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## Promoting mathematical communication and community via Blackboard

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## **Disciplines**

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## **Comments**

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## **Promoting Mathematical Communication and Community via Blackboard**

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### **Abstract:**

Major changes in mathematics pedagogy include writing as pedagogy and the role of community in learning. The classroom community is naturally extended by the use of online discussion boards. In this paper several models for student use of online discussion boards that have been successfully used to promote mathematical discourse are presented. Structured and unstructured examples that are easily adaptable and transportable to a variety of mathematics classroom settings are offered. These assignments facilitate student engagement and interaction outside of the classroom. Assessment, utility, and transferability are offered. Although the authors use the discussion boards provided by Blackboard, this particular software package is not necessary.

**Keywords:** undergraduate mathematics education, communicating and writing mathematics, alternative assessment, online discussion boards

## **Promoting Mathematical Communication and Community via Blackboard**

### **Introduction**

"I thought of a way to introduce the concept of a representative sample to kids. Give them this scenario: You have homework every day of class. You dutifully do your homework every day except for one. This happens to be the only time the teacher collects assignments. She assumes this to be representative of your work and gives a homework grade based on that day. Since you didn't do it, you get a 0 for the year. Have the kids explain whether they think this is fair and why. Appealing to their sense of justice will usually get them interested and thinking."

This student insight was offered as part of an exchange in an online, instructor-mediated discussion board assignment required in a mathematics course for prospective elementary teachers. Many schools have adopted electronic educational platforms such as those offered by Blackboard™ or WebCT™. These course management tools allow instructors to post course information (assignments, grades, etc.) and facilitate course related discourse via online discussion boards. General academic uses of discussion boards vary but include a forum for students to discuss study strategies, get help with course content, or facilitate work on group projects. Academic uses of online discussion boards, however, transcend general academic utility. They are rife with the potential to facilitate students' mathematical communication. The electronic medium provides a conduit for promoting mathematical discourse in a myriad of educational settings, spanning traditional classrooms to distance learning courses.

In recent years the use of communicating and writing mathematics has become an increasingly popular pedagogical tool. This momentum is supported by positions taken in national documents such as the National Council for Teachers of Mathematics *Principles and Standards for School Mathematics* [9] and the *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* [2]. The NCTM advocates that school instructional programs provide opportunities for students to organize their mathematical understanding through communication, use mathematical language to carefully and clearly explain their mathematical thinking, and to examine and assess other's mathematical reasoning

and strategies. The *CUPM Curriculum Guide* provides six general recommendations for mathematics departments; the second of which is “develop mathematical thinking and communication skills” [2, p. 13]. They suggest that courses and programs should foster students’ ability to “read and communicate mathematics with understanding and clarity” [2, p. 16]. Specifically, writing about mathematics helps develop student comprehension and provides instructors with meaningful data about students’ mathematical understanding. In their recommendation, explicit reference is made to communicating in proofs and writing explanations of problem solving processes. Such activities force students to engage in metacognition (thinking about their thinking) and can help them evaluate their thinking process. The *CUPM Curriculum Guide* also advises that mathematics instructors assign smaller writing assignments to aid in evaluating student work and to allow for more frequent engagement in writing.

Carpenter et al [4] view understanding as a dynamic process rather than an unchanging characteristic. Accordingly, they define learning with understanding in terms of mental activities that foster understanding. They propose that mathematical understanding is developed through the following kinds of mental activities: “(a) constructing relationships, (b) extending and applying mathematical knowledge, (c) justifying and explaining generalizations and procedures, and (d) developing a sense of identity related to taking responsibility for making sense of mathematical knowledge” [4, p. 3]. They also indicate that their conception of learning applies to the study of communities of learners. In their research, a variety of educational communities were immersed in the mental activities involved in developing understanding. In school classrooms and professional development communities many members made conjectures and worked together to refute, refine, or validate these conjectures. Consequently, class and teacher communities came to believe that developing knowledge was related to the interactions within the community and that they did not have to depend on the community leader as the facilitator of knowledge. Moreover, in *Education and Learning to Think*, Resnick [10] argues that the “social setting” can cultivate a disposition for higher order thinking. The social setting offers opportunities for demonstrating effective thinking strategies. Expert problem-solvers can model

skilled ways of approaching problems and constructing arguments. Mental activities that are typically hidden are open to scrutiny. When “thinking aloud” in a social setting, members of the community can interact to critique and offer suggestions to improve other’s ideas and strategies. Instead of thinking locally in isolation, unaware of the global significance, individuals comprising the community analyze concepts, solve problems, or dissect an argument together.

