

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

7.G.4

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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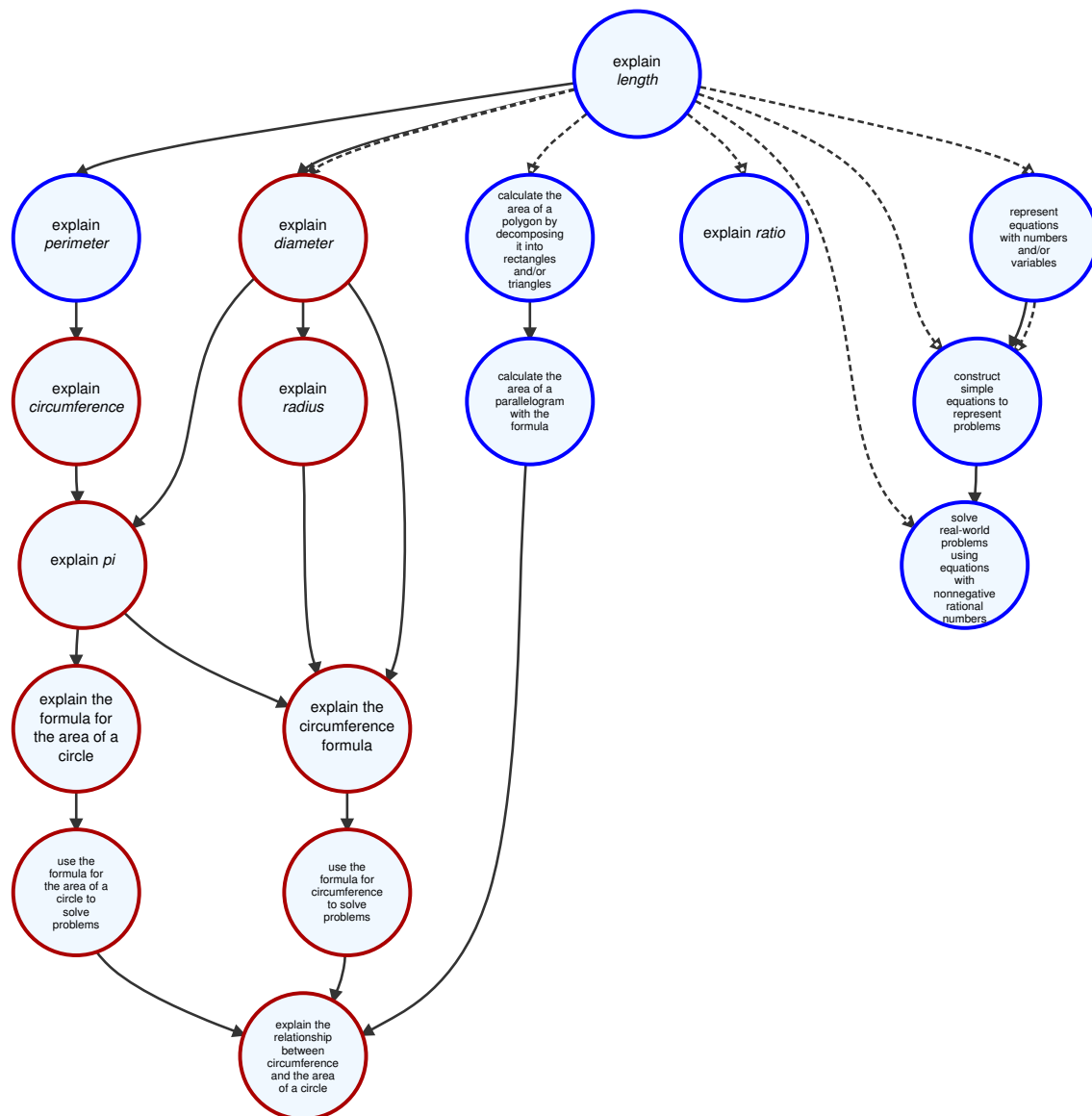
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EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

LEARNING MAP INFORMATION

STANDARDS

7.G.4 Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.



