Proof of the Riemann hypothesis

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Abstract

I treat Riemann hypothesis as a series and proved it.

Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.

However, I realized that a simple formula before expansion can prove it.

key words

Riemann hypothesis, series, non-trivial zero, critical line

1 introduction

s=c+ix, $0 \leq c \leq 1$, x is non-trivial zero value.
If it is $\zeta(s) = 0$, the Eq.(2) holds.
If it is $\zeta(s) \neq 0$, the Eq.(2) does not hold.
This is an obvious matter.

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{s\pi}{2}\right) \Gamma(1-s) \zeta(1-s) \tag{1}$$

which satisfies:

$$\zeta(s) = \zeta(1-s) \tag{2}$$

Eq.(2) holds only for non-trivial zeros.
Even if the real value of s is 1/2, if the imaginary value is not a non-trivial zero value, the plus and minus of the imaginary value are switched.

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The formula below is Riemann's formula, and the formula above is Euler's formula.

$$\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s) \tag{3}$$

which satisfies:

$$\xi(s) = \xi(1-s) \tag{4}$$

For example:
$\{1/2s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.49 + i14.1347\} = -3.71631... \times 10^{-11} + 8.08549... \times 10^{-11}i$
$\{1/2s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.5 + i14.1347\} = 3.47645... \times 10^{-8} + 3.11925... \times 10^{-18}i$
$\{1/2s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.5 - i14.1347\} = 3.47645... \times 10^{-8} - 3.11925... \times 10^{-18}i$
$\{1/2s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.51 + i14.1347\} = 4.03079... \times 10^{-11} - 8.27127... \times 10^{-11}i$

If it is $\xi(s) = 0$, the Eq.(4) holds.
If it is $\xi(s) \neq 0$, the Eq.(4) does not hold.

2 Discussion

$0 \leq \Re(s) \leq 1$

Define

$$\omega(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s}.... \tag{5}$$

$$\zeta(s) = \frac{2^s}{2^s-2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s} = \frac{2^s-2+2}{2^s-2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s} + \frac{2}{2^s-2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s} \tag{6}$$

$$= \omega(s) + \frac{2}{2^s-2}\omega(s) = \omega(s) + \frac{2}{2^s}\frac{2^s}{2^s-2}\omega(s) = \omega(s) + \frac{2}{2^s}\zeta(s) \tag{7}$$

$$\neq \omega(s) + \frac{2}{2^s}[\omega(s) + \frac{2}{2^s}[\omega(s) + \frac{2}{2^s}[\omega(s) + \frac{2}{2^s}\zeta(s)]]] \tag{8}$$

$$= [1 + (\frac{2}{2^s}) + (\frac{2}{2^s})^2 + (\frac{2}{2^s})^3]\omega(s) + (\frac{2}{2^s})^4\zeta(s) \tag{9}$$

The following is the sum of n+1 terms in the series.

$$\neq [1 + \frac{2}{2^s} + (\frac{2}{2^s})^2 + (\frac{2}{2^s})^3 + ... + (\frac{2}{2^s})^n]\omega(s) + (\frac{2}{2^s})^{n+1}\zeta(s) \tag{10}$$

$$= \omega(s)\frac{1-(\frac{2}{2^s})^n}{1-\frac{2}{2^s}} + (\frac{2}{2^s})^{n+1}\zeta(s) = \omega(s)\frac{1-2^{(1-s)n}}{1-2^{1-s}} + 2^{(1-s)(n+1)}\zeta(s) \tag{11}$$

And from Eq.(11)

$$\zeta(1-s) \neq \omega(1-s)\frac{1-2^{sn}}{1-2^s} + 2^{s(n+1)}\zeta(1-s) \tag{12}$$

$$\omega(1-s) = \frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s) \tag{13}$$

$2^s \neq 0$, $2^s - 2 \neq 0$, $1 - 2^{1-s} \neq 0$, $2^{1-s} \neq 0$

If s is non-trivial zeros.

$$Eq.(11) = Eq.(12) \tag{14}$$

This formula $\zeta(s) = \zeta(1-s)$ is not valid except when the real value is 1/2.
When the real value is 1/2, the real value is the same, but the imaginary value is the opposite of plus or minus.
This formula $\zeta(s) = \zeta(1-s)$ is valid only for non-trivial zeros.

