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FLUID MECHANICS

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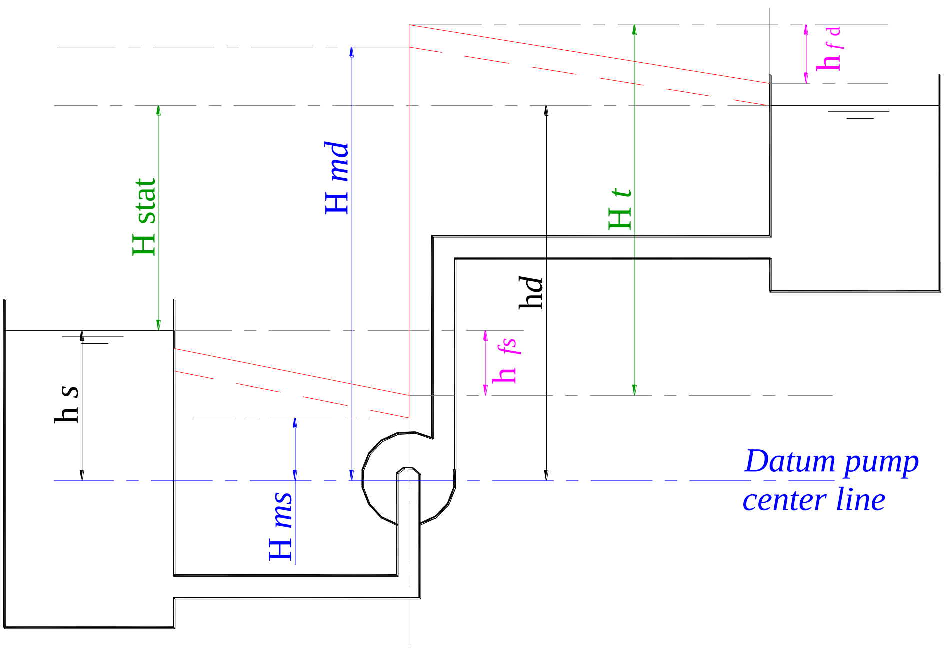


بسم الله الرحمن الرحيم

﴿ قالوا سبحانك لا علم لنا الا ما علمتنا

إنك انت العليم الحكيم ﴾

صدق الله العظيم

الآيه(32) سوره البقره

FLUID MECHANICS

Contents

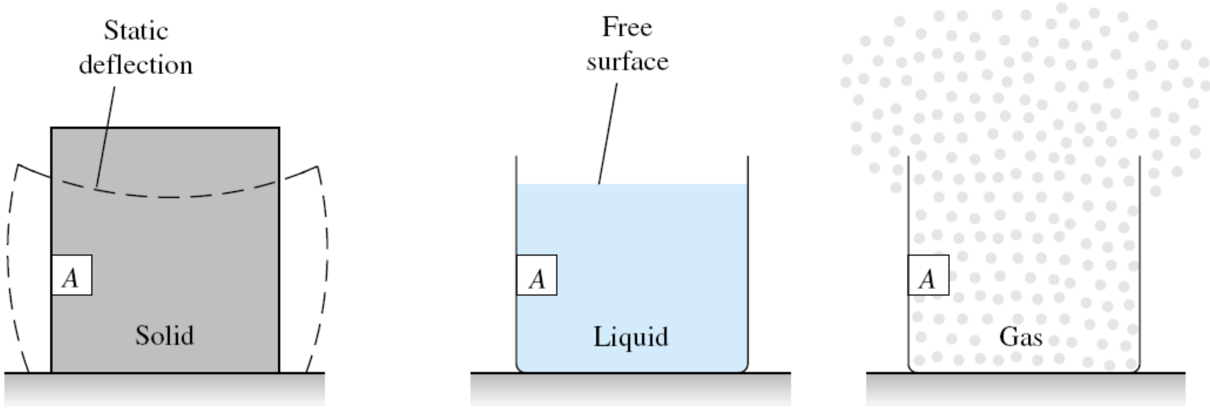
CHAPTER 1

UNITS AND DIMENSIONS & FLUID PROPERTIES

1.1 Fluids الموائع:

Fluid can be defined as a substance which can deform continuously when being subjected to shear stress at any magnitude. This includes any liquid or gas.

يعرف المائع بأنه المادة التى يتغير شكلها باستمرار عند تعرضها لإجهاد قص مهما صُغر هذا الإجهاد، وتشمل السوائل والغازات.



1.2 Fluid mechanics ميكانيكا الموائع :

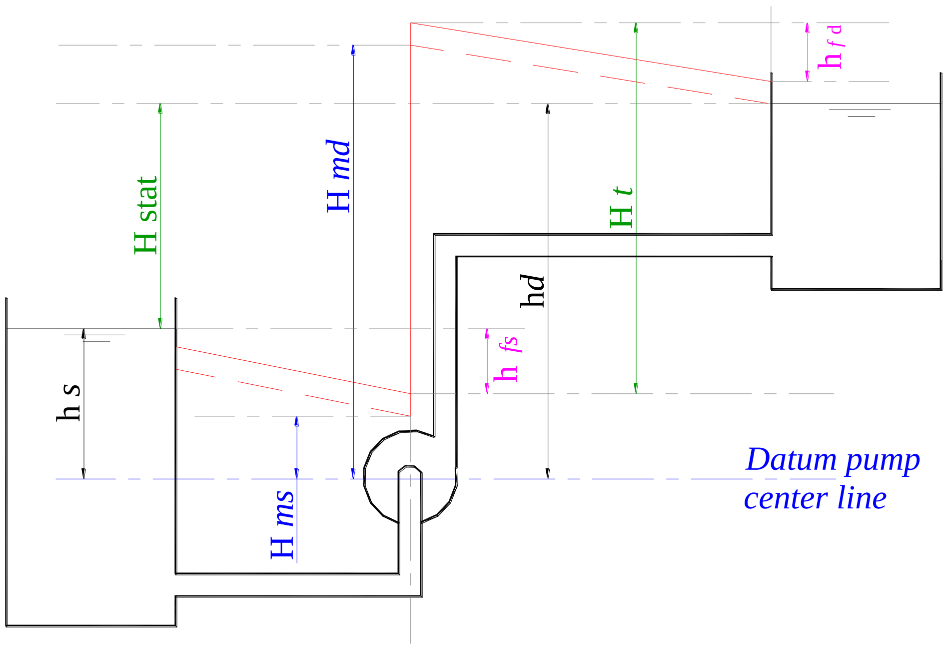
Fluid mechanics is a division in applied mechanics related to the behaviour of liquids or gases which is either in rest or in motion.

هو ذلك الفرع من العلم الذى يهتم بدراسة الموائع المختلفة مثل السوائل والغازات فى جميع حالاتها (السكون أو الحركة).

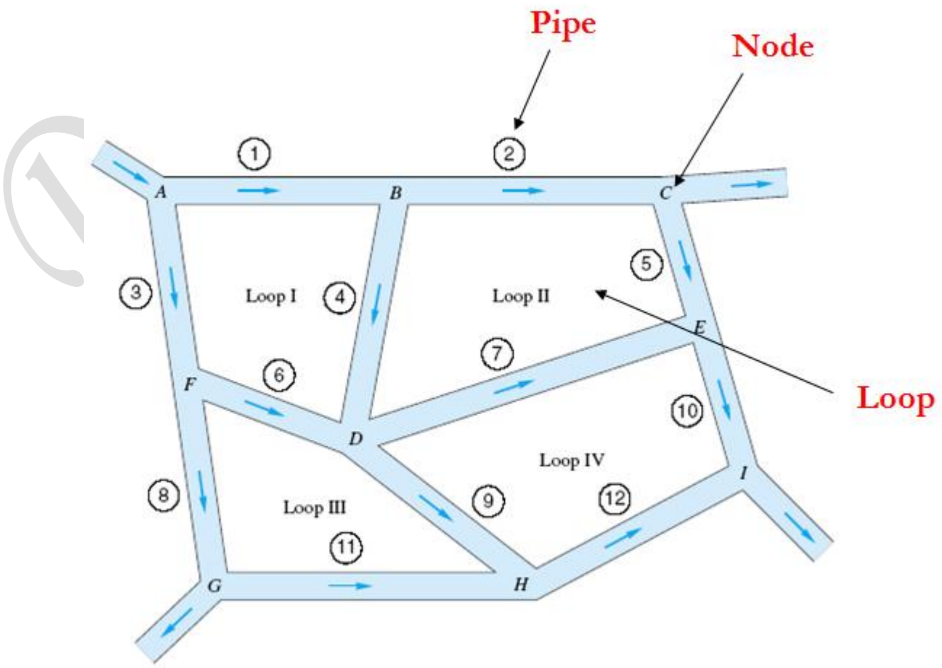
The study related to a fluid in rest or stationary is referred to "fluid statics" استاتيكا الموائع, otherwise it is referred to as "fluid dynamics" ديناميكا الموائع.

