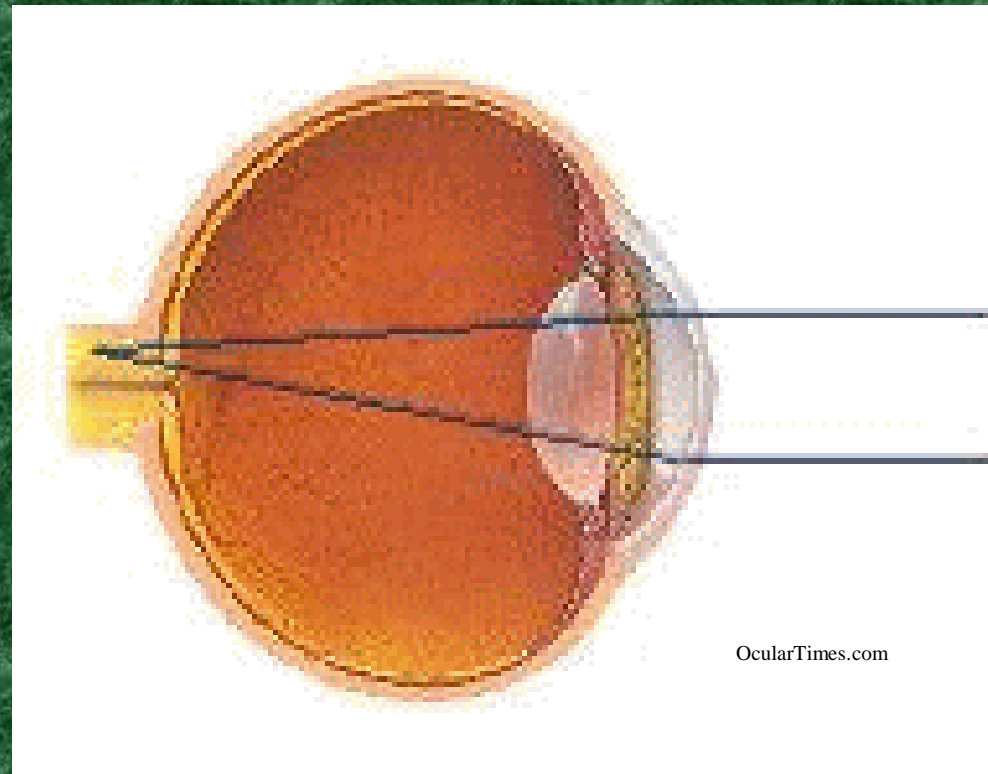
- Hyperopia
  - Farsightedness
  - inability to see near objects
  - positive powers





