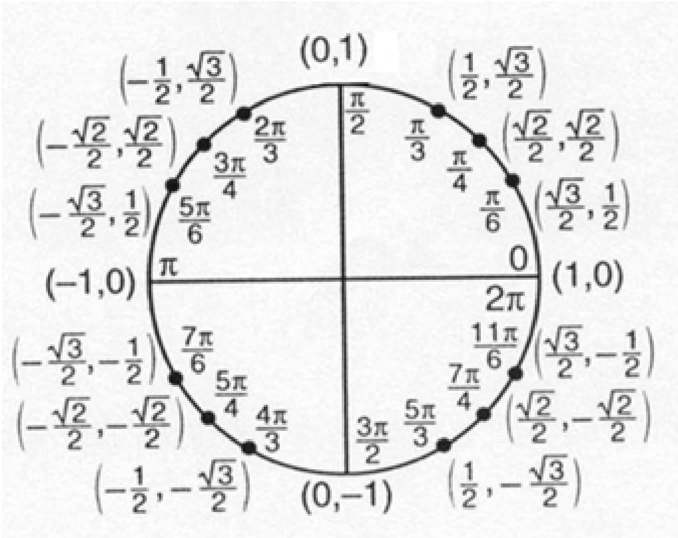
# Sec. 6.1 The Unit Circle

(The relationship between angles (measured in radians) and point on a circle of radius 1.)

# Sec. 6.2 Trigonometric Functions of Real Numbers

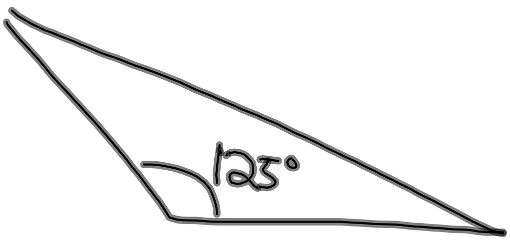
(Defining the trig functions in terms of a number, not an angle.)

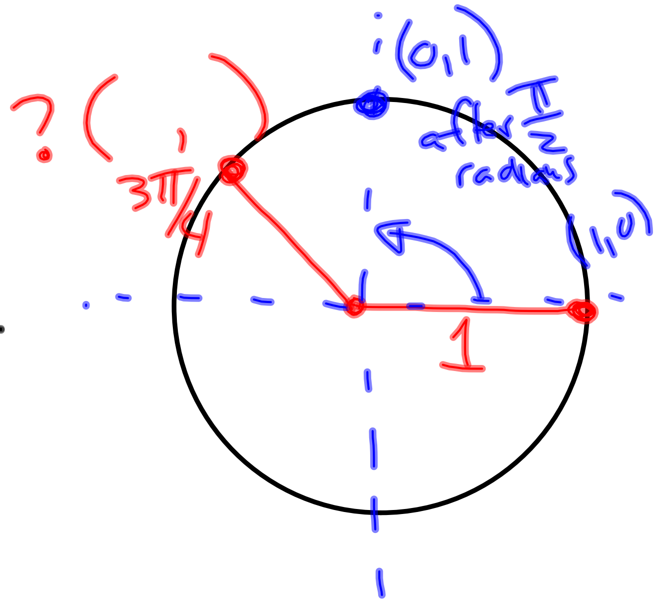



# Sec. 6.1 The Unit Circle
## Terminal Points on the Unit Circle

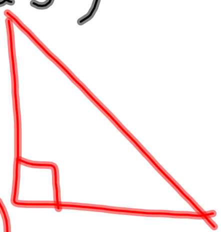
Start at the point (1,0) on a unit circle. Walk (counterclockwise) for a distance of ‘t’ units. The point you end up at is called the “terminal point” P(x,y).

Ex.



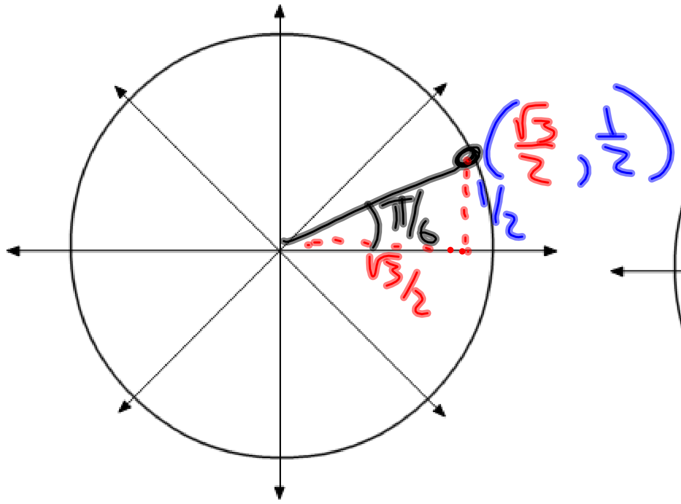

Sin(125°)