1.3 Applications Of Fluid Mechanics In Civil Engineering

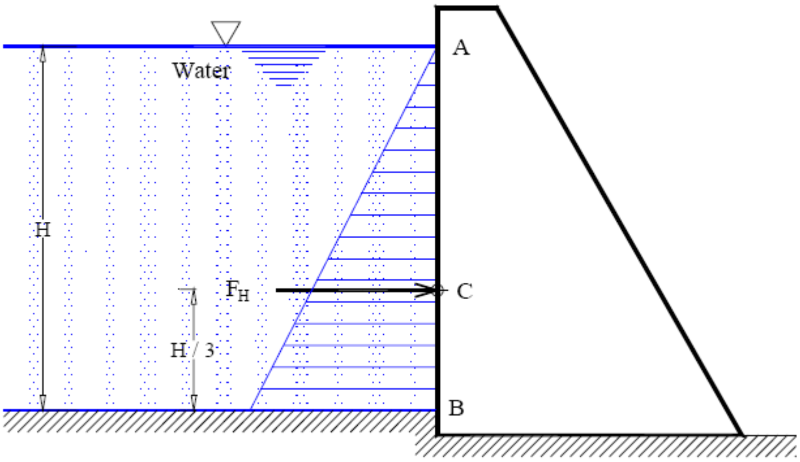
- **Water pipelines:**



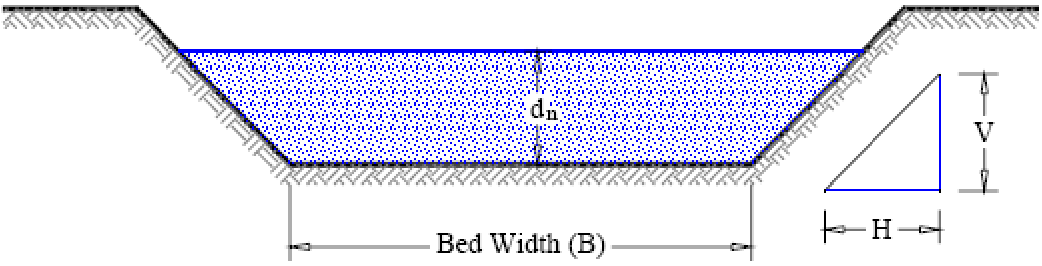
- **Water distribution systems and Sewer systems:**



- **Dams and water control structures:**



- **Rivers and manmade canals:**



- **Coastal and Harbour structures:**

1.4 Units And Dimensions الوحدات والأبعاد :

a- Dimensions:

Length	Time	Mass	Force
L	T	M	F

Mass: is the property of a body of fluid that is a measure of its inertia or resistance to a change in motion. It is also a measure of the quantity of fluid.

Force (Weight): is the amount that a body weights, that is, the force with which a body is attracted towards the earth by gravitation.

- **Types of Dimensions:**

M – L – T System

F – L – T System

Force (F) = Mass (M) x Acceleration (L/T^2)

$$F = MLT^{-2}$$

b- Units:

System	Quantity			
	Length (L)	Time (T)	Mass (M)	Force (F)
International (SI)	M	sec	Kg	N
French (c.g.s)	Cm	sec	gm	dyne
British (English)	Foot (ft)	sec	slug	Pound (Ib)
Kilogram weigth	M	sec	Kg	Kg_w

$$1\ N = 1\ Kg \times 1\ m/s^2$$

$$1\ dyne = 1\ gm \times 1\ cm/s^2$$

$$1\ Ib = 1\ slug \times 1\ ft/s^2$$

- **Some conversion factors:**

a- Length (L):

1 ft = 12 inch 1 inch = 2.54 cm 1 ft = 0.3048 m

b- Mass (M):

1 slug = 14.59 Kg

c- Force (F):

$1\ Kg_w = 9.81$ N $1\ gm_w = 981$ dyne $1\ N = 10^5$ dyne

1 Ib = 4.44 N $1\ Kg_w = 2.205$ Ib   1 Ib = 453.60 gm

Note: Acceleration due to gravity, $g = 9.81\ m/s^2 = 32.20\ ft/s^2$.

Example 1-1

Convert the following:

a) A discharge of 20 ft^3/min. to lit/sec.

$$20\ ft^3/\text{min} = \frac{20 \times (30.48)^3}{60 \times 1000} = 9.439 \text{ lit/sec.}$$

b) A pressure of 30 lb/inch^2 to gm/cm^2.

$$30 \text{ lb/inch}^2 = \frac{30 \times 453.6}{(2.54)^2} = 2109.24 \text{ gm/cm}^2.$$

c) A specific weight of 62.4 lb/ft^3 to kg/lit.

$$62.4 \text{ lb/ft}^3 = \frac{62.4 \times 453.6 \times 1000}{1000 \times (30.48)^3} = 0.999 \approx 1 \text{ kg/lit.}$$

1.5 Main Water properties:

1- Density (ρ) الكثافة :

Density (ρ): mass per unit volume كتلة وحدة الحجوم.

$\rho = M/V = ML^{-3}$ (Dimension)

$\rho = Kg/m^3 = gm/cm^3 = slug/ft^3$ (units)

$\rho_{water} = 1000\ Kg/m^3 = 1\ gm/cm^3 = 1.94\ slug/ft^3$

2- The specific weight (γ) الوزن النوعى:

The specific weight (γ) = weight per unit volume وزن وحدة الحجوم

$\gamma = W/V = FL^{-3}$ (Dimension)

$\gamma = \rho\ g$

$\gamma = Kg_w/m^3 = N/m^3 = dyne/cm^3 = Ib/ft^3$ (units)

$\gamma_{water} = 9810\ N/m^3 = 981\ dyne/cm^3 = 62.42\ Ib/ft^3$

Water reaches a maximum density at 4°C. It becomes less dense when heated.

تبلغ أقصى قيمة للوزن النوعى للمياه عند درجة حرارة 4 مئوية، وتقل باتفاع درجات الحرارة.

Density of sea water about 4% more than that of fresh water. Thus, when fresh water meets sea water without sufficient mixing, salinity increases with depth.

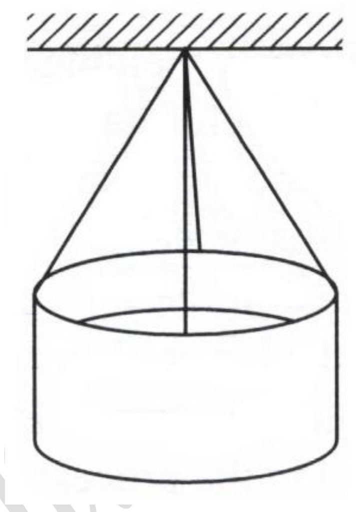
كثافة مياه البحر أعلى من كثافة الماء العذب بحوالى 4 %.

Temperature (°C)	**Density (ρ , *kg/m³*)**	**Specific Weight (ɣ, N/m³)**
0°(ice)	917	8996
0° (water)	999	9800
4 °	1000	9810
10°	999	9800
20°	998	9790
30°	996	9771
40°	992	9732
50°	988	9692
60°	983	9643
70°	978	9594

80°	972	9535
90°	965	9467
100°	958	9398

Example 1-2

A cylindrical water tank (see the figure) is suspended vertically by its sides. The tank has a 2-m diameter and is filled with 40°C water to 1 m in height. Determine the force exerted on the tank bottom.



Solution

The force exerted on the tank bottom is equal to the weight of the water body.

$$F = W = m \cdot g = [\rho \cdot (\text{Volume})] \cdot (g)$$
$$= \left[992 \text{ kg/m}^3 \cdot \left(\frac{\pi \cdot 2^2}{4} \times 1\text{m}^3\right)\right](9.81 \text{ m/sec}^2)$$
$$= 30,572 \text{ kg-m/sec}^2 = 30,572 \text{ N}$$

3- **Specific gravity (S.G), Relative density (R.D) الكثافة النسبية:**

Specific gravity (S.G): the ratio of the specific weight of any liquid to that of water at 4°C.

هى النسبة بين وزن حجم معين من السائل الى وزن نفس الحجم من الماء عند درجة حرارة 4 مئوية.

$S.G = R.D = \rho_{liquid} / \rho_{water} = \gamma_{lquid} / \gamma_{water}$ (no units)

(*for example* $S.G_{Hg} = 13.60$)

<u>Example 1-3</u>

Determine the specific weight, density and specific gravity of a liquid that occupies a volume of 200 lit. , and weighs 178 kg. Will this fluid float on the surface of an oil of specific gravity (0.8)? Provide results in SI units.

<u>Solution</u>

$$\text{Density}(\rho)\text{:}\ \frac{178\times1000}{200} = 890\ \text{kg/m}^3.$$

$$\text{Specific weight}(\gamma)\text{:}\ \frac{178\times9.81\times1000}{200} = 8730.9\ \text{N/m}^3.$$

$$\text{Specific gravity (S.G.):}\ \frac{890}{1000} = 0.89 > 0.80$$

$\therefore$ This fluid will not float on the surface of an oil.

4- <u>Surface Tension الشد السطحى:</u>

Surface tension (σ): A liquid's ability to resist tension.

