

# Double and Half Angle Formulas



# Preliminaries and Objectives

## Preliminaries

- Be able to derive the double angle formulas from the angle sum formulas
- Inverse trig functions
- Simplify fractions
- Rationalize the denominator

## Objectives

- Use the double angle formulas to find specific values

# Double and Half Angle Formulas

$$\sin(2A) =$$

$$\sin\left(\frac{A}{2}\right) =$$

$$\cos(2A) =$$

$$\cos\left(\frac{A}{2}\right) =$$

$$=$$

$$=$$

$$\tan\left(\frac{A}{2}\right) =$$

$$\tan(2A) =$$

$$=$$

# Double and Half Angle Formulas

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

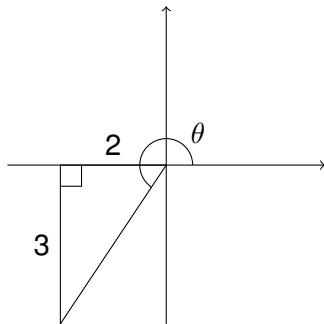
$$= \frac{1 - \cos A}{\sin A}$$

# Double and Half Angle Formulas

Find  $\sin(2\theta)$ ,  $\tan(2\theta)$  and  $\tan(\frac{\theta}{2})$  if  $\tan \theta = \frac{3}{2}$ , where  $\pi < \theta < \frac{3\pi}{2}$

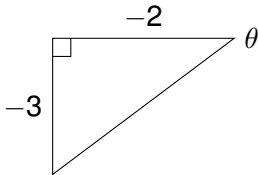
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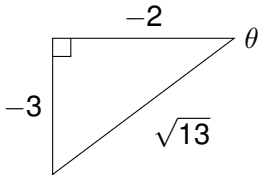
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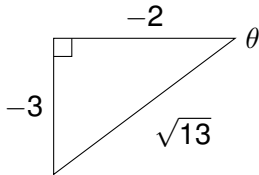


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Solution:

$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$



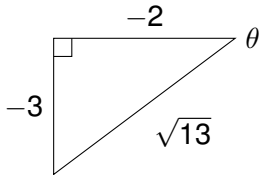
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$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left( \frac{-3}{\sqrt{13}} \right) \left( \frac{-2}{\sqrt{13}} \right) \\ &= \frac{12}{13}\end{aligned}$$



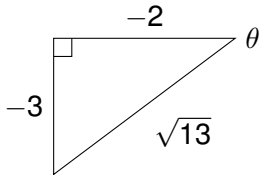
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Solution:

$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$

$$\begin{aligned}\tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2(\frac{3}{2})}{1 - (\frac{3}{2})^2} = \frac{3}{-\frac{5}{4}} = -\frac{12}{5}\end{aligned}$$



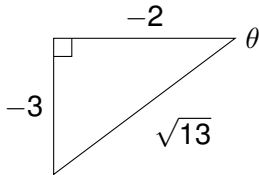
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$$\begin{aligned}\tan\left(\frac{\theta}{2}\right) &= \frac{\sin \theta}{1 + \cos \theta} \\ &= \frac{-\frac{3}{\sqrt{13}}}{1 + \frac{-2}{\sqrt{13}}} = \frac{-2 - \sqrt{13}}{3}\end{aligned}$$



## Previous Answer with slower algebra

$$\begin{aligned}\frac{-\frac{3}{\sqrt{13}}}{1 - \frac{2}{\sqrt{13}}} &= \frac{-\frac{3}{\sqrt{13}} \cdot \sqrt{13}}{\left(1 - \frac{2}{\sqrt{13}}\right) \cdot \sqrt{13}} = \frac{-3}{\sqrt{13} - 2} \\&= \frac{-3}{(\sqrt{13} - 2)} \frac{(\sqrt{13} + 2)}{(\sqrt{13} + 2)} = \frac{-3\sqrt{13} - 6}{13 - 4} \\&= \frac{3(-\sqrt{13} - 2)}{9} = \frac{-2 - \sqrt{13}}{3}\end{aligned}$$

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$$\sin(2\theta) = 2 \sin \theta \cos \theta$$



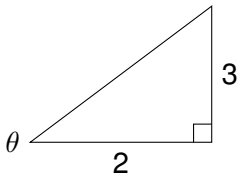
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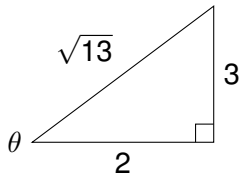
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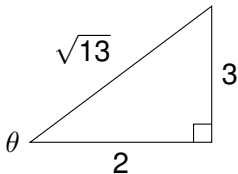
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$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\sin \theta = \frac{3}{\sqrt{13}}, \cos \theta = \frac{2}{\sqrt{13}}$$



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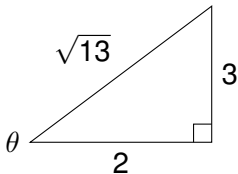
Solution:

Let  $\theta = \tan^{-1} \frac{3}{2}$  so that we are asked  
to find  $\sin(2\theta)$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\sin \theta = \frac{3}{\sqrt{13}}, \cos \theta = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \sin(2\theta) = 2 \left( \frac{3}{\sqrt{13}} \right) \left( \frac{2}{\sqrt{13}} \right) = \frac{12}{13}$$



# Recap

- Draw and label triangles for each given trig value
- Use Pythagorean Theorem to find missing lengths
- Write down the appropriate formula
- Plug in values of trig functions from the triangles in steps 1 and 2
- Simplify

# Credits

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