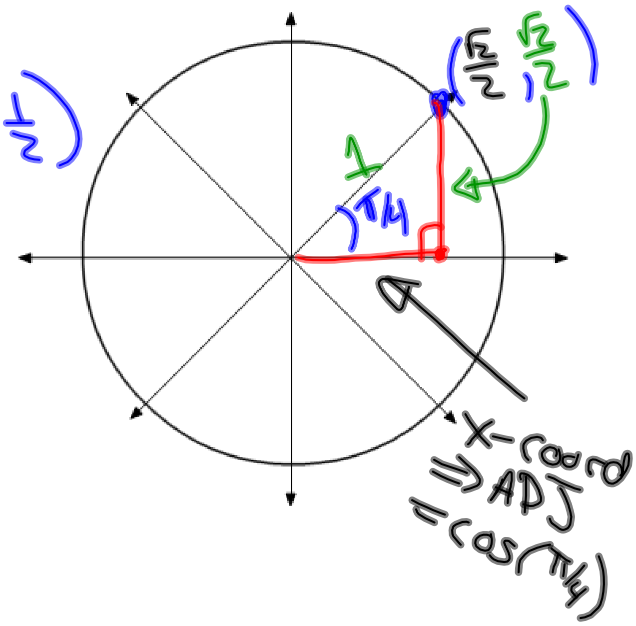


Sin(90°)

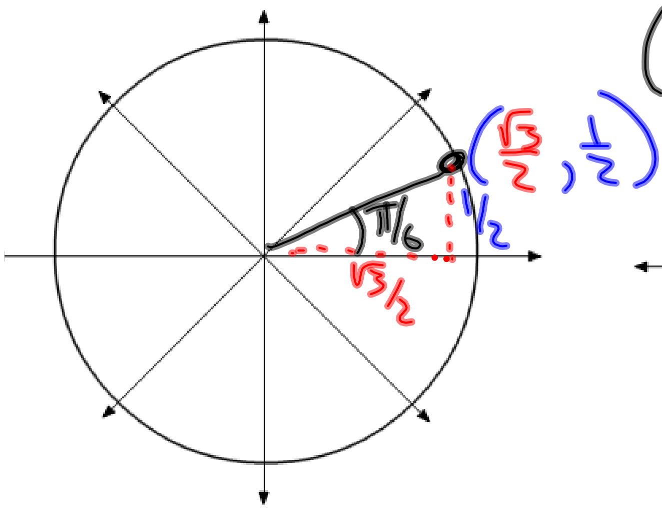
# Sec. 6.1 The Unit Circle
## Terminal Points on the Unit Circle

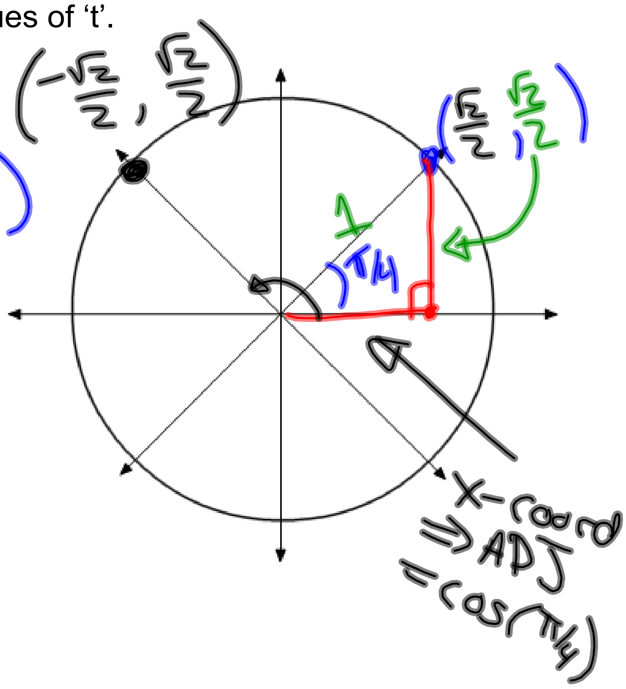
Using Right Triangle Trig Functions (from Sec. 5.2) we can find the terminal points for different values of 't'.




Sec. 6.1 The Unit Circle
Terminal Points on the Unit Circle
Using Right Triangle Trig Functions (from Sec. 5.2) we can find the terminal points for different values of 't'.



## Sec. 6.1 The Unit Circle

### Using Reference Numbers to Find Terminal Points

For values of 't' outside the range [0,π/2] we can find the terminal points based on the 'corresponding' terminal point in the first quadrant.

Let t be a real number. The reference number t' associated with t is the shortest distance along the circle between the terminal point 't' and the x-axis.