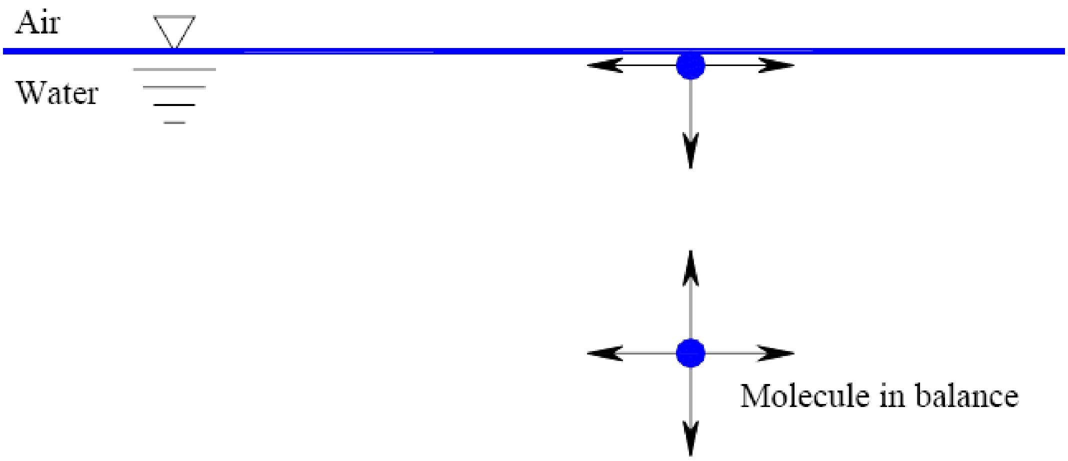
هى الخاصية التى تمكن السائل من مقاومة اجهادات الشد على سطحه.

The surface tension (σ) of a liquid is usually expressed in the units of <u>force per unit length.</u>

At the interface between a liquid and a gas, i.e., at the liquid surface, and at the interface between two immiscible (not mix-able) liquids, the out-of-balance attraction force between molecules forms an imaginary surface film, which exerts a tension force in the surface. This liquid property is known as surface tension.

جميع جزيئات السائل تتجاذب فيما بينها بصورة متزنة ولكن عند سطح السائل الملامس للهواء تكون قوى التجاذب من أعلى ومن اسفل غير متزنة (وتختلف من سائل لآخر)، وعند تلامس

السائل لسطح ما فإن قوى الشد السطحى تكون متساوية عند جميع نقاط ذلك السطح، وتؤثر فى مستوى ذلك السطح.



Molecular forces قوى الجزيئات :

Cohesion تجاذب: Cohesion enables a liquid to resist tensile stress (inner force between liquid molecules)قوى التجاذب تكون بين جزيئات السائل بعضها البعض.

Adhesion التلاصق: adhesion enables it to adhere to another body (attraction force between liquids, and a solid surface).

قوى التلاصق تكون بين جزيئات السائل والسطح الصلب الملامس له.

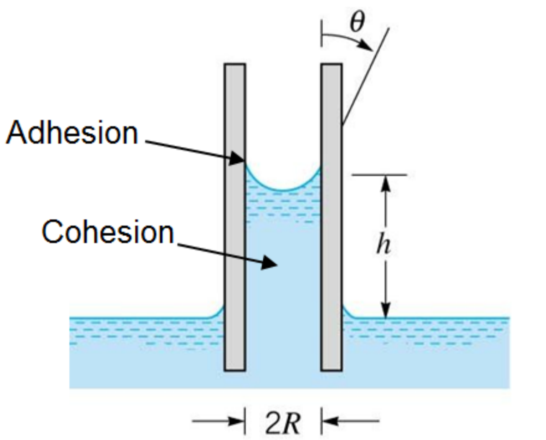
Surface Tension of Water

Temperature (°C)	0°	10°	20°	30°	40°
$\sigma(\times 10^{-2}$N/m)	7.416	7.279	7.132	6.975	6.818
σ(dyn/cm)	74.16	72.79	71.32	69.75	68.18
Temperature (°C)	50°	60°	70°	80°	90°
$\sigma(\times 10^{-2}$N/m)	6.786	6.611	6.436	6.260	6.071
σ(dyn/cm)	67.86	66.11	64.36	62.60	60.71

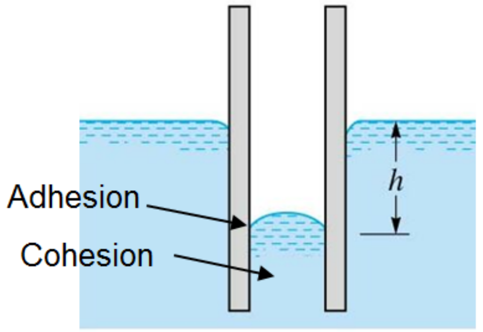
4-1 Capillarity الخاصية الشعرية:

Capillary effect is the rise or fall of a liquid in a small-diameter tube. It is caused by surface tension. خاصية ارتفاع او انخفاض السوائل فى الأنابيب الشعرية

$$\boxed{h = \frac{4\sigma \cos\theta}{\gamma D}}$$



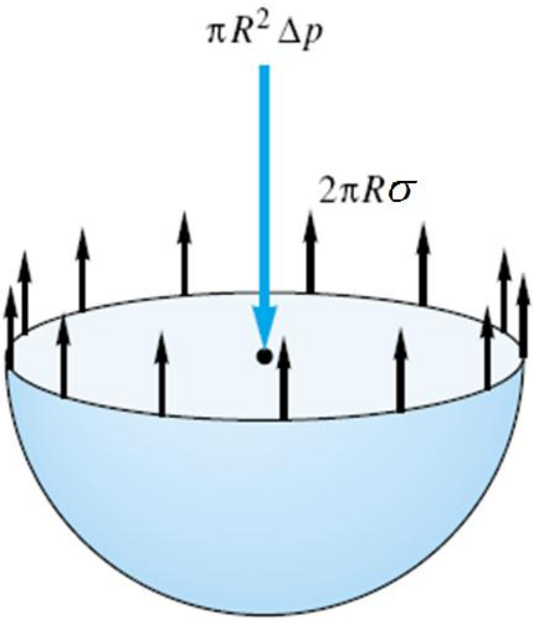
Adhesion > Cohesion



Cohesion > Adhesion

4-2 Water droplets

$$\boxed{\Delta P = \frac{4\sigma}{D}}$$



Example 1-4

Calculate the capillary effect in millimeters in a glass tube of 5 mm diam., when immersed in (i) water and (ii) in mercury (S.G. = 13.6). The temperature of liquid is 20° C and the values of surface tension of water and mercury at this temperature in contact with air are 0.0075 kg/m and 0.052 kg/m respectively. The contact angle for water = 0 and for mercury = 130°.

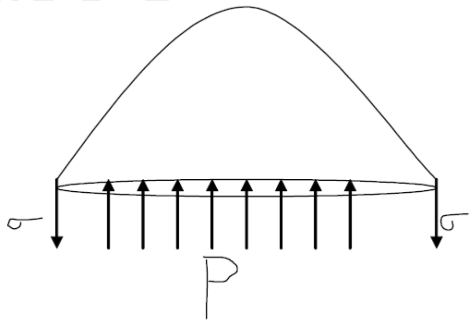
Solution

In water: $h_w = \frac{4 \times 0.0075 \times 9.81 \times \cos(0)}{9810 \times 5 \times 10^{-3}} = 6 \times 10^{-3} m = 6\ mm.$

In mercury: $h_m = \frac{4 \times 0.052 \times 9.81 \times \cos(130)}{13.6 \times 9810 \times 5 \times 10^{-3}} = -1.97 \times 10^{-3} m$ = -1.97 mm.

Example 1-5

Calculate the internal pressure of a 25 mm diameter soap bubble if the tension in the soap film is 0.5N/m.