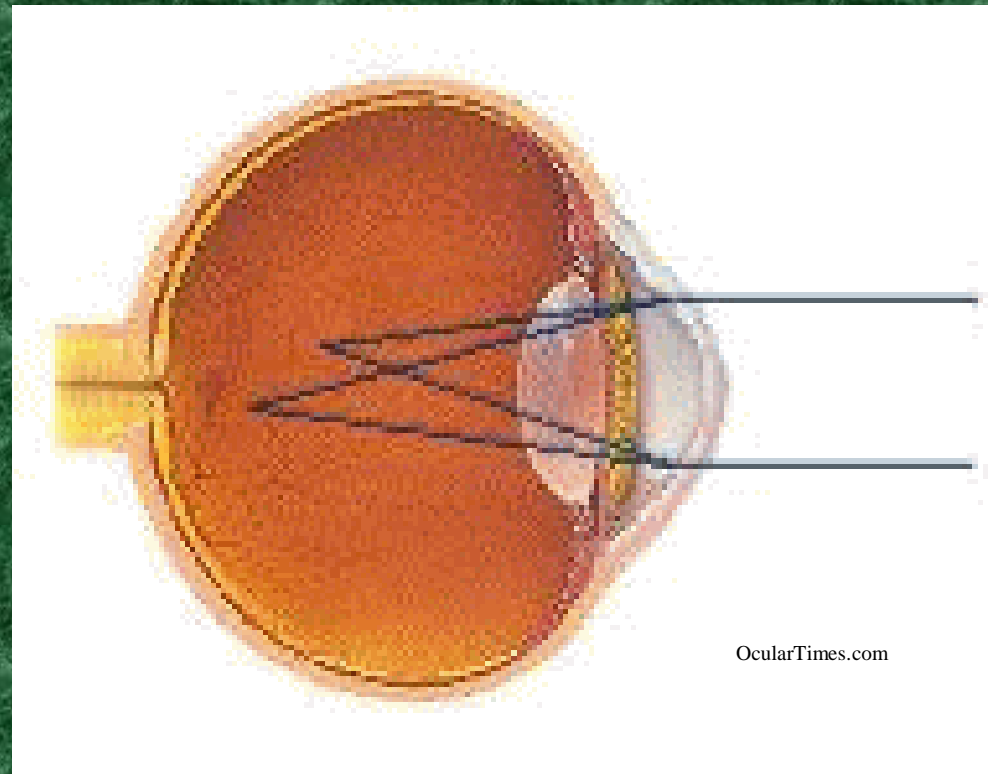
# Optometric Terms

- **Presbyopia**
  - loss of lens elasticity
  - occurs with age
  - corrected with bifocals



# Optometric Terms

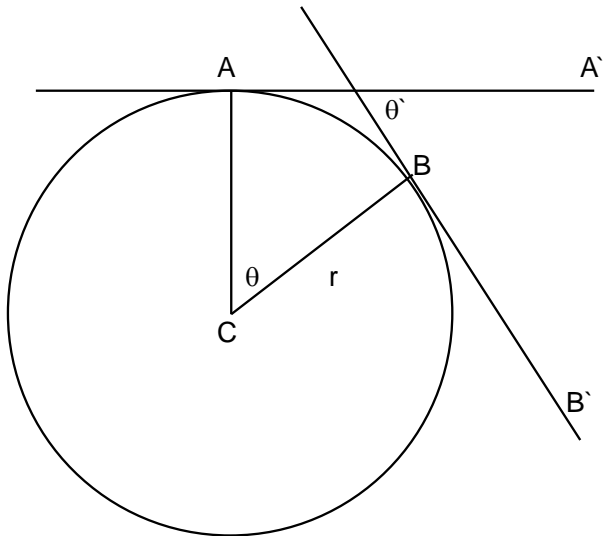
- **Astigmatism**
  - cornea or lens is irregularly shaped
  - scattered light
  - images are distorted



# Optometric Formulas

- **Curvature**

- The curvature of a surface is defined as the angle through which the surface turns in a unit length of arc.



$$\text{Curvature} = \frac{\theta}{\text{arc AB}}.$$

$$\text{Since } \theta, \text{ in radians, } = \frac{\text{arc AB}}{r},$$

$$\text{curvature (R)} = \frac{1}{r}.$$

# Optometric Formulas

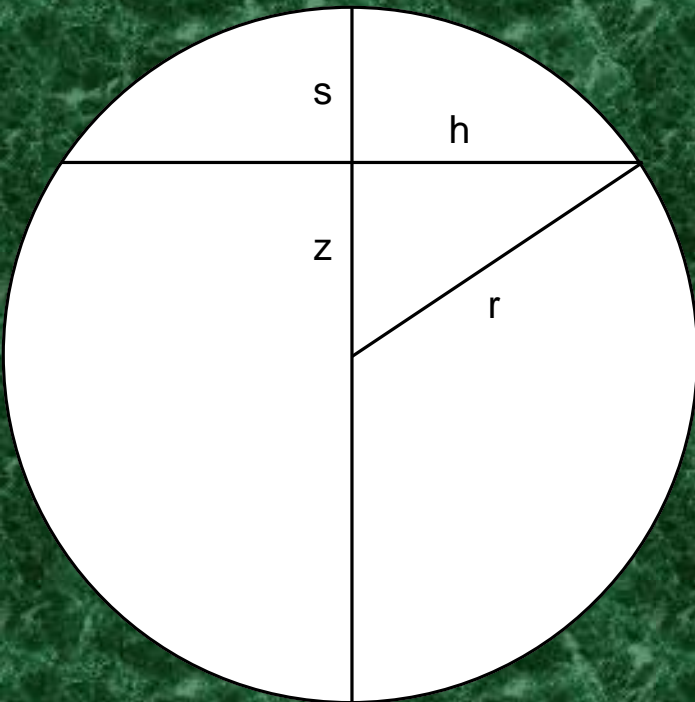
- **Units of Measure**
  - The **Diopter, D** ( $\text{m}^{-1}$ )
    - There is not a specific unit for curvature.
    - Since  $r$ , when applied to lenses will be small, the reciprocal is used.