Node Name	Node Description
CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA	Calculate the area of a parallelogram using the formula $A = bh$, where b is the base of the parallelogram and h is the height of the parallelogram.
CALCULATE THE AREA OF A POLYGON BY DECOMPOSING IT INTO RECTANGLES AND/OR TRIANGLES	Using words, drawings, models, etc., decompose a polygon (including special quadrilaterals) into rectangles and/or triangles and calculate the area of the polygon by summing the areas of the rectangles and triangles.
CONSTRUCT SIMPLE EQUATIONS TO REPRESENT PROBLEMS	Construct simple equations (one-step or two-step) to represent real-world or mathematical problems.
EXPLAIN CIRCUMFERENCE	Make known your understanding through words, drawings, concrete models, etc., that circumference is the perimeter of a circular area.
EXPLAIN DIAMETER	Make known your understanding through words, drawings, concrete models, etc., that the diameter of a circle is any straight line that passes through the center and touches the circle at each endpoint.
EXPLAIN LENGTH	Make known your understanding through words, drawings, manipulatives, etc., that length is the distance along a path between two points on that path.
EXPLAIN PERIMETER	Make known your understanding through words, drawings, concrete models, etc., that perimeter is a path (length) that surrounds or encloses a plane area.
EXPLAIN PI	Make known your understanding through words, drawings, concrete models, etc., that pi is the ratio of the circumference of a circle to its diameter.
EXPLAIN RADIUS	Make known your understanding through words, drawings, concrete models, etc., that the radius of a circle is any straight line from the circumference of the circle to the center.
EXPLAIN RATIO	Make known your understanding through words, drawings, manipulatives, etc., that a ratio represents a multiplicative comparison of two quantities or the joining of two quantities into a composed unit. For example, the ratio of eyes to nose on a person is 2:1, because for every two eyes there is one nose.
EXPLAIN THE CIRCUMFERENCE FORMULA	Make known your understanding through words, drawings, manipulatives, etc., that the formula of the circumference of a circle is $2(\pi)r$ or $(\pi)d$.
EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE	Make known your understanding through words, drawings, manipulatives, etc., that the formula for the area of a circle is $(\pi)r^2$.
EXPLAIN THE RELATIONSHIP BETWEEN CIRCUMFERENCE AND THE AREA OF A CIRCLE	Make known your understanding through words, drawings, manipulatives, etc., of the relationship between circumference and area of a circle.
REPRESENT EQUATIONS WITH NUMBERS AND/OR VARIABLES	Through writing or an appropriate assistive technology, represent equations with numbers, variables, or both.
SOLVE REAL-WORLD PROBLEMS USING EQUATIONS WITH NONNEGATIVE RATIONAL NUMBERS	Use equations with nonnegative rational numbers to solve real-world problems.
USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS	Use the formula for area of a circle to solve real-world or mathematical problems.
USE THE FORMULA FOR CIRCUMFERENCE TO SOLVE PROBLEMS	Use the formula for circumference to solve real-world or mathematical problems.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
<i>EXPLAIN AREA</i>	Make known your understanding through words, drawings, manipulatives, etc., that area is a two-dimensional quantity representing the amount of space in a surface.	Postrequisite of <i>EXPLAIN LENGTH</i>

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

TEACHER NOTES

This unit includes the following documents:

- ▶ Learning Map Information
- ▶ Instructional Activity (two lessons)
- ▶ Instructional Activity Student Handout (for Lesson 1 and Lesson 2)
- ▶ Instructional Activity Supplement (for Lesson 2)
- ▶ Student Activity
- ▶ Student Activity Solution Guide

In this unit, students will learn about the radius, diameter, circumference, and area of a circle. The learning activities provide the context for students to notice properties and create mathematically accurate and meaningful definitions for properties of and measurements in a circle (Sinclair, 2012). As recommended by Van de Walle (2014), students will measure and explore relationships among radius, diameter, and circumference, which will lead them to discover that pi represents the ratio of the circumference of a circle to the diameter. Students need to develop an awareness that pi is an irrational value which must always be approximated when making calculations. Using a rounded value of 3.14 for pi means all values calculated using the rounded value are approximations. In addition, reporting values where an exact value for pi is used to calculate (e.g., pi button on a calculator) requires rounding, and therefore those values are approximations as well.

In the first lesson, students will measure the radius, diameter, and circumference of several different circular objects. Series of guiding questions steer students toward understanding the relationship between the radius and the diameter, as well as the relationship between the diameter and the circumference of a circle. By examining circles of different sizes, organizing the measurements in tables, and computing the ratio of each circle's circumference to its diameter, students will develop an understanding of how the ratio circumference/diameter remains constant regardless of a circle's size. By exploring the relationships among the measurable features of circles, students will develop a strong foundation for understanding two formulas for circumference of a circle ($C = \pi d$ and $C = 2\pi r$) and an understanding of why both formulas are accurate.

The second lesson addresses the area of a circle. This lesson builds on students' previous experiences with area of polygons to establish a conceptual foundation for the formula for the area of a circle. A key skill in geometry is the ability to create, attend to, and learn how to work with imagery (Sinclair, 2012). Students will use drawings of shapes to describe and analyze properties of shapes and to consider different ways to view shapes. The lesson uses imagery throughout, beginning with an activity to guide the development of the formula for area based on what students know about the circumference of a circle and the area of a parallelogram. An important conceptual prerequisite is understanding that decomposing and rearranging the parts of a two-dimensional shape preserves the area. Building on this knowledge, students will decompose circles into sectors and rearrange the sectors into shapes that approximate parallelograms. As the number of sectors increases and the size of each sector decreases, the resulting shapes look more like parallelograms. This activity promotes students' use of imagery through opportunities to reimagine how a circle's area can be

represented while preserving its original size. By connecting the area of a circle to the area of a parallelogram, students can establish meaningful connections to their prior study of area of parallelograms in 6.G.1.

The learning map section for this sequence of activities includes prerequisite knowledge of length, perimeter, and the ability to represent and solve equations. Building on an understanding of length, students can learn to explain diameter and radius. An understanding of perimeter provides the foundation for students to explain circumference. Knowledge of circumference and diameter together permit students to explore relationships between them to develop an understanding of pi as the ratio of the circumference of a circle to the diameter. Then students should be ready to explore and explain the formulas for area and circumference of circles, use the formulas to solve real-world and mathematical problems, and explain the relationship between circumference and area.

REFERENCES

- Sinclair, N., Pimm, D., & Skelin, M. (2012). Geometry: The big ideas and essential understandings. *Developing essential understanding of geometry for teaching mathematics in grades 6-8* (pp. 7–68). Reston, VA: NCTM.
- Van de Walle, J., Bay-Williams, J., Karp, K., & Lovin, L. (2014). Exploring measurement concepts. *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8: Volume I (Second ed.)* (pp. 298–324). Upper Saddle River, NJ: Pearson Education Limited.