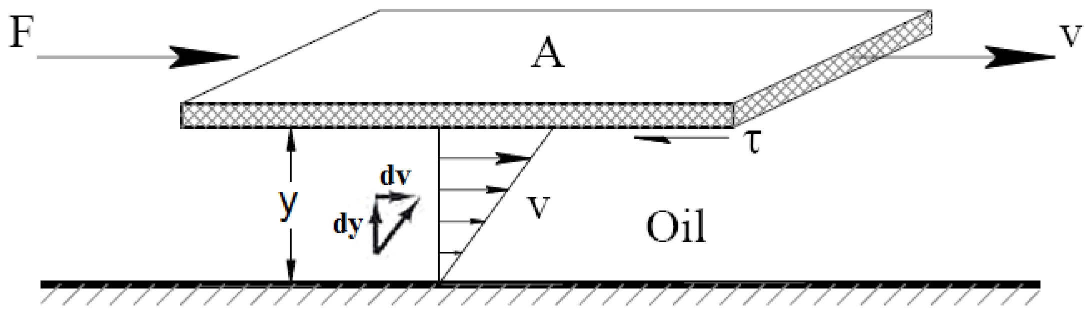
Solution

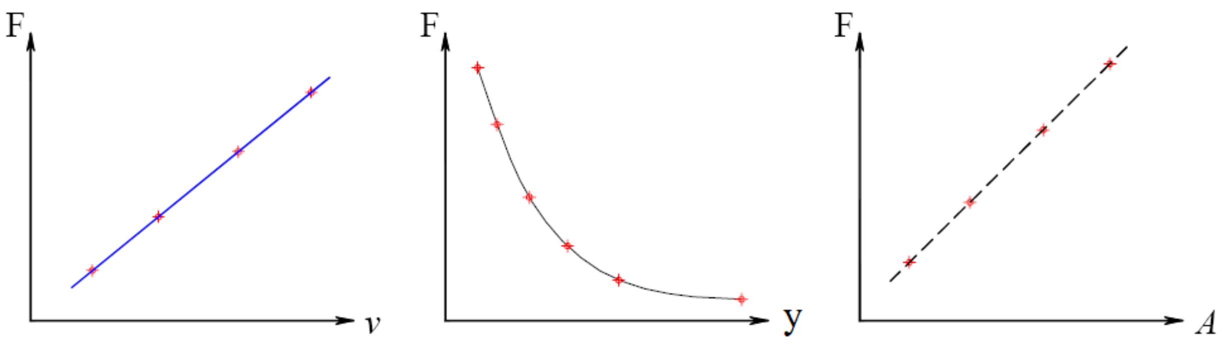
$$p \times \frac{\pi \times D^2}{4} = 2 \times \sigma \times \pi \times D$$

$$p = 160\ N/m^2$$

5- Viscosity اللزوجة:

Consider that oil fills the space between two parallel plates at a distance "**y**" a part. A horizontal force "**F**" is applied to the upper plate and moves it to the right at velocity "**v**" while the lower plate remains stationary. The shear force "**F**" is applied to overcome the oil resistance "**τ**", and it must be equal to "**τ**" because there is no acceleration involved in the process.





y & A = constants

$F \propto v$

v & A = constants

$F \propto \frac{1}{y}$

y & v = constants

$F \propto A$

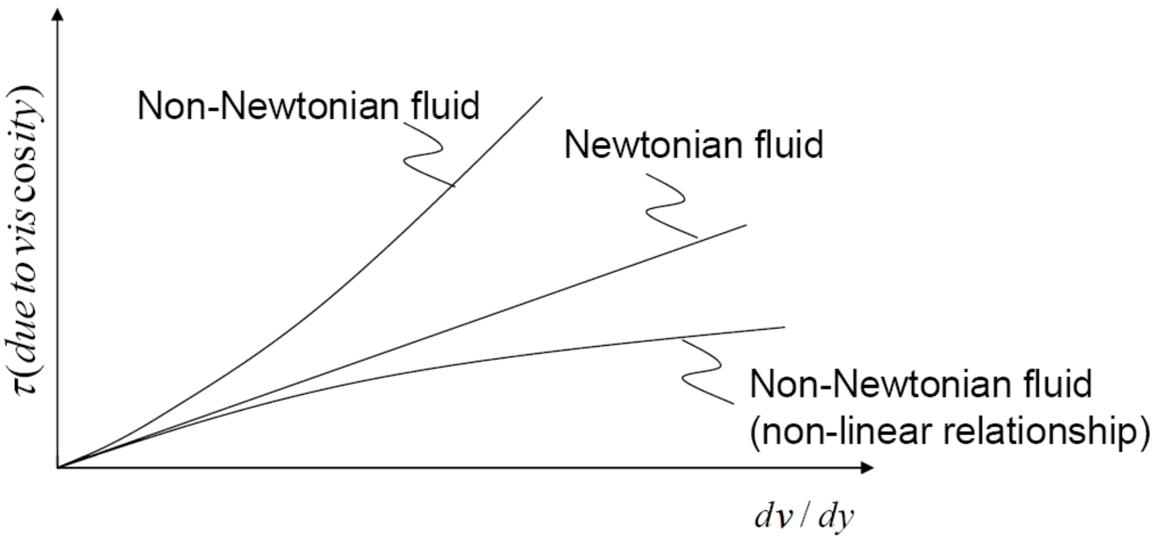
$$F \propto A\frac{v}{y} \quad \rightarrow \quad F = \mu A\frac{v}{y}$$

The proportionally constant, μ , is called "dynamic viscosity of the fluid" "the absolute viscosity of the fluid".

$$\frac{F}{A} = \mu\frac{v}{y} \quad \rightarrow \quad \tau = \mu\frac{v}{y}$$

$$\mu = \frac{\text{Force}}{\text{Area}} \Big/ \frac{\text{Velocity}}{\text{Distance}} = \frac{\text{Force} \times \text{Time}}{\text{Area}} = \frac{\text{Mass}}{\text{Length} \times \text{Time}}$$

The perviouse equation is commonly known at **Newton's law of viscosity**. Most liquids abide by this relationship and are called **Newtonian fluids**. Liquids that do not abide by this linear relationship are known as **non-Newtonian fluids**. These include most house paints and blood.



The absolute viscosity has the dimension of "force per unit area (stress) times the time interval considered". It is usually measured in the unit of "**<u>poise</u>**".

$$\textbf{1 poise = 0.1 N • sec/m}^2$$

6- <u>Kinematic viscosity:</u>

Kinematic viscosity, υ, is obtained by dividing the absolute viscosity by the mass density of the fluid at the same temperature;

$$\nu = \mu / \rho = M\,L^{-1}\,T^{-1} / M\,L^{-3} = L^2\,T^{-1}$$

The kinematic viscosity unit is cm^2/sec "Stoke"

The absolute viscosities and the kinematic viscosities of pure water and air are shown as functions of temperature in the next table.

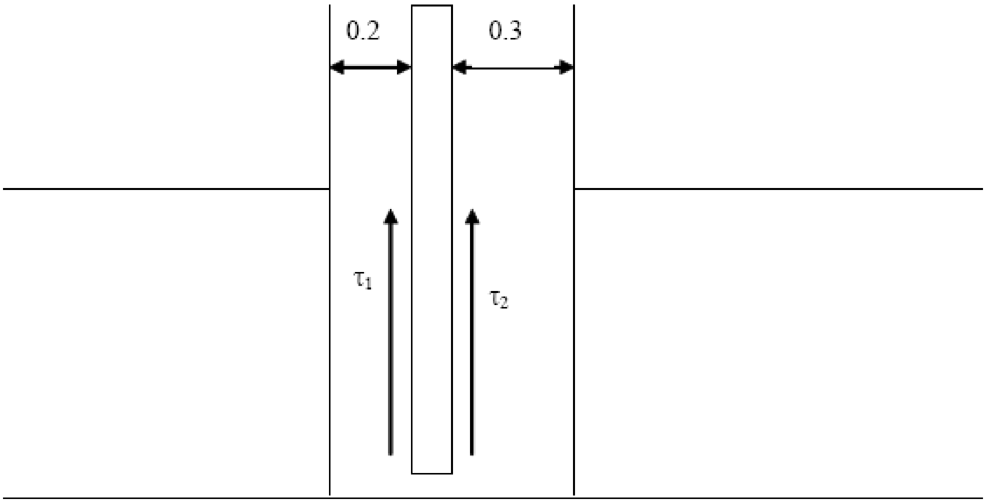
يلاحظ أن معامل اللزوجة للسوائل يقل مع ارتفاع درجة الحرارة والعكس صحيح مع الغازات. وذلك لأن زيادة درجة الحرارة فى السوائل يقلل ظاهرة الالتصاق وتقلل اللزوجة ، بينما فى الغازات تزيد سرعة الجزيئات وبالتالى زيادة تبادل الزخم وبالتالى اللزوجة.

	Water		Air	
Temperature (°C)	Absolute Viscosity	Kinematic Viscosity	Absolute Viscosity	Kinematic Viscosity
	$N \bullet$ sec/m^{2}	cm^{2}/sec	$N \bullet$ sec/m^{2}	cm^{2}/sec
0	1.781 x l0-^{3}	.785 x 10-^{6}	1.717 x 10-5	.329 x 10-^{5}
5	1.518 x l0-^{3}	.519 x 10-6	1.741 x l0-^{5}	.371 x l0-^{5}
10	1.307 x 10-^{3}	.306 x l0-^{6}	1.767x 10-^{5}	.417 x l0-^{5}
15	1.139 x l0-^{3}	1.139 x l0-^{6}	1.793 x l0-^{5}	.463 x 10-^{5}
20	1.002 x l0-^{3}	1.003 x l0-^{6}	1.817 x l0-^{5}	.509 x 10-^{5}
25	0.890 x 10-^{3}	3.893 x 10-^{6}	1.840 x l0-^{5}	.555 x10-^{5}
30	0.798 x l0-^{3}	3.800 x 10-6	1.864 x 10-'	.601 x 10-^{5}
40	0.653 x 10-^{3}	3.658 x 10-6	1.910 x l0-^{5}	.695 x 10-^{5}
50	0.547 x l0-^{3}	1553 x 10-^{6}	1.954 x l0-^{5}	.794 x l0-^{5}
60	0.466 x 10-^{3}	3.474 x 10-6	2.001 x 10-5	.886 x l0-^{5}
70	0.404 x 10-^{3}	3.413 x 10-^{6}	2.044 x l0-^{5}	.986 x 10-^{5}
80	0.354 x 10-^{3}	3.364 x I0-^{6}	2.088 x 10-^{5}	-.087 x 10-^{5}

90	0.315 x l0^{-3}	3.326 x l0^{-6}	2.131 x l0^{-5}	2.193 x l0^{-5}
100	0.282 x 10^{-3}	1294 x 10^{-6}	2.174 x l0^{-5}	-.302 x l0^{-5}

Example 1-6

A thin plate weighting 125gm takes 3.3 sec. to fall a distance of one meter in a liquid with a specific gravity of 0.9 between two vertical boundaries 0.5 cm apart, the plate moves at a distance of 0.2 cm from one of them. If the surface area of the plate is 1.5 m^2. What are the dynamic viscosity of the liquid in poise and the kinematic viscosity in stokes.