Not only are the communication recommendations advocated by the *NCTM Standards* [9] and the *CUPM Curriculum Guide* [2] easily adapted to an online discussion board format, but such an environment also parallels that described by Resnick [10] in support of higher order thinking and Carpenter et al [4] used to foster learning with understanding. Furthermore, current computer software like equation editor or the equation features in Blackboard make it possible for students to include fairly sophisticated mathematical notation without resorting to calculator syntax which can be difficult to follow. Likewise, students can attach word processing or spreadsheet files illustrating mathematical ideas such as different or multiple representations of a functional relationship.

Additionally, publicly accessible writings allow for many minds to read and evaluate the writing for clarity and completeness. If terms are used in a nonstandard way, if notation is unclear, or if assumptions are not explained, there is a tendency for one reader to ignore it by projecting one’s own ideas or understanding, rather than reading what is actually communicated (or not communicated) by the student (for example, see [2] or [7]). With multiple readers, it is more likely than someone will question an unclear or incomplete statement.

In this paper several structured and unstructured prototypes of student use of online discussion boards that have been successfully used to promote mathematical discourse within a community of learners are presented. Assessment specifics, utility, and transferability are suggested. Although the authors used Blackboard, particular software packages are not necessary. Numerous other resources that support online discussion boards are available online.

### **Assignments**

Using Blackboard for Class Notes:

We use discussion boards as a way to collect class notes. Rather than hand out printed notes each class, which can give some students the idea that they do not have to attend or write anything down when they do attend, each student is assigned at least one day of class for which they are responsible. Within some reasonable time after class (24 hours is usually sufficient) that student is required to post a summary of her class notes. If the class is small, each student can be assigned multiple class periods. If the class is large, students can be assigned in groups of two to post summaries.

Other students in the class may have a different perspective on what happened in class, with a slightly different emphasis on the important ideas or how the ideas are related. They can respond to the posting by adding notes or thoughts of their own. Students who miss class can consult the discussion board for notes, rather than searching out someone (usually the instructor) for copies of notes. This speeds up their ability to catch up after missing class, and encourages them to be more proactive as learners. The instructor can monitor the postings for accuracy and pose further questions, such as "Did anyone have an alternative way of solving example 2 that wasn't discussed in class?"

This activity is consistent with the goals described in the Illustrated Resources section of the CUPM guide [2]. Not only are students reading mathematics, they are also reflecting on the class discussion, vetting their notes for "big ideas" and "illustrative examples", thus forcing students to review their notes and synthesize the information once class had ended. Moreover, both the students and the professor have perpetual access to an electronic archive of what has been discussed in the course. At the end of the semester, the instructor has an accurate record of the course that can assist in developing the final exam, documenting one's teaching, or preparing for the next time the course is taught.

Assessing the class notes activity can be as simple as including some points in the participation grade for each post, or it can be as complex as grading it for grammar, mechanics, and content. We recommend, however, that the lighter version be used, since we want to encourage different perspectives on what was important from class and how it relates to other

materials. One can address the correctness and completeness of the postings without getting into specific details. In general, if the student posts the summary on time (within 24 hours of class) and it is reasonably complete and accurate, we recommend full credit. Any errors or misconceptions can then be addressed either by other students in the discussion board or by the instructor by demonstrating the skills of questioning, *weaving* and *summarizing* [5, p. 196]. Other students can then receive some small amount of participation credit for responding to posts, raising questions, and providing meaningful feedback.

Student summaries can be incorporated into any mathematics course and variations are easy to come by. One successful variant is used in a number theory class. Students were each assigned a problem requiring math induction and a portion of an introductory “review” chapter for which they had to read, take notes, and summarize their reading. Students posted the solution to their problem and their notes for the rest of the class. This forces them to re-examine material and brush up on a proving strategy that students often struggle with. As a result the class then had many induction examples to work from and notes on the background material. Alternatively, instructors can provide students with a list of possible items to include in the summary, such as key terms, illustrative examples, proofs, and conceptual discussions. Depending on the course or the student population, one might choose a different set of information for the summary. For example, in courses for mathematics majors, the summaries could highlight mathematical structures and argumentation; whereas in courses for prospective mathematics teachers, suggested emphasis might include aspects regarding the presentation of the material and how it connected with other mathematics content.