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will develop an understanding of diameter, radius, circumference, and pi and the relationships among them, and will solve problems using the circumference formula.

PRIMARY ACTIVITY

Students will measure circular objects and create definitions to support their understanding of diameter, radius, circumference, and pi.

OTHER VOCABULARY

Students will need to know

- ▶ Circle
 - ▶ Distance
 - ▶ Perimeter
-

MATERIALS

- ▶ Several circular objects of different sizes (up to 10)
 - ▶ String
 - ▶ Rulers or tape measures with centimeter markings
 - ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
-

IMPLEMENTATION

Begin the lesson by having students measure the circular objects available and record their results as described in the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#). The following table is an example row of student work.

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OBJECT	DISTANCE FROM THE CENTER OF THE CIRCLE TO THE EDGE	DISTANCE ACROSS THE WIDEST PART OF THE CIRCLE	DISTANCE AROUND THE CIRCLE	DISTANCE AROUND THE CIRCLE ÷ DISTANCE ACROSS THE WIDEST PART OF THE CIRCLE
small paper plate	9 centimeters	17.9 centimeters	57.6 centimeters	about 3.218 centimeters

Emphasize the importance of measuring in centimeters as accurately as possible for each object. Students should measure each object to the nearest tenth of a centimeter.

Model the measurements students will make using a circular object. Tape measures or string can be used to measure the distance around the circle (circumference) and to measure straight distances from the center of the circle to the edge (radius) or across the widest part of a circle (diameter).

Check the students' work as they fill in the table to ensure their measurements are accurate, particularly in the column where students divide the circumference by the diameter to calculate approximations of pi. Note that the value in this column should be close to pi (approximately 3.14159) but will not be exact due to human error in calculation.

Require students to answer Questions 1–7 to summarize their work in the table and look for patterns in their data in pairs. Students should write either $d = 2r$ or $r = \frac{1}{2}d$ for Question 3, $C = \pi d$ for Question 6, and $C = 2\pi r$ for Question 7.

Provide scaffolding when necessary using the following guiding questions. As students describe and show the diameter and radius of a circle, they should articulate the fact that both the diameter and radius pass through the center of the circle. When students describe how to determine the distance around an object, it is important that they describe a process that measures the entire continuous distance (no gaps) and does not measure any portion of the perimeter more than one time (no overlaps).

GUIDING QUESTIONS

Elicit student thinking:

- ▶ What can you tell me about this circle?
- ▶ Can you describe different properties of this circle?

Determine if the student can **EXPLAIN DIAMETER**:

- ▶ What do you picture when you hear the word “diameter”?
- ▶ Can you show me where the diameter of this circle is?
- ▶ How would you describe the diameter of a circle in your own words?

Determine if the student can **EXPLAIN RADIUS**:

- ▶ What do you picture when you hear the word “radius”?
- ▶ Can you show me where the radius of this circle is?
- ▶ How would you describe the radius of a circle in your own words?

Determine if the student can **EXPLAIN PERIMETER**:

- ▶ What do we call the distance around a rectangle?
- ▶ How can you determine the distance around a shape?
- ▶ Can you show me the “perimeter” of this circle?

GUIDING QUESTIONS

Determine if the student can **EXPLAIN CIRCUMFERENCE**:

- ▶ What shape are we referring to when we talk about circumference?
- ▶ What is the difference between perimeter and circumference?
- ▶ How would you describe the circumference of a circle in your own words?

Determine if the student can **EXPLAIN π** :

- ▶ How did we determine the value of π ?
- ▶ How would you describe π ?

Determine if the student can **EXPLAIN THE CIRCUMFERENCE FORMULA**:

- ▶ What is the circumference formula? Is there another option for the circumference formula?
- ▶ How do these formulas relate to your measurements?

Once most students have completed Questions 1–7, review student answers as a class and ensure students correct their work when needed. Inform students that π is approximately 3.14159 and that it is an irrational number because it does not end or repeat. Make sure students realize that operations involving π result in an approximate value, because either π or the result must be rounded. Refer to the **TEACHER NOTES** for more information regarding approximations and π .

Practice a few questions as a class where students are provided either radius, diameter, or circumference and asked to find a different measurement. Discuss what is given, what students are trying to find, where each is located on the circle, and which equation is most efficient based on the situation. Following are some example questions.

- ▶ If the radius of a circle is 2 inches, what is the diameter? What is the circumference?
- ▶ If the diameter of a circle is 10 centimeters, what is the circumference? What is the radius?
- ▶ If the circumference of a circle is 25 feet, what is the diameter? What is the radius?
- ▶ You are building a circular fence around your garden. If the radius of garden is 3 feet, how much fencing do you need?

NOTE: Real-world situations in this unit provide opportunities to ask students to extend the measurement calculation and determine how much of

the given material would need to be purchased. In the preceding example, there is an opportunity to ask students how many sections of fence they would have to purchase if it is sold in 6-foot sections. This leads to a discussion that in real-world situations, rounding up is necessary to ensure there is enough material to complete the job.

Require students to practice Questions 8–13 to find either circumference, radius, or diameter in real-world and mathematical problems.