Solution

t=3.3 sec , d=100 cm, γ_L=0.9 , A=1.5 m^2

assume acceleration = 0

$$v = \frac{d}{t} = \frac{100}{3.3} = 30.3 \text{ cm/sec}$$

$A \times (\tau_1 + \tau_2) = W$

Dynamic viscosity = $\mu = 32.4 \times 10^{-3}$ poise

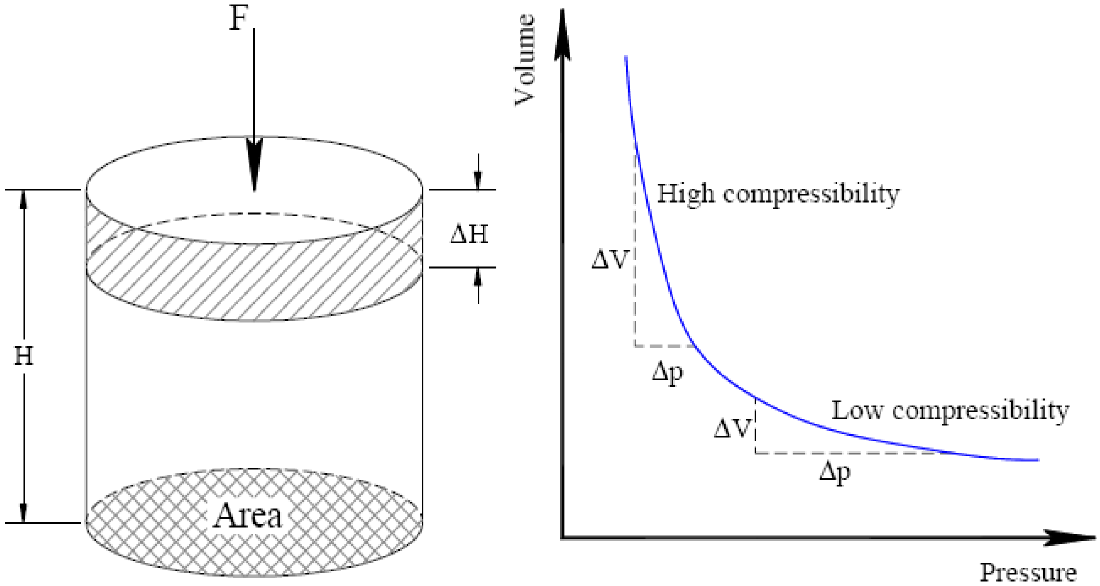
$$\text{Kinematic viscosity} = \frac{\mu}{\rho} = \frac{32.4 \times 10^{-3}}{0.9} = 36 \times 10^{-3} \ (\text{cm}^2/\text{sec})$$

7- Compressibility of liquids قابلية السوائل للانضغاط:

Compressibility (change in volume due to change in pressure) is inversely proportional to its volume modulus of elasticity (Bulk Modulus of Elasticity "K"). قابلية التغير فى الحجم نتيجة التغير فى الضغط

$$K = \frac{-dp}{d\cancel{V}/\cancel{V}} \qquad \text{units lb/in}^2\text{, or N/m}^2$$

$$\text{Compressibility} \rightarrow \beta = \frac{1}{K} \qquad (\text{m}^2/\text{N, or ft}^2/\text{lb})$$



Water is often considered **incompressible**, but it does have a finite, low compressibility. يعتبر الماء عمليا سائل غير قابل للانضغاط

Quantity	Eqn	F-L-T	M-L-T	S.I.	C.G.S.	English	Other
Length	L	L	L	m	cm	ft (‘)	inch (“)
Mass	m	$F\,L^{-1}\,T^{2}$	M	Kg	gm	slug	
Volume	$V = L^3$	L^3	L^3	m^3	cm^3 (cc)	ft^3	gallon
Velocity (Speed)	$v = \frac{L}{t}$	$L\,T^{-1}$	$L\,T^{-1}$	m/sec	cm/sec	ft/sec	
Acceleration	$a = \frac{L}{t^2}$	$L\,T^{-2}$	$L\,T^{-2}$	m/sec^2	cm/sec^2	ft/sec^2	
Gravity	g	$L\,T^{-2}$	$L\,T^{-2}$	m/sec^2	cm/sec^2	ft/sec^2	
Force	$F = m.a$	F	$M\,L\,T^{-2}$	$Kg.m/sec^2$ N	$gm.cm/sec^2$ dyne	$slug.ft/sec^2$ lb	
Density	$\rho = \frac{m}{V}$	$F\,L^{-4}\,T^{2}$	$M\,L^{-3}$	Kg/m^3	gm/cm^3	$slug/ft^3$	
Specific Weight	$\gamma = \frac{W}{V} = \rho g$	$F\,L^{-3}$	$M\,L^{-2}\,T^{-2}$	N/m^3	$dyne/cm^3$	lb/ft^3	
Specific Volume	$\frac{1}{\rho}$	$F^{-1}\,L^{4}\,T^{-2}$	$M^{-1}\,L^{3}$	m^3/Kg	cm^3/gm	$ft^3/slug$	
Specific Gravity	$S.G. = \frac{\rho_{liquid}}{\rho_{water}}$	- - -	- - -	- - -	- - -	- - -	- - -
Pressure (Stress)	$p = \frac{F}{A} = \rho g h$	$F\,L^{-2}$	$M\,L^{-1}\,T^{-2}$	N/m^2 Pa	$dyne/cm^2$	lb/ft^2 (psf) lb/in^2 (psi)	Bar, Torr atmosphere (mm Hg) (m water)

Quantity	Eqn	F-L-T	M-L-T	S.I.	C.G.S.	English	Other
Discharge	$Q = A.v$	$L^3 T^{-1}$	$L^3 T^{-1}$	m^3/sec (cum ecs)	cm^3/sec	ft^3/sec (cfs)	
Momentum	$M = m.v = F.t$	F T	$M L T^{-1}$	Kg.m/sec N.sec	gm.cm/sec dyne.sec	slug.ft/sec lb.sec	
Energy (Work) Torque (Moment)	$W = F.L$	F L	$M L^2 T^{-2}$	N.m Joule	dyne.cm erg	lb.ft	
Rotation (N, n)	$n = \frac{\omega}{2\pi}$	T^{-1}	T^{-1}	r.p.m. r.p.s.	r.p.m. r.p.s.	r.p.m. r.p.s.	
Angular Velocity	$\omega = \frac{v}{r}$	T^{-1}	T^{-1}	rad/sec	rad/sec	rad/sec	
Power	$P = F.v$	$F L T^{-1}$	$M L^2 T^{-3}$	N.m/sec Watt	dyne.cm/sec	lb.ft/sec	Horse
Bulk Modulus	$K = \frac{\Delta p}{-\frac{\Delta V}{V}}$	$F L^{-2}$	$M L^{-1} T^2$	N/m^2	$dyne/cm^2$	lb/ft^2	
Coeff. of Viscosity (Dynamic Viscosity)	$\mu = \tau.\frac{y}{v}$	$F L^{-2} T$	$M L^{-1} T^{-1}$	N.sec/m^2 Kg/m/sec Pa.sec	dyne.sec/cm^2 gm/cm/sec Poise	lb.sec/ft^2 slug/ft/sec	
Kinematic Viscosity	$\nu = \frac{\mu}{\rho}$	$L^2 T^{-1}$	$L^2 T^{-1}$	m^2/sec	cm^2/sec Stoke	ft^2/sec	
Surface Tension	$\delta = \frac{Work}{Area}$	$F L^{-1}$	$M T^{-2}$	Kg/sec^2 N/m	gm/sec^2 dyne/cm	slug/sec^2 lb/ft	