Another simple modification of this assignment is to ask the student posting class notes to fill in the details for any mathematical arguments or problem solutions given in class. Many instructors intentionally leave out steps and this method ensures that at least one student filled in the gaps, while simultaneously providing a resource for the rest of the class. Other variations could combine these methods with strategies for getting feedback after class by requiring summaries to address questions often posed in the “minute paper” as described by Angelo and



Cross [1]. Questions such as “What was the most important thing you learned today?”, “What two things are you confused about?” or “What important questions remain unanswered?” supply valuable feedback on student understanding. They also open the discussion to other students, thus promoting mathematical dialogue. For more information on the “minute paper”, including variations, see [1] or [11].

Using Blackboard to Look for Mathematics:

We have students use online discussion forums to post mathematical observations in their daily lives relevant to the mathematical content studied in class. The objective is that students clearly identify and explicitly articulate mathematical ideas related to their observations. This allows students to see mathematics in the real world and gain an appreciation of the same, as well as hone their written communication skills. The intent is to facilitate a student focused mathematical dialogue, driven by student contributions and interests, thus promoting a community of learners reflecting the different interests, abilities, and learning styles of the individuals that comprise the community. Relevant mathematical offerings and connections in all forms are encouraged. Observations may come from other mathematics courses, courses outside of math, or non-academic contexts. Students describe a wide variety of real-life situations in which they have identified or applied the concepts studied in class. They respond to peers’ observations, modify the scenario to make a different comment, and extend or generalize the original statement. Students also pose their own questions to which their classmates can respond.

Discussion board forum topics are aligned with course content, necessitating different forum topics for different courses. For example, in a course focusing on differential calculus, forums might include power functions, exponential functions, trigonometric functions, logarithmic functions, general functions, the derivative, and the definite integral. However, there is no unique way to create forums or to use online discussion boards to promote students’ mathematical discourse. Variants follow from the content, students’ backgrounds, course outcomes, and

desired objectives of the assignment. The assignment can be as open-ended or as directed as desired.

Model examples and questions stemming from class discussions are posted online as necessary, to stimulate dialogue. In many classrooms a large facet of the teacher's role is as a moderator, and asking good questions is invaluable. Strategies used in classroom discourse apply to online discussions. These questions include, but are not limited to: questions for clarification, probing questions, and questions that direct students to extend, generalise, or make connections between the mathematics in different postings. The moderator also contributes by pointing out questions or issues that have not been resolved, as well as summarizing, synthesizing, and generalizing observations that have similar themes. These student-teacher interactions communicate high standards, provide students with regular timely feedback, and facilitate student-student interactions.

Student comments are assessed on the quantity of submissions and the quality of a variety of factors including: mathematical content, mathematical language, clear and thorough explanations, spelling and grammar, and appropriate citations (when applicable). Students are required to post a minimum number of messages, ranging from almost weekly to biweekly. Partitioning the term into cutoff points creates regularly distributed deadlines. Messages posted after each cutoff count toward the next round. Formal feedback is given after each cutoff point.

The only content requirement is that the mathematical content is easily related to or identified in real-world phenomena. Consequently, this activity is a staple in many courses, from the calculus sequence to courses required for prospective teachers. Student assessment data is more authentic and more illustrative of student understanding than that generated from traditional exams or homework.

Using Blackboard for Directed Questioning:

We use online discussion boards for directed questioning in different forms in numerous mathematics courses. The use of directed tasks or questioning can take virtually any form and may be integrated throughout a course or used in specific circumstances. We use directed

questioning throughout the semester in a calculus course. Students are divided into small groups and each group is required to respond to five questions through the term as well as respond to the contributions of their classmates. Students are given approximately two weeks to respond to each question. Questions parallel the course content or major course activities. While most questions are mathematical in nature, some focus on general academic support. For example, in preparation for an upcoming exam students are asked to describe their study strategies. Students are graded individually on the number of messages submitted and the variety of questions to which they. Students are awarded additional points for posting their own questions or responding to a peer's question. This assignment can easily be extended to require that each group come to a consensus, finalizing each question with an agreed upon group response.

We use another directed task to encourage student curiosity and develop students' capacity to answer their own questions. After certain units students are asked to post a question that they are interested in. Students then research the question and post a response. A fact based quiz is given on the content of these question and response messages to ensure that students read these submissions. If desired, the instructor can provide a list of questions from which the students may select topics of interest. Similarly, students share the results of their research on key ideas. For instance, in a discrete mathematics course, students use online discussion boards to post the results of their explorations of key concepts like vacuous truth, tautology, contradiction, paradox, and function. Students research each term, describe the concept in their own words, and provide an illustrative example and citation. When possible, student submissions include an application of the concept to relevant real-world context. In both cases, student messages are graded on correctness and completeness of the on-time postings.

