Practice Explanations

I mentioned in class yesterday that the test would be mostly, but not entirely, calculations. The other questions will be explanations, and/or perhaps applications of theorems. Explanations are, in my opinion, an important but under-utilized method of teaching and testing mathematics. Explaining concepts helps solidify understanding, and requires a deeper understanding of the material than calculations alone. Unfortunately, the homework does not include explanations, since it is online.

Some students, understandably, expressed some concern, especially since most of you have probably not had to do many or any explanations in earlier math classes. This worksheet is to help give you some practice. It will not be graded, but it should give you an idea of the kind of questions I could/would ask on the midterms and final. I will post answers to them on Monday, to give you some time to try them on your own. These questions were not as carefully thought about in regards to difficulty and length as they would be on a midterm.

To be clear, explanations are not proofs, but they are similar in many circumstances. One main difference is that in an explanation, an example is usually sufficient/useful, while in a proof, you must deal with all cases generally. The other (related) main difference is that a proof must be rigorous, while an explanation allows a certain amount of hand waving. Also, a good explanation always includes words, like you were explaining to another person.

That should be the goal of any explanation you write. It should be clear enough so that, if you gave it to someone who was in the class, but didn't know the specific thing you were explaining, that they could learn that material from your explanation.

1. Suppose y_1 and y_2 are both solutions to the same homogenous, second order, linear differential equation. Explain why, as long as y_1 and y_2 are not multiples of each other, that $y = c_1y_1 + c_2y_2$ can satisfy any initial condition by choosing c_1 and c_2 correctly.

   (Since this is an explanation (and not a proof), you may use an example to show why this is true, rather than trying to show it in the general case. Remember, though, that an explanation is more than just a calculation. You should try to explain this as if to someone who didn't know why.)

2. Suppose the characteristic equation of the differential equation $ay'' + by' + cy = 0$ has two (distinct) real roots. When does the solution $y(t)$ go to zero at infinity (i.e., when is $\lim_{t\to\infty} y(t) = 0$)? Why do we know this?

3. Suppose $B(x, y)$ is a function such that $B_x(x, y) = M(x, y)$ and $B_y(x, y) = N(x, y)$. Explain why $B(x, y) = c$ is the solution of the differential equation $M(x, y) + N(x, y)\frac{dy}{dx} = 0$. Also, explain why we know that $\frac{\partial}{\partial y}M = \frac{\partial}{\partial x}N$.