CHAPTER 2

FLUID PRESSURE AND HYDROSTATIC FORCE

2.1 Hydrostatic Pressure:

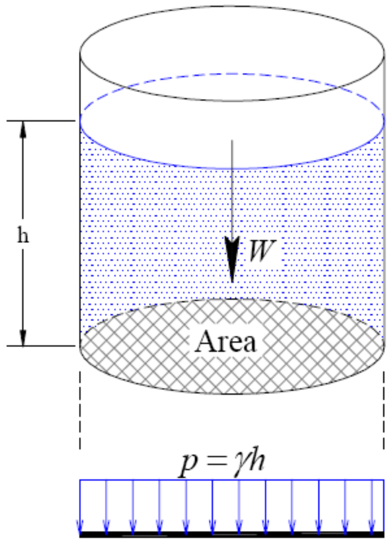
Force = weight of fluid, W

$$W = \gamma \cancel{V} = \gamma(A \times h)$$

Pressure = Force / area

$$p = \frac{W}{A} = \frac{\gamma(A \times h)}{A}$$

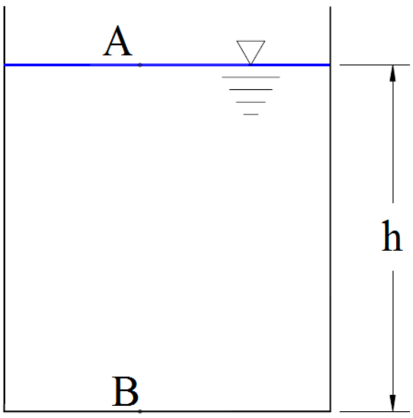
$$p = \gamma h$$



For liquids at rest, the pressure at all points in a horizontal plane is the same. جميع النقاط التى تقع فى مستوى افقى واحد فى سائل واحد لها نفس الضغط

2.2 Absolute and Gage Pressures

If the water body has a free surface that is exposed to atmospheric pressure, P_{atm}. Point A is positioned on the free surface such that $P_A = P_{atm}$



$$\boxed{(P_B)_{abs} = P_A + \gamma h = P_{atm} + \gamma h = \text{absolute pressure}}$$

The difference in pressure heads at two points in liquid at rest is always equal to the difference in elevation between the two points.

$$\boxed{(P_B/\gamma) - (P_A/\gamma) = \Delta(h)}$$

Gage pressure: is the pressure measured with respect to atmospheric pressure (using atmospheric pressure as a base).

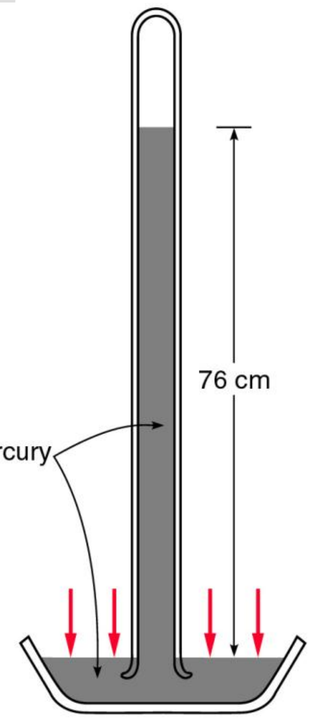
Atmospheric pressure:

P_{atm} = 10.33m high column of water = 10.33 γ_ω = 10.33 x 1 x 9810 =

= 0.76m high column of Hg = 0.76 γ_{Hg} = 0.76 x 13.60 x 9810 =

= 101.3 x 10^3 N/m^2 (Pa) ≈ 1 x 10^5 N/m^2 (Pa) ≈ 1 bar

= 14.70 Ib/in^2 = 2116 Ib/ft^2

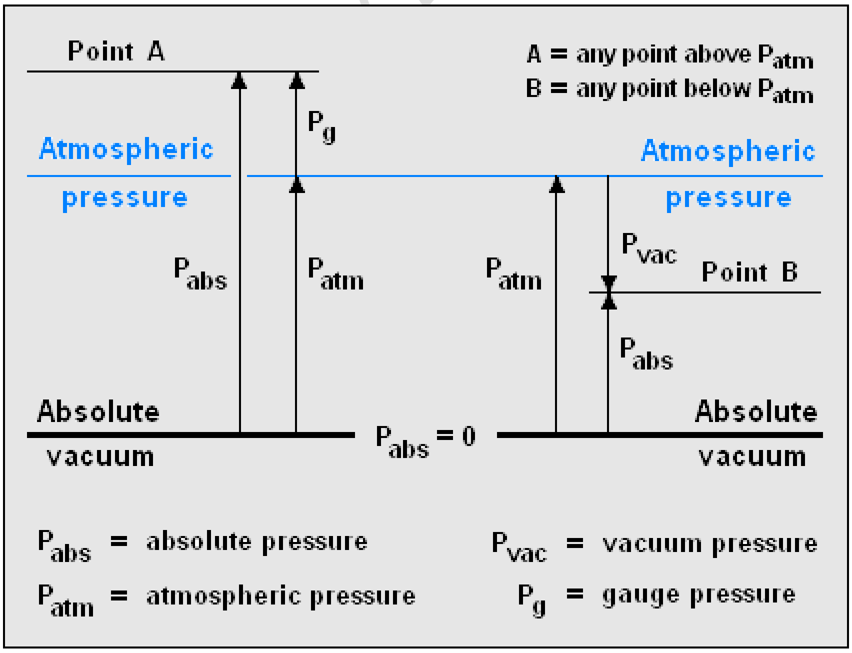


Absolute pressure: $P_{abs} = P_{gage} + P_{atm}$

If "P_{abs}" is negative it is called "**Vacum Pressure**".

the water body has a free surface that is exposed to atmospheric pressure, P_{atm}. Point A is positioned on the free surface such that $P_A = P_{atm}$

Pressure head, h = P/ γ

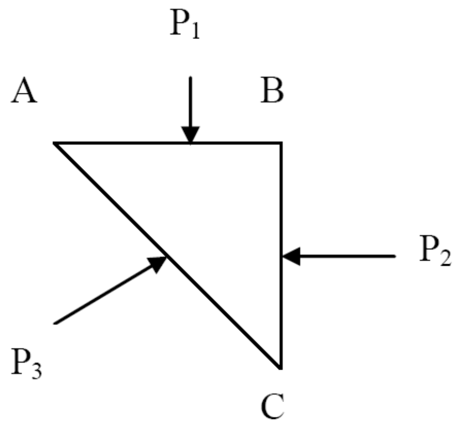


2.3 Pascal's law :

Pascal's law states that the pressure intensity at a point in a fluid at rest is the same in all directions.

قيمة شدة الضغط الهيدروستاتيكى عند نقطة معينة فى سائل ساكن متساوية فى جميع الاتجاهات

Consider a small prism of fluid of unit thickness in the z-direction contained in the bulk of the fluid as shown below. Since the cross-section of the prism is equilateral triangle, P_3 is at an angle of 45° with the x-axis. If the pressure intensities normal to the three surfaces are P_1, P_2, P_3, as shown then since:-



Force = Pressure x Area

Force on face AB = P_1 x (AB x I)

BC = P_2 x (BCx 1)

AC = P_3 x (ACx 1)

Resolving forces vertically:

P_1 x AB = P3 x AC cos θ

But AC cos θ = AB Therefore $P_1 = P_3$

Resolving forces horizontally:

P_2 x BC = P_3 x AC sin θ

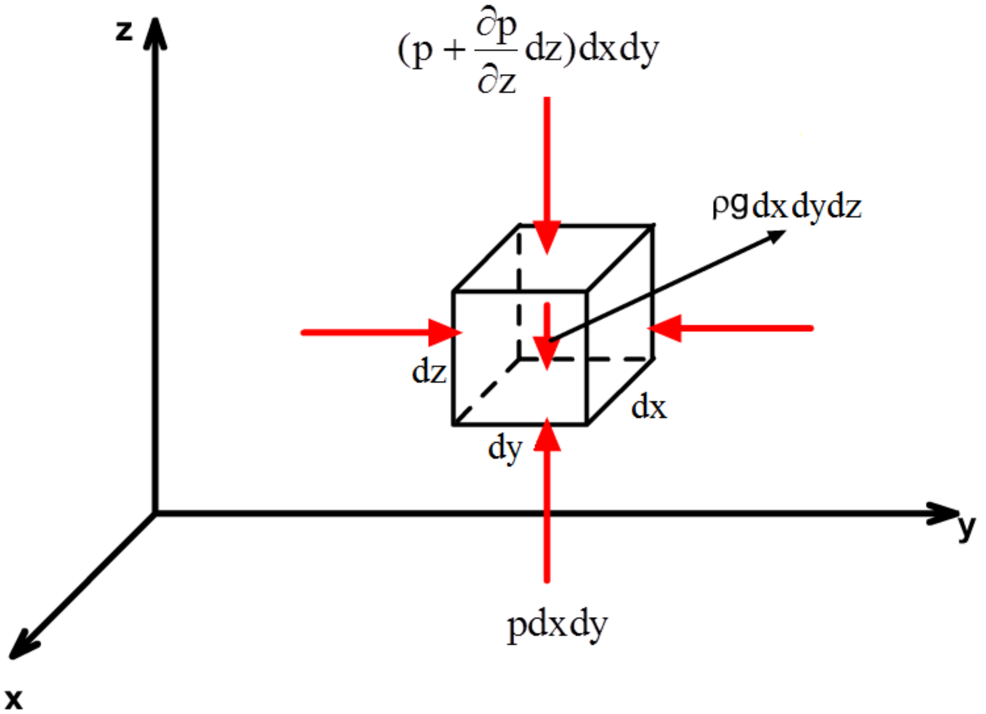
But AC sin θ = BC Therefore $P_2 = P_3$

Hence $P_1 = P_2 = P_3$,

In words: the pressure at any point is equal in all directions.