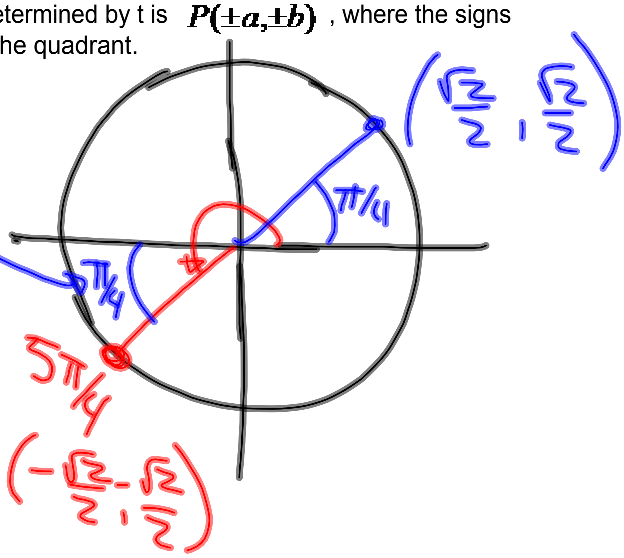
- Find the reference number t'
- Find the terminal point Q(a,b) determined by t'
- The terminal point determined by t is $P(\pm a, \pm b)$ , where the signs are chosen based on the quadrant.

$t = \dfrac{5\pi}{4}$

$t' =$

$Q(x, y) =$

$P(x, y) =$

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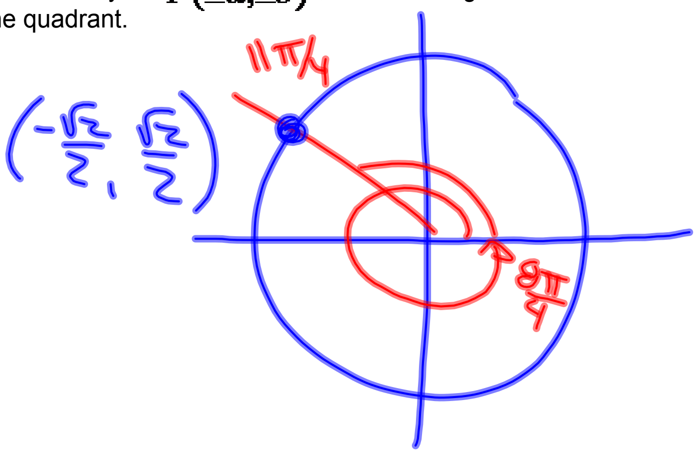
- Find the reference number t'
- Find the terminal point Q(a,b) determined by t'
- The terminal point determined by t is $P(\pm a, \pm b)$ , where the signs are chosen based on the quadrant.

$t = \dfrac{11\pi}{4}$

$t' =$

$Q(x, y) =$

$P(x, y) =$

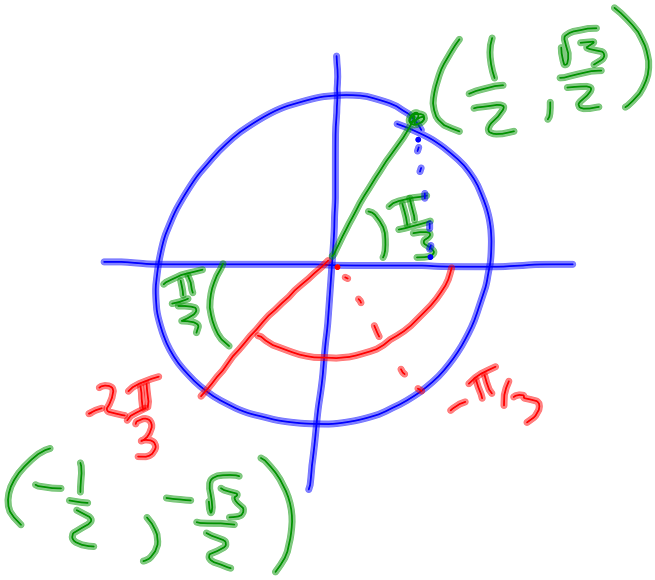


$t = -\frac{2\pi}{3}$

$t' =$

$Q(x, y) =$

$P(x, y) =$




$\sin(\pi/3) = \frac{OPP}{1} = \frac{Y\text{-value}}{1} = \frac{\sqrt{3}}{2}$

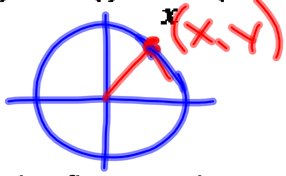
$\sin\left(-\frac{2\pi}{3}\right) = ? = \frac{Y\text{-value}}{1} = -\frac{\sqrt{3}}{2}$