# Optometric Formulas

- **Sagitta Unit of Curvature**

- Interesting relationship between curvature and the sides of a right triangle.



$$r^2 = h^2 + z^2 \Rightarrow z^2 = r^2 - h^2$$

$$\Rightarrow z = \pm \sqrt{r^2 - h^2}.$$

Since  $s = r - z$ ,

$$s = r \pm \sqrt{r^2 - h^2}$$

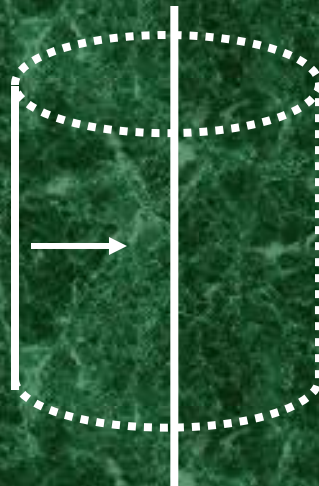
# Surfaces of Revolution

- **Spherical Surface**
  - Used to correct stigmatic eyes.
  - Generated by rotating a circle or arc about one of its diameters



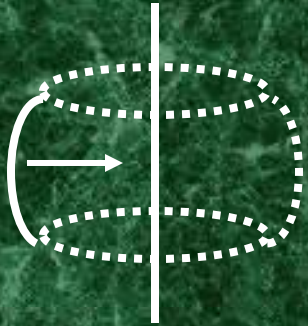
# Surfaces of Revolution

- Cylindrical Surface
  - Used to correct astigmatism.
  - Generated by rotating a straight line about another line parallel to it.