GUIDING QUESTIONS

Determine if the student can **USE THE FORMULA FOR CIRCUMFERENCE TO SOLVE PROBLEMS**:

- ▶ What information do you have about this circle?
- ▶ What are you trying to determine? Can you describe this measurement in relation to the circle?
- ▶ What equation or process is most efficient for answering this question?
- ▶ How would you simplify/solve this equation or carry out this process?

Students should be required to show all work as they complete the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT** and write their answers in complete, thorough, well-thought out sentences.

At the end of the activity, teachers should provide students with an example requiring them to use either $C = \pi d$ or $C = 2\pi r$ to solve a problem as an exit ticket for the day.

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

Lesson 1

For each object, make the measurements described in the following table. Use the string to help measure when needed. Be sure to measure in centimeters and be as precise as possible.

OBJECT	DISTANCE FROM THE CENTER OF THE CIRCLE TO THE EDGE	DISTANCE ACROSS THE WIDEST PART OF THE CIRCLE	DISTANCE AROUND THE CIRCLE	DISTANCE AROUND THE CIRCLE ÷ DISTANCE ACROSS THE WIDEST PART OF THE CIRCLE

1. What do you notice about the distance across the widest part of the circle and the distance from the center of the circle to the edge?
2. The distance across the widest part of the circle is called the *diameter*. Write a definition of *diameter* in your own words.
3. The distance from the center of the circle to a point on the edge is defined as the *radius*. What is the relationship between the length of the radius and the length of the diameter in each row of your table? Write an equation that relates the diameter, d , to the radius, r .
4. What do you notice about the distance around the circle divided by the diameter of the circle? (These numbers are in the last column of the table.)
5. The ratio of the distance around the circle to the diameter of the circle is defined as *pi* (π). Using the values in one row of your table, explain how the diameter, the distance around the circle, and π are related mathematically.
6. The distance around the circle is defined as the *circumference*. Write an equation that relates the circumference, C , to the diameter, d .
7. Using what you know about the relationship between the radius of the circle and the diameter, write an equation that relates the circumference, C , to the radius, r .

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will develop a formula for the area of a circle using what they know about the radius and circumference of a circle, as well as what they know about the area of a parallelogram.

PRIMARY ACTIVITY

Students will decompose a circle to develop the formula for the area of a circle.

OTHER VOCABULARY

Students will need to know the meaning of the following

- ▶ Radius
 - ▶ Circumference
 - ▶ Area of a parallelogram
-

MATERIALS

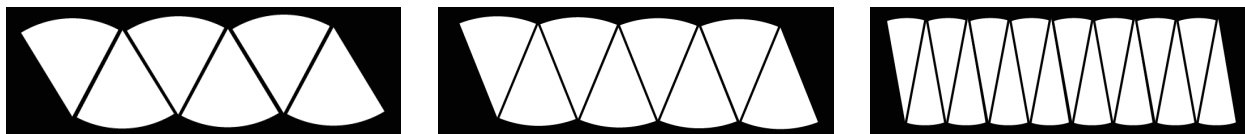
- ▶ Scissors
 - ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ [INSTRUCTIONAL ACTIVITY SUPPLEMENT](#) (recommend one copy for every student)
-

IMPLEMENTATION

Begin the lesson by having students cut out the circles and follow the directions in the [INSTRUCTIONAL ACTIVITY SUPPLEMENT](#) to deconstruct each circle into a figure which resembles a parallelogram. Students should be familiar finding the area of a parallelogram, which is knowledge this lesson will build on.

Ensure students arrange the pieces of each circle in a manner similar to what is shown in the following images and keep the pieces on their desks for reference throughout the lesson.

EXAMPLE IMAGES

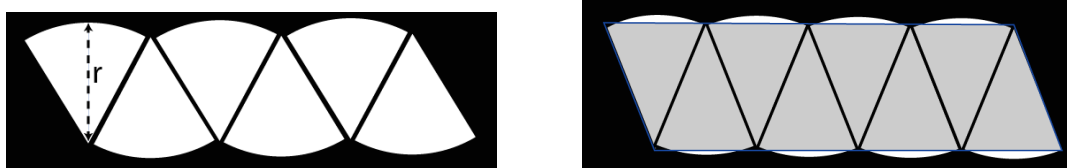


Discuss the fact that as the circle is cut into smaller and smaller sectors, the sectors can be arranged into a figure that begins to appear to have straighter sides and look more like a parallelogram. **Ask** students what they think would happen if they could cut the circle into an infinite number of sectors and arrange them in a similar fashion (they would eventually form a rectangle).

Ask students what they think is true about the area of this figure compared to the area of the original circle (they should state that they are the same).

Draw the radius on one of the sectors as shown in the following image (left). **Ask** students what measurement this would be, approximately, in a parallelogram (height). If students struggle to answer, you could draw in a parallelogram as shown in the following image (right) to **scaffold** the question.

EXAMPLE IMAGES



Ask students what the base of the parallelogram would be related to the original circle. **Scaffold** their thinking with questions to guide them towards the realization that it would be half the circumference of the circle (half the circle is on top; half the circle is on bottom). Questions could include, “Can you describe where the circumference of the circle falls on the decomposed circle?” and, “If the whole circumference is represented somewhere in this image, what fraction of the circumference falls here (the bottom base of the parallelogram)?” **Note** that as the sectors get smaller, the shape gets closer and closer to a rectangle.