Pressure Variation in A static fluid

For a static fluid, pressure varies only with elevation within the fluid. This can be shown by consideration of equilibrium of forces on a fluid element



Newton's law (momentum principle) applied to a static fluid

$\Sigma \underline{F} = m\underline{a} = 0$ for a static fluid

i.e., $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$

$$\Sigma F_z = 0$$

$$pdxdy - (p + \frac{\partial p}{\partial z} dz)dxdy - \rho g dxdydz = 0$$

$$\frac{\partial p}{\partial z} = -\rho g = -\gamma$$

Basic equation for pressure variation with elevation

$$\Sigma F_y = 0$$

$$pdxdz - (p + \frac{\partial p}{\partial y} dy)dxdz = 0$$

$$\frac{\partial p}{\partial y} = 0$$

$$\Sigma F_x = 0$$

$$pdydz - (p + \frac{\partial p}{\partial x} dx)dydz = 0$$

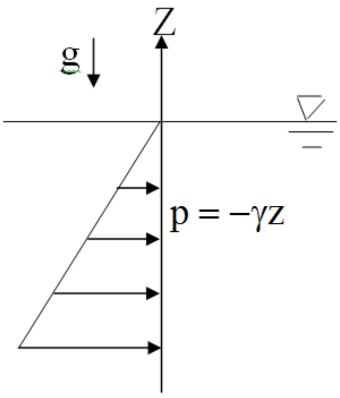
$$\frac{\partial p}{\partial x} = 0$$

For a static fluid, the pressure only varies with elevation z and is constant in horizontal xy planes.

فى حالة السوائل الساكنة تتغير قيمة الضغط فى الاتجاه الراسى فقط بينما تكون ثابتة فى المستوى الأفقى

Pressure Variation for a Uniform-Density Fluid

$$\frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad \rho = \text{constant for liquid}$$



$$\Delta p = -\gamma \Delta z$$

$$p_2 - p_1 = -\gamma\left(z_2 - z_1\right)$$

Alternate forms:

$$p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{ constant}$$

$$p + \gamma z = \text{constant} \qquad \text{(piezometric pressure)}$$

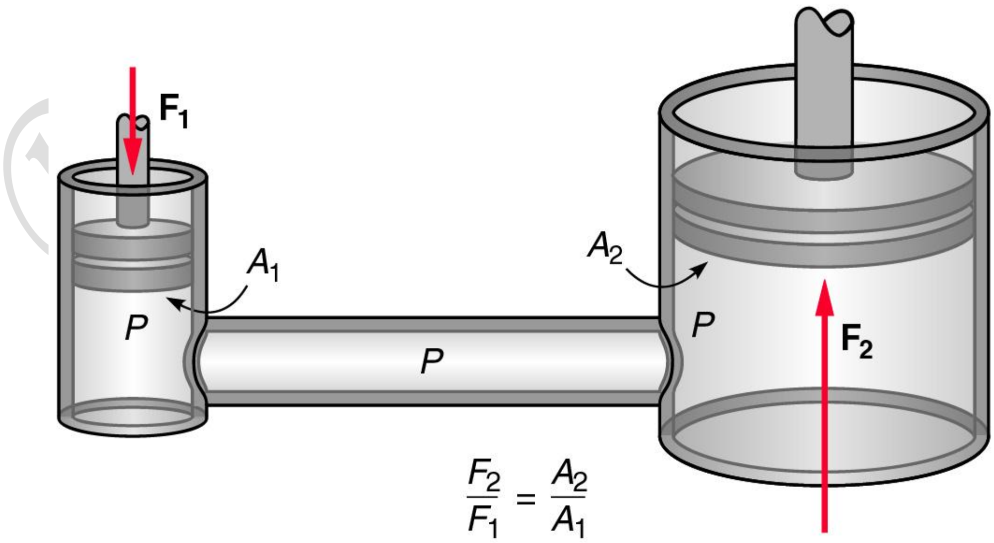
$$p(z=0) = 0 \qquad \text{gage}$$

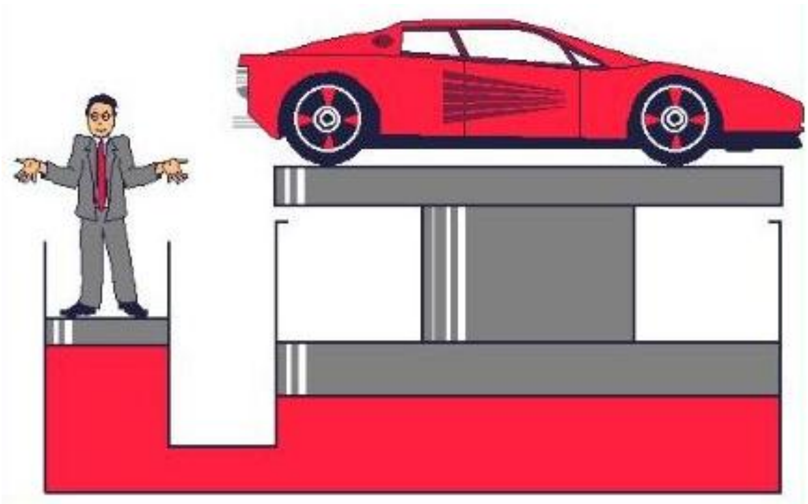
i.e., $p = -\gamma z$ increase linearly with depth

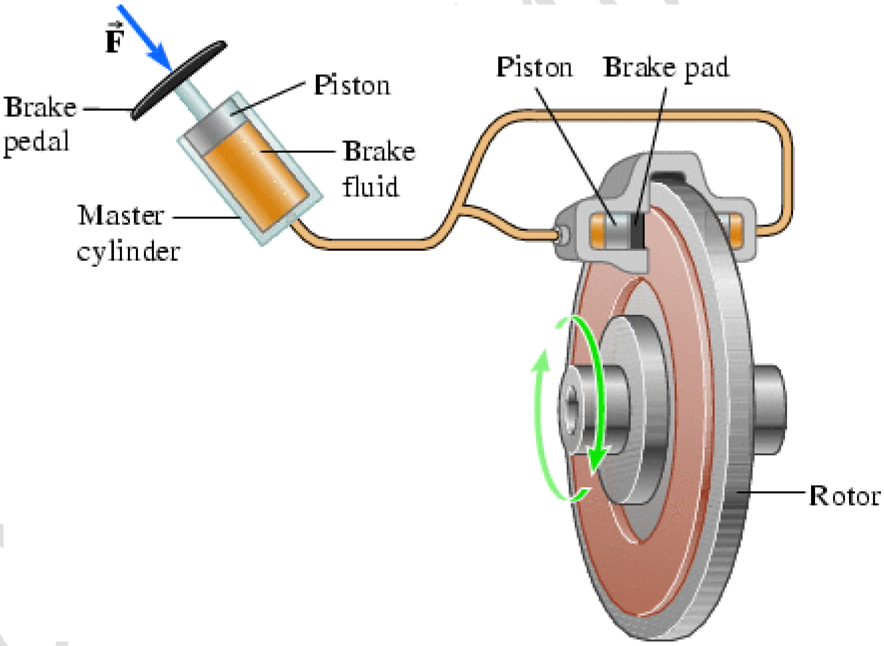
$$\frac{p}{\gamma} + z = \text{ constant} \qquad \text{(piezometric head)}$$

Pascal's Application:

A small force F_1 applied to a piston with a small area produces a much larger force F_2 on the larger piston. This allows a hydraulic jack to lift heavy objects.

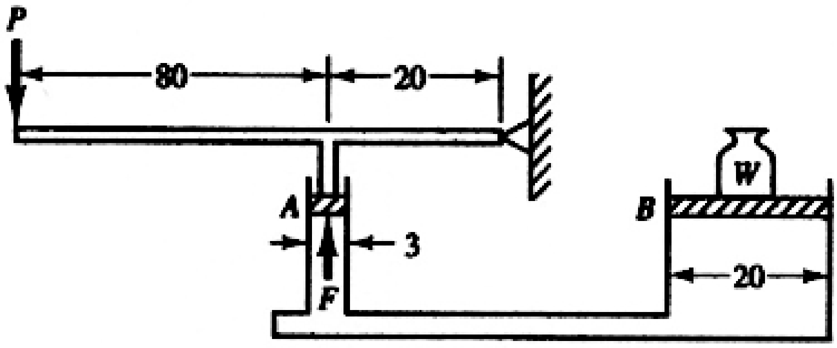





F
Brake pedal
Piston
Brake fluid
Master cylinder
Piston
Brake pad
Rotor

Example 2-1

The diameters of cylindrical pistons A and B are 3 cm and 20 cm, respectively. The faces of the pistons are at the same elevation and the intervening passages are filled with an incompressible hydraulic oil. A force “P” of 100 N is applied at the end of the lever, as shown in the figure. What weight “W” can the hydraulic jack support?