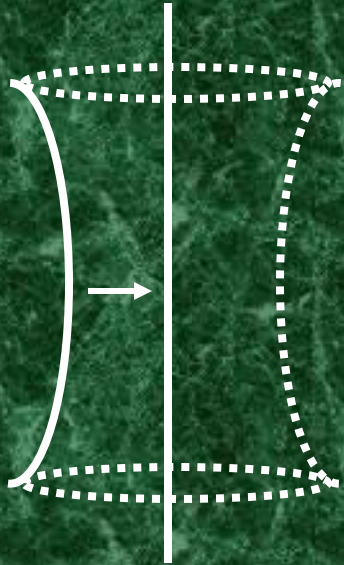


\*No Curvature

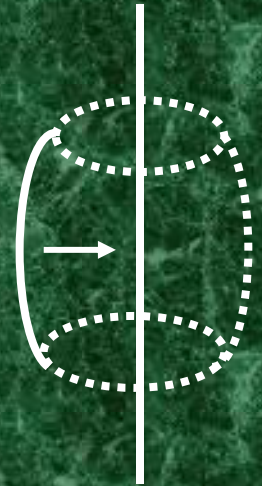
# Surfaces of Revolution



~Donut or Tire Surface



Barrel Surface~

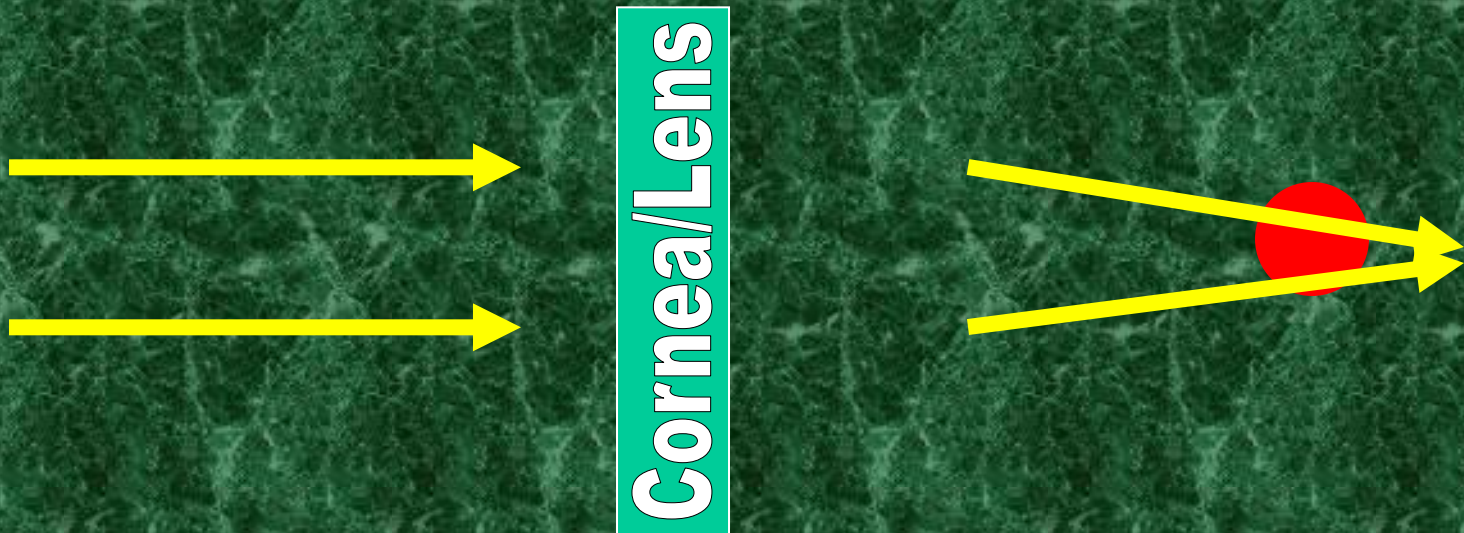


~Capstan Surface



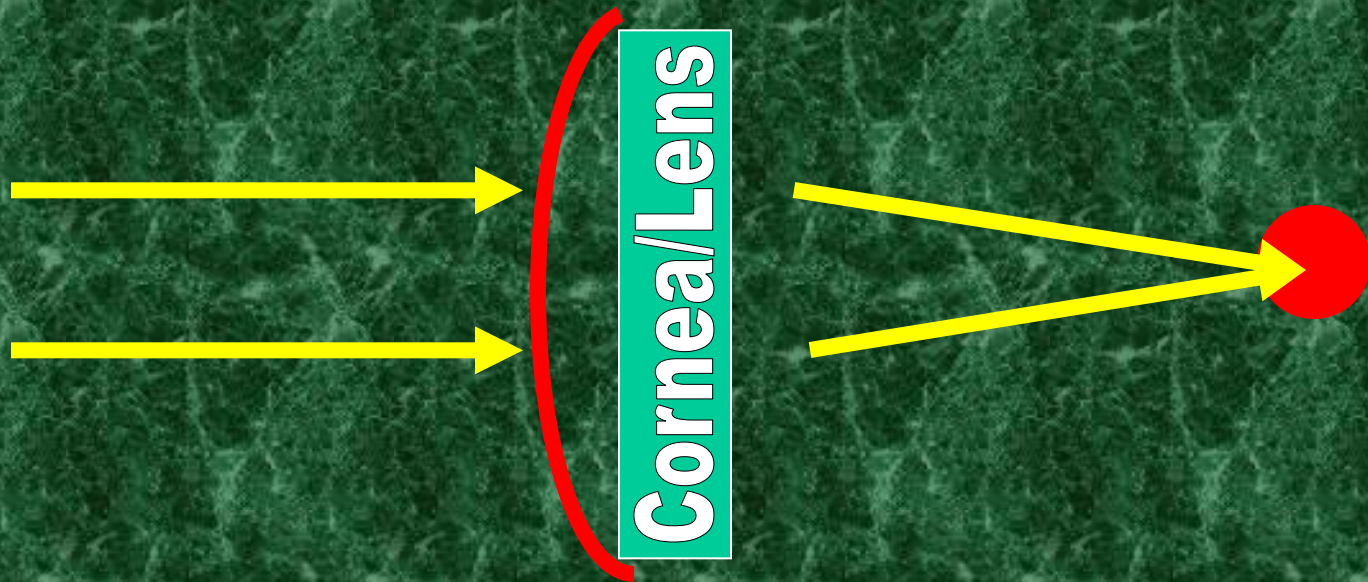
# Refraction Correction

- Use of Convex Lens to Correct Hyperopia
  - Thin on edges, thick in middle
  - Converging Lens