Remind (or ask) students what the formula is for area of a parallelogram, and use this to derive the formula for the area of a circle. An example process is shown following this paragraph. **Scaffold** this process by going one step at a time, referring back to the figures students created, discussions from earlier in the lesson, as well as what students learned in the first lesson about circumference.

$$\text{Area} = \text{base} \cdot \text{height}$$

$$\text{Area} = \frac{1}{2} \text{circumference} \cdot \text{radius}$$

$$\text{Area} = \frac{1}{2} (2 \cdot \pi \cdot \text{radius}) \cdot \text{radius}$$

$$A = \frac{1}{2} (2 \cdot \pi \cdot r) \cdot r$$

$$A = \pi \cdot r \cdot r$$

$$A = \pi \cdot r^2$$

A video summarizing this process can be found online using this link:

<https://www.youtube.com/watch?v=YokKp3pwVFc>. Note that although the video describes the area of a rectangle rather than a parallelogram, the formulas are essentially equivalent for the purpose of deriving the area of a circle.

GUIDING QUESTIONS

Elicit student thinking:

- ▶ What can you tell me about the mathematical meaning of the word "area"?
- ▶ [Point to a drawing of any shape or indicate an object such as the student's desk] Can you show me what part of this drawing/object represents the area?

Determine if the student can **EXPLAIN RADIUS**:

- ▶ [Point to a drawing of a circle] What do you picture when you hear the word "radius"?
- ▶ [Point to a drawing of a circle] Can you show me where the radius of this circle is?
- ▶ [Point to the student's circle that has been cut apart and rearranged to resemble a parallelogram] When the circle is cut and rearranged, can you show me a radius of the circle? Where could you draw the radius so it also represents the height of the "parallelogram"?

Determine if the student can **EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE:**

- ▶ How would you describe the area of a circle?
- ▶ Does the area of a circle change if we cut it up and rearrange it?
- ▶ What formula do we use to find the area of a circle?
- ▶ How is the formula for the area of a circle related to the formula for the area of a parallelogram?

Determine if the student can **EXPLAIN THE RELATIONSHIP BETWEEN CIRCUMFERENCE AND THE AREA OF A CIRCLE:**

- ▶ How did we use the circumference of a circle to figure out the formula for the area of the circle?
- ▶ Why did we only use half the circumference in our work to find the formula for the area of a circle?

Now that a formula for the area of a circle has been established, students will practice solving problems using the formula.

Practice a few questions as a class where students are provided either radius, diameter, area, or circumference and asked to find a different measurement. **Discuss** what is given, what students are trying to find, where each is located on the circle, and which equation is most efficient based on the situation. Following are some example questions.

- ▶ If the radius of a circle is 2 inches, what is the diameter? What is the area?
- ▶ If the diameter of a circle is 10 centimeters, what is the radius? What is the area?
- ▶ If the area of a circle is 28.3 square feet, what is the radius?
- ▶ You are building a circular fence around your garden and will fill the circular space with dirt. If the radius of a circular garden is 5 feet, how much ground will be covered with dirt? How much fencing will you need?

Require students to practice questions in the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT** to find either area, circumference, radius, or diameter in real-world and mathematical problems.

GUIDING QUESTIONS

Determine if the student can **USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS**:

- ▶ What information do you have about this circle? Where is this located on the circle?
- ▶ What measurement of the circle are you trying to find? [Given a student response, follow their response with the following question] Can you describe where that measurement is located on the circle?
- ▶ What equation or process should you use to answer this question?
- ▶ How would you simplify/solve this equation?

Students should be required to show all work as they complete the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT** and write their answers in complete sentences for Questions 4–6 to ensure they understand what their answers mean.

At the end of the activity, teachers should provide students with an example which requires them to use the area formula for a circle to solve a problem as an exit ticket for the day.