There is no limit to the ways in which directed questioning can be used or the number of good questions and easily accessible resources for the same. To illustrate, at the end of each chapter in *Calculus: Single Variable* by Hughes-Hallett et al [8], there is section entitled "Check your understanding". This section provides plenty of fodder for instructor-driven questions or an

easy reference for students to choose their own questions of interest to respond to using discussion boards. Furthermore, directed questioning can also be used in conjunction with other activities as a vehicle to motivate or moderate mathematical dialogue. In *The Art of Problem Posing*, Brown and Walter [3] extol the virtues of problem posing independent of its vital role in problem solving. They argue that often we do not fully understand the meaning of a solution without posing and investigating additional problems and questions. The authors provide general questions as well as a set of strategies for creating problems to develop students' mathematical habits of mind.

### **Lessons Learned**

The contents of this paper result from our various experiments with the use of online discussion boards to promote mathematical discourse and what we have learned. We have employed the assignments described above in many different mathematics courses for several years now and (hopefully) have learned from our experiences. We offer the following general information in the hope that we can save others some of the trials and errors that we have experienced.

#### **Role of the Instructor:**

Grabe and Grabe [5] describe the four roles that educators play when facilitating discourse in online discussion boards. These roles parallel those in the traditional classroom and include the following. Educators must be a *technical assistant*, helping solve mundane problems with getting online and posting. Creating a log of student questions and instructor responses in a commonly available FAQ is one method for decreasing the amount of time spent in this role. One may also create a forum where students can get technical assistance. It is also critical that the instructor get students online in a non-threatening way, allowing them to get familiar with the system before requiring participation for a grade. We both have students post an "introductory message" in which students share some insight into who they are ("my favorite TV show is *Lost*", "I have three cats", etc.). No mathematical content is required as the intent is to ensure that students are online and to support the instructor's *social role*. It is the instructor's responsibility

to help the students “meet” each other in order to get comfortable communicating. Engagement in this role is largely at the beginning of a semester or series of projects. Throughout the semester instructors need to maintain a *managerial role*, which involves five tasks: identifying the purpose for the discussion, defining the roles of the instructor and the students, keeping the discussion fresh, establishing basic expectations for students, and responding to productive, disruptive, or passive behavior. The final role educators play is the *pedagogical role*. This includes evaluating student performance in the discussions, directing student thinking through questioning strategies, and modeling expectations.

#### Anonymity:

Although most discussion board software gives administrators the option of allowing anonymous postings, we recommend against this for three reasons. First, grading the discussion boards is nearly impossible when the posts are anonymous. Second, there is a greater accountability in the quality of student responses. As with all activities, if you want students to take this more seriously, we have found that it is worth making this a part of the course grade. The third reason is that when students know their name is associated with the post, they tend to act more responsibly. This cuts down on inappropriate online behavior.

#### Too much information:

Depending on the extent to which the discussion boards are used, the discussion board interface can become unwieldy with too many threads or forums. This is easily avoided with a little planning and organization. It is also important to not subject students to an information overload, either in terms of upfront information or in expecting them to read too much through the course. The first can be circumvented by posting follow-up instructions, supplemental information as an organic consequence to students’ questions. Second, in a large class discussions can generate many messages. Expecting students to read and respond to all of them, in addition to their regular homework and readings, can be too much. Most electronic discussion boards allow the moderator to create groups of participants. The discussions can then

take place in several smaller groups, rather than in one large group. This gives more students a chance to participate, as their voices are more likely to be heard.

#### Student Engagement:

The looking for math assignment is an unusually open-ended assignment and as such, students often have difficulty with the first posting. The introductory message is especially useful to increase student comfort and stimulate dialogue. Perhaps it is because they have never been asked to perform such a task or because they do not know "what the instructor wants". We liken it to perusing a fabulous menu when one is ravenously hungry. Everything looks so tasty that it is difficult to narrow down the choices. Once students cut their teeth with their first submission and begin to own this assignment, it becomes much easier for them to contribute to the electronic conversation. The instructor can post example models and starter questions to stimulate dialogue. It is also especially easy to procrastinate in the online environment. Thus we recommend regularly distributed deadlines to establish a routine and increase the time students spend on task.

