

## The laws of indices

### Introduction

A **power**, or an **index**, is used to write a product of numbers very compactly. The plural of index is **indices**. In this leaflet we remind you of how this is done, and state a number of rules, or laws, which can be used to simplify expressions involving indices.

### 1. Powers, or indices

We write the expression

$$3 \times 3 \times 3 \times 3 \quad \text{as} \quad 3^4$$

We read this as 'three to the power four'.

Similarly

$$z \times z \times z = z^3$$

We read this as 'z to the power three' or 'z cubed'.

In the expression  $b^c$ , the **index** is  $c$  and the number  $b$  is called the **base**. Your calculator will probably have a button to evaluate powers of numbers. It may be marked  $x^y$ . Check this, and then use your calculator to verify that

$$7^4 = 2401 \quad \text{and} \quad 25^5 = 9765625$$

### Exercises

1. Without using a calculator work out the value of

a)  $4^2$ ,    b)  $5^3$ ,    c)  $2^5$ ,    d)  $\left(\frac{1}{2}\right)^2$ ,    e)  $\left(\frac{1}{3}\right)^2$ ,    f)  $\left(\frac{2}{5}\right)^3$ .

2. Write the following expressions more concisely by using an index.

a)  $a \times a \times a \times a$ ,    b)  $(yz) \times (yz) \times (yz)$ ,    c)  $\left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right)$ .

### Answers

1. a) 16,    b) 125,    c) 32,    d)  $\frac{1}{4}$ ,    e)  $\frac{1}{9}$ ,    f)  $\frac{8}{125}$ .

2. a)  $a^4$ ,    b)  $(yz)^3$ ,    c)  $\left(\frac{a}{b}\right)^3$ .

### 2. The laws of indices

To manipulate expressions involving indices we use rules known as the **laws of indices**. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own! The three most important laws are given here:

**First law**

$$a^m \times a^n = a^{m+n}$$

When expressions with the same base are multiplied, the indices are added.

**Example**

We can write

$$7^6 \times 7^4 = 7^{6+4} = 7^{10}$$

You could verify this by evaluating both sides separately.

**Example**

$$z^4 \times z^3 = z^{4+3} = z^7$$

**Second Law**

$$\frac{a^m}{a^n} = a^{m-n}$$

When expressions with the same base are divided, the indices are subtracted.

**Example**

We can write

$$\frac{8^5}{8^3} = 8^{5-3} = 8^2 \quad \text{and similarly} \quad \frac{z^7}{z^4} = z^{7-4} = z^3$$

**Third law**

$$(a^m)^n = a^{mn}$$

Note that  $m$  and  $n$  have been multiplied to yield the new index  $mn$ .

**Example**

$$(6^4)^2 = 6^{4 \times 2} = 6^8 \quad \text{and} \quad (e^x)^y = e^{xy}$$

It will also be useful to note the following important results:

$$a^0 = 1, \quad a^1 = a$$

**Exercises**

1. In each case choose an appropriate law to simplify the expression:

a)  $5^3 \times 5^{13}$ ,    b)  $8^{13} \div 8^5$ ,    c)  $x^6 \times x^5$ ,    d)  $(a^3)^4$ ,    e)  $\frac{y^7}{y^3}$ ,    f)  $\frac{x^8}{x^7}$ .

2. Use one of the laws to simplify, if possible,  $a^6 \times b^5$ .

**Answers**

1. a)  $5^{16}$ ,    b)  $8^8$ ,    c)  $x^{11}$ ,    d)  $a^{12}$ ,    e)  $y^4$ ,    f)  $x^1 = x$ .

2. This cannot be simplified because the bases are not the same.