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

Lesson 2

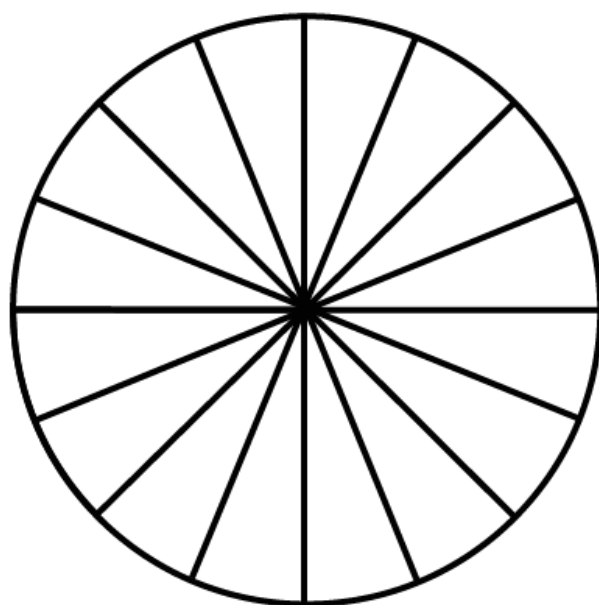
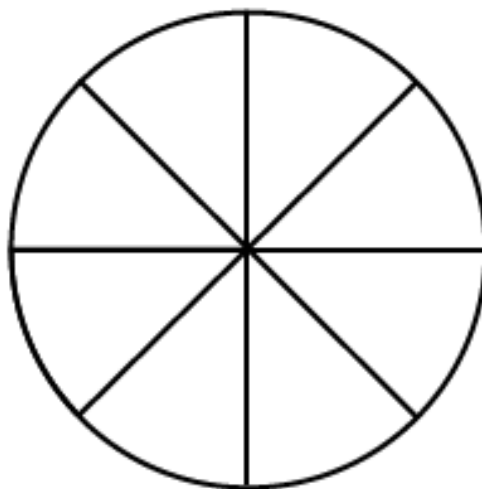
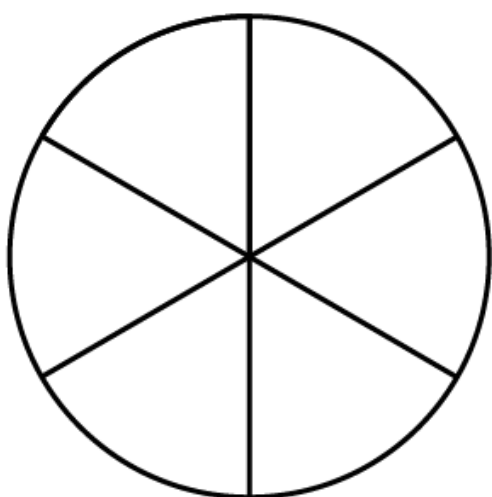
1. The diameter of a circle is 4 feet. What is the radius? What is the area?
2. The radius of a circle is 5 centimeters. What is the diameter? What is the area?
3. The area of a circle is 113 square yards. What is the radius? What is the diameter?
4. The diameter of a circular swimming pool is 20 feet. What is the area of the surface of the pool? What is the circumference of the pool? Write your answers as complete sentences.
5. The area of a pizza is 153.9 square inches. What is the radius of the pizza? Write your answer as a complete sentence.
6. You know the area of the garden inside a circular fence is 50 square feet. How long is the fence surrounding the garden? Write your answer as a complete sentence.

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

The circles below will help illustrate the relationship between the area of a circle and the area of a parallelogram. Cut out the circles one at a time and arrange the pieces so they lay next to each other, alternating up and down, to form a figure resembling a parallelogram. Be sure to keep the pieces of each circle separate as you work. In the end, you should have formed three “parallelograms” (one per circle).



EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

Lessons 1 – 2

1. Define each of the following parts of a circle in your own words.

a. Radius

b. Diameter

c. Circumference

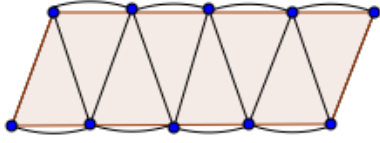
d. Area

2. Describe how the following terms are related to each other within a circle.

a. Diameter and radius

b. Diameter, circumference, and pi

- c. Circumference, radius, and area. (A complete circle has been rearranged into sectors below. Use the image to describe, in general, how the circumference and radius can be used to determine the area.)



-
3. Write all possible formulas for each of the following measurements in a circle.

a. Circumference

b. Area

-
4. If the radius of a circle is 2.5 inches, find each of the following measurements. Show all work for each measurement.

a. Diameter

b. Circumference

c. Area

5. If the diameter of a circle is 8 feet, find each of the following measurements. Show all work for each measurement.

a. Radius

b. Area

c. Circumference

6. If the circumference of a circle is 6 centimeters, find each of the following measurements. Show all work for each measurement.

a. Diameter

b. Radius

c. Area

7. If the area of a circle is 28.3 square centimeters, find the circumference. Show all work.

-
8. You are fencing in and mulching a circular area of your yard for a garden. The diameter of the circular area needs to be 7 feet. Determine how many feet of fencing you need and how many square feet the mulch must cover. Show all work and write your answers as complete sentences.

-
9. You are trying to decide where to buy pizza. Pizza Shop A advertises a pizza with a 14-inch diameter. Pizza Shop B advertises a pizza with a 42-inch circumference. Both pizzas are the same price. Which Pizza Shop gives you more pizza for the price? Show your work and explain the process you used to determine your answer.

EXPLAINING AREA AND CIRCUMFERENCE OF A CIRCLE

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 2

1. Define each of the following parts of a circle in your own words.

a. Radius

CORRECT ANSWER

For this question and others in this activity, answers will vary but should resemble the response provided.

The radius of a circle is the distance from the center of the circle to a point on the circumference of the circle.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The radius of a circle is the distance across the widest part of a circle.	confuses radius with diameter and does not use specific vocabulary	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>
The radius of a circle is the distance between any two points on the circle.	confuses radius with chord	EXPLAIN <i>RADIUS</i>

b. Diameter

CORRECT ANSWER

The diameter of the circle is any straight line segment that passes through the center of the circle with endpoints lying on the circle.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The diameter is a line that goes across the circle.	does not specify a line segment or that the segment must go through the center of the circle.	EXPLAIN <i>DIAMETER</i>
The diameter is the distance from the center of the circle to the edge of the circle.	confuses radius with diameter	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>
The diameter of a circle is the distance between any two points on the circle.	confuses diameter with chord	EXPLAIN <i>DIAMETER</i>

c. Circumference

 CORRECT ANSWER

The circumference of a circle is the perimeter of the circle or the distance around a circular area.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Perimeter.	does not specify that circumference is specifically the perimeter of a circle	EXPLAIN <i>CIRCUMFERENCE</i>
The circumference is the distance across a circle.	confuses circumference with diameter	EXPLAIN <i>CIRCUMFERENCE</i> and EXPLAIN <i>DIAMETER</i>
The circumference is the space inside a circle.	confuses circumference with area	EXPLAIN <i>CIRCUMFERENCE</i>

d. Area

 CORRECT ANSWER

The area of a circle is the two-dimensional space contained within the circle.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Area is the distance around a circle.	confuses circumference with area	EXPLAIN <i>CIRCUMFERENCE</i>