The class notes activity works best if the instructor posts class summaries for the first two or three classes. This serves as a model for the type of summaries that students should strive for. After students take over, it is necessary for the instructor to read and post responses to each of the first few summaries from students in order to model the follow-up process. The biggest problem encountered is that students can "over-post" and include everything they wrote down on the discussion board as their post, rather than summarizing their notes. This leads some students "tuning out" (even during active learning and hands-on explorations) and not writing anything down or engaging the class at all. To counter this, we are more explicit about the contents of the class summaries. In particular, students are told that each class summary should contain the following information: (1) The date of the class period you are posting notes for should be the subject line of the posting, (2) The essential question or idea of the class period - this should be a short statement, not more than 2-3 sentences, (3) An overview of the activities conducted in class - at most a short sentence on each, (4) A list of key terms and concepts - no

definitions or descriptions, just a list of them, (5) A 3-2-1 Reflection: 3 things you learned in class, 2 questions you have about the concepts and activities, 1 connection between that class and either the real world/daily life or what you will be doing as a prospective teacher. If students meet these criteria they earn 70% credit. To promote dialogue, students earn full credit students by responding to the summaries of their classmates.

#### Assessment:

Examples of student work from previous semesters are used as models for current students, illustrating what to strive for and what to avoid. Examples are supplemented by annotations demonstrating the strengths and weaknesses and suggestions for improvement. Providing an assessment rubric and examples helps convey high standards. Formal and informal feedback is given regularly to communicate expectations, facilitate discourse, and for assessment. As with any assignment, online discussions are enhanced when grades depend on regular participation. In an on-going assignment the instructor does not face a large stack of papers, but instead may offer feedback a little at a time. The second author reviews online messages intermittently through the week whereas the first author integrates it into her morning coffee routine. It is useful to not make the grading scale too restrictive.

Rather than focusing solely on the number of messages submitted, it is better to emphasize the quality of the responses, the interaction (including responding to and extending others' ideas), and provide incentives for students to introduce their own questions. This is supported by Graham et al [6] who suggest that students' grades should depend on participation and posted messages should be assessed on the quality, not the length or number of submissions. For the detailed assessment rubrics that are used please visit the authors' websites.

An easy way to encourage students to review the discussions at the end of the semester while providing the instructor valuable support in the grading process is to have the students submit a short paper (1-2 pages) detailing their participation in the discussion boards. In the reflection students can point out what was helpful and how it helped them, illustrating their commentary with posted message of which they are particularly proud and ones that

demonstrate their achievement of the instructor's stated objectives. This provides a chance for the students to think about their work and minimizes "surprises" in the grading process. It also helps to prevent busy instructors from missing what may be the most significant contributions a particular student makes, since those will surely be highlighted in the paper. Students can also be asked specific questions that assess the value of the discussion boards to their learning.

Student Feedback of the use of online discussion boards:

Student comments suggest that each of the assignments described can contribute positively to students' mathematical learning. Students indicate that they appreciate the easy access to class notes. The perspectives and questions of their peers help students to refine their explanations and observations. While writing about mathematics is often new to students, they see the value in explaining their work and reasoning. Students who are required to write about mathematics in real-life contexts liked the chance to connect what they learned in class to everyday occurrences and demonstrated a widespread appreciation for the applicability of mathematics. A sample student comment illustrating this phenomenon follows.

"This helped me to sort out my own ideas and to be able to put mathematical concepts into words. I also liked using Blackboard to post math ideas from our own lives or problems we have thought of. I think that this pushed us to look beyond math in the classroom and to see it within our world, which is very beneficial. The only hard part was the first Blackboard posting. I think that people were confused as to what we were supposed to write. (I was at first, but it became easier to see math in my life.)"

### **Conclusion**

The online discussion board medium plays a significant role in each of the writing assignments described above. Students develop their critical thinking skills by interacting in a socially constructivist environment. They have time to reflect on and synthesize the mathematical content or apply concepts to real-life contexts. This time-delayed environment allows students to organize their ideas before submitting comments. Each student has the same opportunity to contribute to the discussion. Students too reticent to speak up in class may find their voice in this



time-delayed environment. Peer to peer learning is enhanced as students are learning from one another as well as the teacher. All of which serve to create an active learning environment in which students can improve their writing and the quality of their mathematical discourse and reasoning.

In alignment with recommendations by the *NCTM Standards* [9] and the *CUPM Curriculum Guide* [2], students communicate mathematical ideas and identify appropriate connections. In communicating mathematical ideas or justifying their reasoning to their peers, students gain insights into their thinking. It is easy to envision the utility of online discussion boards as a natural extension of the classroom experience. As the accessibility of online discussion boards expands so increases their pedagogical function. Through problem posing students describe mathematical ideas, as well as extend, analyze modifications of problem contexts, and generalize phenomena and mathematical behaviors. Online discussion boards serve as powerful mechanism for exploring and communicating mathematical concepts and fostering academic community in any mathematics course.

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