Sec. 6.2 Trigonometric Functions

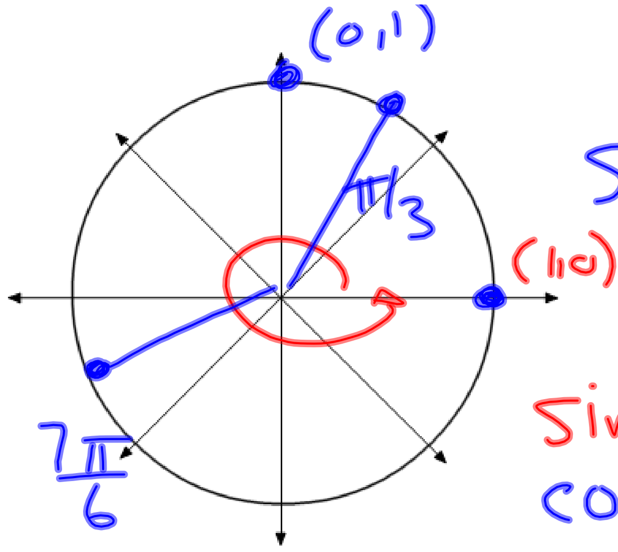
Let t be any real number and P(x,y) the terminal point determined by it. Define the trig functions as follows:

$\sin(t) = y = \frac{OPP}{1}$ $\quad \cos(t) = x = \frac{ADJ}{1}$ $\quad \tan(t) = \frac{y}{x} \quad (x \neq 0) \; \frac{OPP}{ADJ}$

$\csc(t) = \frac{1}{y} \quad (y \neq 0)$ $\quad \sec(t) = \frac{1}{x} \quad (x \neq 0)$ $\quad \cot(t) = \frac{x}{y} \quad (y \neq 0)$




Note that if P(x,y) is in the first quadrant, we can make a right triangle and let ‘x’ be the length of the adjacent and ‘y’ be the length of the opposite. The hypotenuse has length ‘1’. So this is no different that the definitions we had before. But now they are more general.




$\sin(0) = 0$

$\cos(0) = 1$

$\sin(2\pi) = 0$ (y-value)

$\cos(2\pi) = 1$

$\sin(7\pi/6) = -\frac{1}{2}$

$\cos(7\pi/6) = -\frac{\sqrt{3}}{2}$

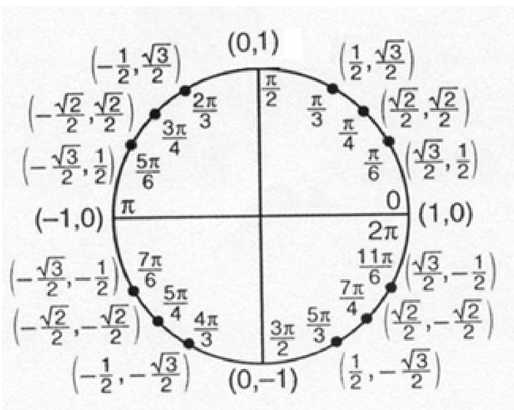
$\sin(\pi/2) = 1$ (Top of circle)

$\cos(\pi/2) = 0$

# Sec. 6.2 Trigonometric Functions

## Special Values

| $t$ | $\sin t$ | $\cos t$ | $\tan t$ | $\csc t$ | $\sec t$ | $\cot t$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | – | 1 | – |
| $\pi/6$ | $1/2$ | $\sqrt{3}/2$ | $1/\sqrt{3}$ | 2 | $2/\sqrt{3}$ | $\sqrt{3}/1$ |
| $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |
| $\pi/3$ | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}/1$ | $2/\sqrt{3}$ | 2 | $1/\sqrt{3}$ |
| $\pi/2$ | 1 | 0 | – | 1 | – | 0 |



# Sec. 6.2 Trigonometric Functions

Domains:

sin, cos

tan, sec (x value in denominator)

cot, csc (y-value in denominator)

Quadrants where Trig Functions are positive

# Sec. 6.2 Trigonometric Functions

Even and Odd Properties:

$$\underset{odd}{\sin(-t) = -\sin(t)} \quad \underset{even}{\cos(-t) = \cos(t)} \quad \underset{odd}{\tan(-t) = -\tan(t)}$$

Fundamental Identities

$$\csc(t) = \frac{1}{\sin(t)} \quad \sec(t) = \frac{1}{\cos(t)} \quad \cot(t) = \frac{1}{\tan(t)}$$

$$\tan(t) = \frac{\sin(t)}{\cos(t)} \quad \cot(t) = \frac{\cos(t)}{\sin(t)}$$

$$\sin^2(t) + \cos^2(t) = 1 \quad \tan^2(t) + 1 = \sec^2(t) \quad 1 + \cot^2(t) = \csc^2(t)$$