2. Describe how the following terms are related to each other within a circle.

a. Diameter and radius

 CORRECT ANSWER

The length of the diameter is twice the length of the radius.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The length of the diameter is half the length of the radius.	confuses diameter with radius	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>

b. Diameter, circumference, and pi

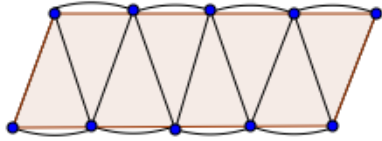
 CORRECT ANSWER

Pi is the ratio of the circumference of a circle to the circle's diameter.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
They are all lengths in a circle.	does not understand the relationship between circumference and diameter or that pi is a ratio rather than a length	EXPLAIN <i>DIAMETER</i> , EXPLAIN <i>CIRCUMFERENCE</i> , and EXPLAIN <i>PI</i>
Pi is the ratio of the diameter of a circle to the circumference of the circle.	does not use the correct ordering of parts in the description of a ratio	EXPLAIN <i>RATIO</i>

- c. Circumference, radius, and area. (A complete circle has been rearranged into sectors below. Use the image to describe, in general, how the circumference and radius can be used to determine the area.)



CORRECT ANSWER

The base of the figure (which resembles a parallelogram) is represented by half the circumference of the circle, and the height of the figure is represented by the radius of the circle. Therefore, using the formula for the area of a parallelogram, the area of a circle equals half the circumference times the radius.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Circumference is the distance around a circle and area is space inside a circle.	knows the meaning of circumference and area of a circle, but cannot describe how they are related to each other	EXPLAIN THE RELATIONSHIP BETWEEN CIRCUMFERENCE AND THE AREA OF A CIRCLE

3. Write all possible formulas for each of the following measurements in a circle.

- a. Circumference

CORRECT ANSWER

$$C = \pi \cdot d$$

or

$$C = 2 \cdot \pi \cdot r$$

**While these are the anticipated equations students will provide, an alternate form of either equation would indicate knowledge of the relationships among the variables.*

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$C = \pi \cdot r^2$	misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE CIRCUMFERENCE FORMULA
Only one equation provided.	does not understand the relationship between radius and diameter well enough to create both equations	EXPLAIN <i>DIAMETER</i> , EXPLAIN <i>RADIUS</i> , and EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE

b. Area

 CORRECT ANSWER

$$A = \pi \cdot r^2$$

**While this is the anticipated equation students will provide, an alternate form of the equation would indicate knowledge of the relationship among the variables.*

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$A = 2 \cdot \pi \cdot r$ or $A = \pi \cdot d$	misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE

4. If the radius of a circle is 2.5 inches, find each of the following measurements. Show all work for each measurement.

a. Diameter

 CORRECT ANSWER

The diameter is 5 inches.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The diameter is 6.25 inches.	squares the radius instead of doubling the radius	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>
The diameter is 1.25 inches	halves the radius instead of doubling the radius	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>

b. Circumference

 CORRECT ANSWER

If students approximate pi using 3.14:

The circumference is approximately 15.7 inches.

If students use the pi button on the calculator:

The circumference is approximately 15.71 inches.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The circumference is 19.63 inches.	calculates the area instead of the circumference; misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE CIRCUMFERENCE FORMULA
The circumference is 7.85 inches.	multiplies pi by the radius instead of the diameter (or forgets to multiply by 2)	USE THE FORMULA FOR CIRCUMFERENCE TO SOLVE PROBLEMS

c. Area

CORRECT ANSWER

Whether students approximate pi using 3.14 or use the pi button on the calculator:

The circumference is approximately 19.63 square inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The area is 15.7 square inches.	calculates the circumference instead of the area; misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE
The area is 7.85 square inches.	forgets to square the radius	USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS

5. If the diameter of a circle is 8 feet, find each of the following measurements. Show all work for each measurement.

a. Radius

CORRECT ANSWER

The radius is 4 feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The radius is 16 feet.	doubles the diameter instead of dividing the diameter in half	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>RADIUS</i>

b. Area

CORRECT ANSWER

If students approximate pi using 3.14:

The area is approximately 50.24 square feet.

If students use the pi button on the calculator:

The area is approximately 50.27 square feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The area is 25.12 square feet.	calculates the circumference instead of the area; misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE FORMULA FOR THE AREA OF A CIRCLE
The area is 12.56 square inches.	forgets to square the radius	USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS
The area is 200.96 square inches.	uses the diameter instead of the radius in the area formula	USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS

c. Circumference

CORRECT ANSWER

If students approximate pi using 3.14:

The circumference is approximately 25.12 feet.

