Mathematica Code For the Moments and Cross-Moments of the MNTS Distribution

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Abstract

This describes the two Mathematica notebooks used to calculate the moments and cross-moments of the elements of a Multivariate Normalized Tempered Stable (MNTS) distributed random vector. Details of the distribution and the expressions involved can be found in M. Kolossiatis, J.E. Griffin and M.F.J. Steel, Modelling overdispersion with the Normalized Tempered Stable Distribution, CRiSM Working Paper, University of Warwick, 2009.
In the following, we assume that $\mathbf{W} = (W_1, W_2, \ldots, W_n) \sim \text{MNTS}(\nu_1, \nu_2, \ldots, \nu_{n+1}; \kappa)$.

1 Moments

The notebook called momentsMNTS.nb contains the commands in Mathematica for calculating the N-th moment (where $N \in \mathbb{N}$) of any element of $\mathbf{W}$, say W_i. In the code, NN denotes the moment to be calculated (i.e. we calculate $E\left(W_i^{NN}\right)$ here), nu denotes the corresponding ν_i, s denotes the sum $S = \sum_{j=1}^{n+1} \nu_j$ and k is used for the parameter κ. After defining the expressions used in the calculation of the moment and the values of the parameters, an algebraic expression for the moment is given by the the command

$$result = moment[NN, nu, s, k];$$

Note: the semicolon used at the end of this command is just to suppress the output (since for moments of high order, these expressions can be quite long) and can be removed, if required.
This algebraic expression can be further simplified (using FullSimplify[result]) or numerically approximated to an arbitrary number of decimal places (using N[result,q], with q denoting the number of decimal places to be used).

2 Cross-Moments

The notebook CrossmomentsMNTS.nb contains the Mathematica commands for the cross-moments of two elements of $\mathbf{W}$, say W_i and W_j. As before, s denotes the sum $S = \sum_{j=1}^{n+1} \nu_j$

and k is the parameter κ. We also have $nu1$ and $nu2$ denoting ν_i and ν_j, respectively, and $NN1, NN2 \in \mathbb{N}$ denoting the corresponding moments of W_i and W_j $\Big($i.e. we calculate $E\left(W_i^{NN1} W_j^{NN2}\right)\Big)$. After defining the expressions used in the calculation of the cross-moment and the values of the parameters, an algebraic expression for the cross-moment is given by the command

$$result = crossmoment[NN1, NN2, nu1, nu2, s, k];$$

This result can be further simplified or numerically approximated, using the two commands mentioned above.