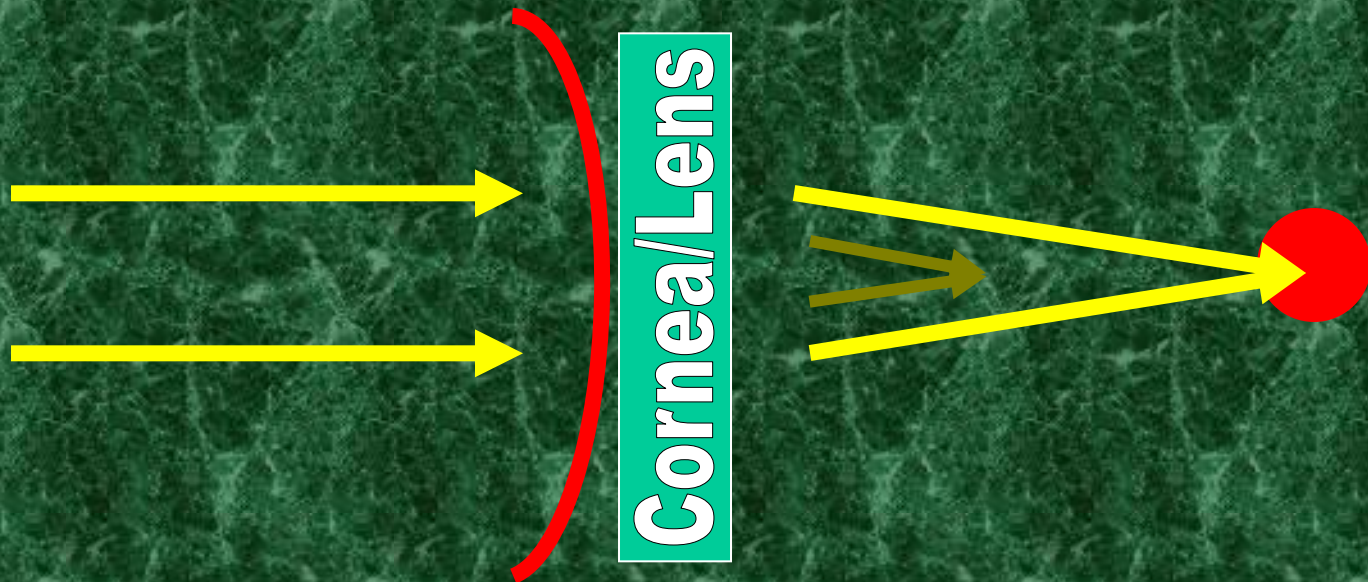
# Refraction Correction

- Use of Convex Lens to Correct Hyperopia
  - Thin on edges, thick in middle
  - Converging Lens



# Refraction Correction

- **Use of Concave Lens to Correct Myopia**
  - Thick on edges, thin in middle
  - Diverging Lens



# Refraction Correction

- How does the lens correct vision?
- What do the numbers on a prescription mean?

**Example:**

	<b>Sph</b>	<b>Cyl</b>	<b>Axis</b>
<b>OD</b>	<b>-4.25</b>	<b>-0.25</b>	<b>075</b>
<b>OS</b>	<b>-4.50</b>		



# Conclusion

- **Future of Optometrics**
  - **Use of Lasers**
    - Currently present in ophthalmology practices
    - Correction and treatment of refractive errors and ocular disease
  - **Invention of new lens materials**
    - Shatter resistant material
    - Thinner lenses for high prescriptions
    - Contact Lens Materials

# Conclusion

- **Why study Optometry?**
  - Do you know anyone with glasses or contact lenses?
  - Our sight, a gift from God, is precious.