If students use the pi button on the calculator:

The circumference is approximately 25.13 feet.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The circumference is 50.24 feet.	calculates the area instead of the circumference; misunderstanding of variable relationships, causing confusion between the area and circumference formulas	EXPLAIN THE CIRCUMFERENCE FORMULA
The circumference is 12.56 inches.	multiplies the radius by pi instead of the diameter (or forgets to multiply by 2)	USE THE FORMULA FOR CIRCUMFERENCE TO SOLVE PROBLEMS

6. If the circumference of a circle is 6 centimeters, find each of the following measurements. Show all work for each measurement.
- a. Diameter

 CORRECT ANSWER

The diameter is approximately 1.91 centimeters.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The diameter is 12 centimeters.	treats the circumference like the radius and doubles it to find the diameter	EXPLAIN <i>CIRCUMFERENCE</i> or EXPLAIN THE CIRCUMFERENCE FORMULA
The diameter is approximately 0.95 centimeters.	determines the radius instead of the diameter; confuses radius with diameter	EXPLAIN THE CIRCUMFERENCE FORMULA, EXPLAIN <i>RADIUS</i> , or EXPLAIN <i>DIAMETER</i>

- b. Radius

 CORRECT ANSWER

The radius is approximately 0.95 centimeters.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The radius is approximately 1.91 centimeters.	finds the diameter instead of the radius	EXPLAIN <i>RADIUS</i> or USE THE FORMULA FOR CIRCUMFERENCE TO SOLVE PROBLEMS

c. Area

 CORRECT ANSWER

Whether students approximate pi using 3.14 or use the pi button on the calculator:

The area is approximately 2.86 square centimeters.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The area is approximately 11.46 square centimeters.	uses the measurement for diameter instead of radius in the area formula	USE THE FORMULA FOR THE AREA OF A CIRCLE TO SOLVE PROBLEMS

7. If the area of a circle is 28.3 square centimeters, find the circumference. Show all work.

 CORRECT ANSWER

If the area is 28.3 square centimeters, then the radius is approximately 3 centimeters. Therefore, the circumference is approximately 18.84 centimeters (if students use an approximation of 3.14 for pi), or approximately 18.85 centimeters (if students use the pi button on the calculator).

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The circumference is approximately 3 centimeters.	finds the radius instead of the circumference	EXPLAIN <i>RADIUS</i> and EXPLAIN <i>CIRCUMFERENCE</i>
The circumference is approximately 9.42 centimeters.	uses the radius instead of the diameter when finding the circumference (or forgets to multiply the length of the radius by 2)	USE THE FORMULA FOR <i>CIRCUMFERENCE</i> TO SOLVE PROBLEMS

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8. You are fencing in and mulching a circular area of your yard for a garden. The diameter of the circular area needs to be 7 feet. Determine how many feet of fencing you need and how many square feet the mulch must cover. Show all work and write your answers as complete sentences.

 CORRECT ANSWER

I will need about 21.98 feet of fencing to surround the circular garden. (If students use the pi button instead of an approximation of 3.14, the result will be approximately 21.99 feet.)

I will need about 38.47 square feet of mulch to fill in the space inside the circular garden. (If students use the pi button instead of an approximation of 3.14, the result will be approximately 38.48 square feet.)

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student gives 21.98 square feet as the amount of mulch needed and 38.47 feet as the length of fencing needed.	confuses the area and circumference formulas or the meaning of these terms	EXPLAIN <i>AREA</i> and EXPLAIN <i>CIRCUMFERENCE</i>
Student gives 153.86 square feet as the amount of mulch needed.	uses 7 for the radius instead of 3.5	USE THE FORMULA FOR THE <i>AREA</i> OF A CIRCLE TO SOLVE PROBLEMS

9. You are trying to decide where to buy pizza. Pizza Shop A advertises a pizza with a 14-inch diameter. Pizza Shop B advertises the same type of pizza with a 42-inch circumference. Both pizzas are the same price. Which Pizza Shop gives you more pizza for the price? Show your work and explain the process you used to determine your answer.

CORRECT ANSWER

Answers may vary. Four likely responses are provided, but all responses should indicate that Pizza Shop A gives more pizza for the price.

1. I calculated the area of each pizza so I could compare them. The area of the pizza from Pizza Shop A is approximately 153.86 square inches. The area of the pizza from Pizza Shop B is approximately 140.45 square inches. Therefore, Pizza Shop A gives more pizza for the price.
2. I calculated the circumference of the pizza from Pizza Shop A to compare it to the circumference of the pizza from Pizza Shop B. The circumference of the pizza from Pizza Shop A is approximately 43.96 inches. Because this is greater than the circumference of the pizza from Pizza Shop B, Pizza Shop A gives more pizza for the price.
3. I calculated the radius of each pizza so I could compare them. The radius of the pizza from Pizza Shop A is 7 inches. The radius of the pizza from Pizza Shop B is approximately 6.69 inches. Therefore, Pizza Shop A gives more pizza for the price.
4. I calculated the diameter of the pizza from Pizza Shop B to compare it to the diameter of the pizza from Pizza Shop A. The diameter of the pizza from Pizza Shop B is approximately 13.38 inches. Because this is less than the diameter of the pizza from Pizza Shop A, Pizza Shop A gives more pizza for the price.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Pizza Shop B gives more pizza for the price because 42 inches is more than 14 inches.	compares two parts of the circle that cannot be compared to determine which pizza is bigger without additional calculations	EXPLAIN <i>DIAMETER</i> and EXPLAIN <i>CIRCUMFERENCE</i>