$$\omega(s)\frac{1-(\frac{2}{2^s})^n}{1-\frac{2}{2^s}} + (\frac{2}{2^s})^{n+1}\zeta(s) \neq \omega(1-s)\frac{1-(2^s)^n}{1-2^s} + (2^s)^{n+1}\zeta(1-s) \tag{15}$$

$$(1-\frac{2}{2^s})\zeta(s)\frac{1-(\frac{2}{2^s})^n}{1-\frac{2}{2^s}} + (\frac{2}{2^s})^{n+1}\zeta(s) \neq \frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\frac{1-(2^s)^n}{1-2^s} + (2^s)^{n+1}\zeta(1-s) \tag{16}$$

$$\zeta(s)[1-(\frac{2}{2^s})^n] + (\frac{2}{2^s})^{n+1}\zeta(s) \neq \zeta(1-s)[1-2^{sn}] + (2^s)^{n+1}\zeta(1-s) \tag{17}$$

$$\zeta(s)[1-(\frac{2}{2^s})^n + (\frac{2}{2^s})^{n+1}] \neq \zeta(1-s)[1-2^{sn} + 2^{s(n+1)}] \tag{18}$$

$$\zeta(s)[1-2^{(1-s)n} + 2^{(1-s)(n+1)}] \neq \zeta(1-s)[1-2^{sn} + 2^{s(n+1)}] \tag{19}$$

Calculation was performed here.

Example:

$$\zeta(s)[1-2^{(1-s)n}+2^{(1-s)(n+1)}],\{s=0.5+i14.1347\}=3.13536...\times10^{-6}-0.0000196934...i$$

$$\zeta(1-s)[1-2^{sn}+2^{s(n+1)}],\{s=0.5+i14.1347\}=3.13536...\times10^{-6}+0.0000196934...i$$

$$\zeta(s)[1-2^{(1-s)n}+2^{(1-s)(n+1)}],\{s=0.4+i14.1347\}=-0.0814778...-0.0136953...i$$

$$\zeta(1-s)[1-2^{sn}+2^{s(n+1)}],\{s=0.4+i14.1347\}=0.0753372...-0.0113547...i$$

$$\zeta(s)[1-2^{(1-s)n}+2^{(1-s)(n+1)}],\{s=0.6+i14.1347\}=0.0753372...+0.0113547...i$$

$$\zeta(1-s)[1-2^{sn}+2^{s(n+1)}],\{s=0.6+i14.1347\}=-0.0814778...+0.0136953...i$$

$$\zeta(s)[1-2^{(1-s)n}+2^{(1-s)(n+1)}],\{s=0.5+i15\}=0.14711...+0.704752...i$$

$$\zeta(1-s)[1-2^{sn}+2^{s(n+1)}],\{s=0.5+i15\}=0.14711...-0.704752...i$$

If the real value of s is 1/2 even if s is not a non-trivial zero imaginary value, the real value will match, and the imaginary value will be the opposite of plus or minus.

As above, in Eq.(19), if s is 1/2+it(t is not non-trivial imaginary value), both sides have the same real value, the imaginary value is the opposite of plus or minus.

If s=1/2. The left and right values are the same in Eq.(19).
However, what happens when it is a complex number is a problem.

$\zeta(s)=\zeta(1-s)$ holds only when s is non-trivial zeros.
If s is not non-trivial zero, the left and right expressions are never equal.

The calculations so far are based on the assumption that $\zeta(s)=\zeta(1-s)$ holds.
In other words, the above formula holds only when s is non-trivial zero.
If s=1/2+it(t is not non-trivial imaginary value), the real value are equal. The plus and minus of the imaginary value are switched.

To be precise, $\zeta(s)=\zeta(1-s)$ is valid only for non-trivial zeros.
This is because the value of ζ at a non-trivial zero value is zero.
This is an expression showing the possibility that there are non-trivial zero values at equal intervals from 1/2 to the same imaginary value on both sides of $\Re(1/2)$.

Riemann hypothesis asks whether all non-trivial zeros are real parts 1/2.
It was shown that the non-trivial zero of Riemann hypothesis is not possible except for the real part 1/2.

The above indicates that the value of s when $\zeta(s)=\zeta(1-s)$ is 1/2.
That is, when the value of s is other than 1/2, a non-trivial zero value cannot be obtained.

$$\Re(s) = \frac{1}{2} \tag{20}$$

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References

[1] B.Riemann.: Uber die Anzahl der Primzahlen unter einer gegebenen Grosse, Mon. Not. Berlin Akad pp.671-680, 1859

[2] John Derbyshire.: Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003

[3] S.Kurokawa.: Riemann hypothesis, Japan Hyoron Press, 2009

[4] Marcus du Sautoy.: The Music of The Primes, Zahar Press, 2007

[5] T.Takami.: Consideration of the Riemann hypothesis, viXra:1905.0546

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