Measurements in cm

Solution

Balancing the moments produced by P and F, we obtain for the equilibrium condition.

$$P \cdot (80 + 20) = F \cdot (20)$$
$$100 \cdot (100) = F \cdot (20)$$

Thus,

$$F = 500 \text{ N}$$

From Pascal's law, the pressure P_A applied at A should be the same as that of P_B applied at B. We may write

$$P_A = \frac{F}{\frac{1}{4}(\pi \cdot 3^2)}; \quad P_B = \frac{W}{\frac{1}{4}(\pi \cdot 20^2)}$$

$$\frac{500}{\frac{1}{4}(\pi \cdot 3^2)} = \frac{W}{\frac{1}{4}(\pi \cdot 20^2)}$$

$$\therefore W = 500 \times \frac{20^2}{3^2} = 22{,}222 \text{ N}$$

2.4 Free Surface of Water

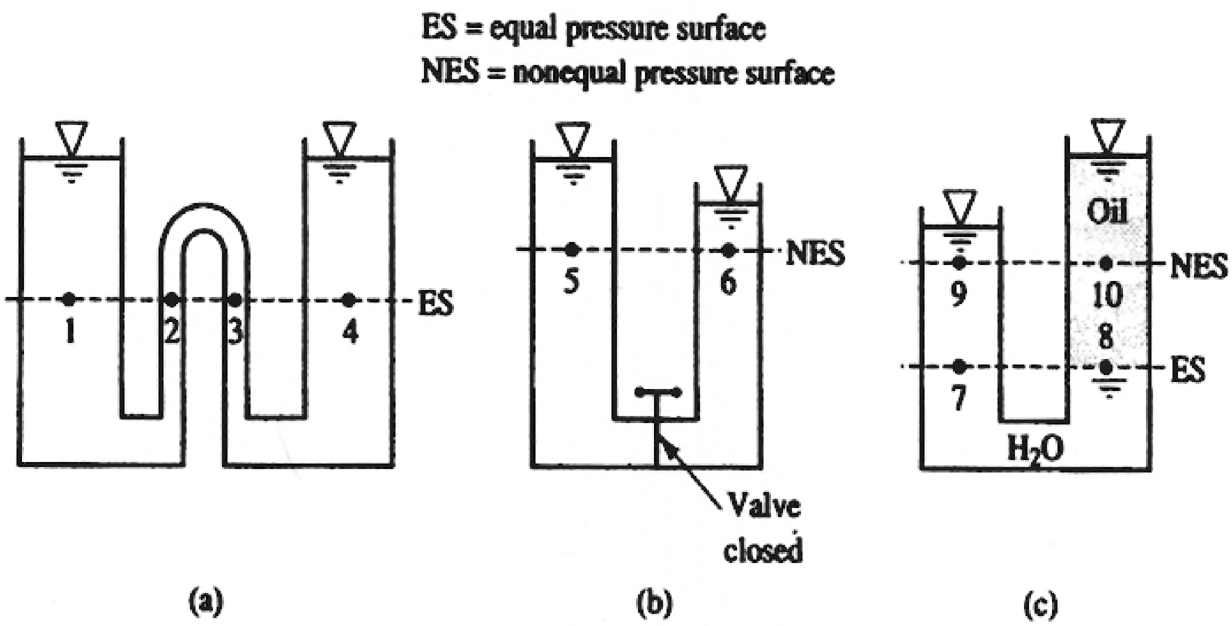
Free surface of liquid in a vessel may be subjected to:

- atmospheric pressure (open vessel) or,
- any other pressure that is exerted in the vessel (closed vessel).

2.5 Surface of Equal Pressure

- The hydrostatic pressure in a body of liquid varies with the **vertical distance** measured from the free surface of the water body.

All points on **a horizontal surface** in the liquid have the same pressure.



2.6 Manometers:

__**A manometer:**__ Is a tube bent in the form of a U containing a fluid of known specific gravity. The difference in elevations of the liquid surfaces under pressure indicates the difference in pressure at the two ends.

The liquid used in a manometer is usually **heavier than the fluids to be measured.** It must not mix with the adjacent liquids (i.e., immiscible liquids).

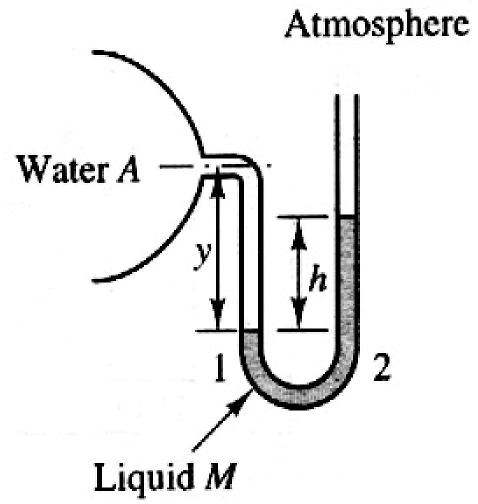
The most used liquids are:

- Mercury (specific gravity = 13.6),
- Water (sp. gr. = 1.00),
- Alcohol (sp. gr. = 0.9), and
- Other commercial manometer oils of various specific gravities.

Two types of manometers:

1. An open manometer: has one end open to atmospheric pressure and is capable of measuring the gage pressure in a vessel.

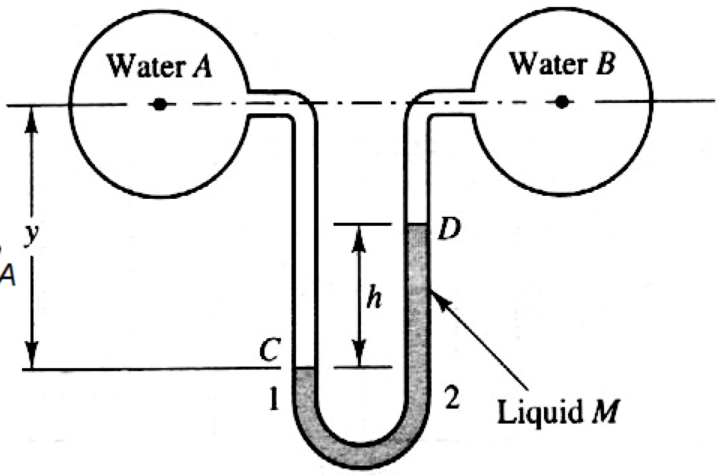
$$P_2 = P_1$$
$$\gamma_M \cdot h = \gamma_W \cdot y + P_A$$
$$\boxed{P_A = \gamma_M \cdot h - \gamma_W \cdot y}$$



2. A differential manometer: connects each end to a different pressure vessel and is capable of measuring the pressure difference between the two vessels.

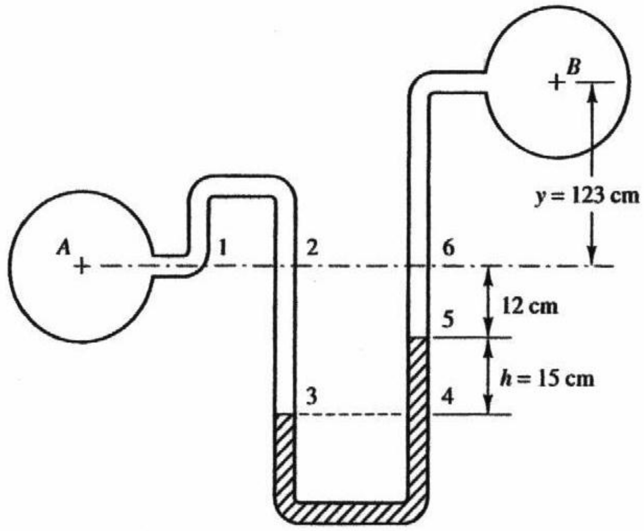
Inverted U-tube manometer is used when there is a small pressure difference or a light liquid such as oil is used.

$$P_2 = P_1$$
$$\gamma_M \cdot h + \gamma_w \cdot (y - h) + P_B = \gamma_W \cdot y + P_A$$
$$\boxed{\Delta P = P_A - P_B = h\,(\gamma_M - \gamma_w)}$$



Example 2-2

Determine the pressure difference ΔP



Solution

$$P_3 = P_4$$
$$P_A = P_1 = P_2$$

The pressures at points 3 and 4 are, respectively,

$$P_3 = P_2 + \gamma_W \cdot (27\text{ cm}) = P_A + \gamma_W \cdot (27\text{ cm})$$
$$P_4 = P_B + \gamma_W \cdot (135\text{ cm}) + \gamma_M \cdot (15\text{ cm})$$

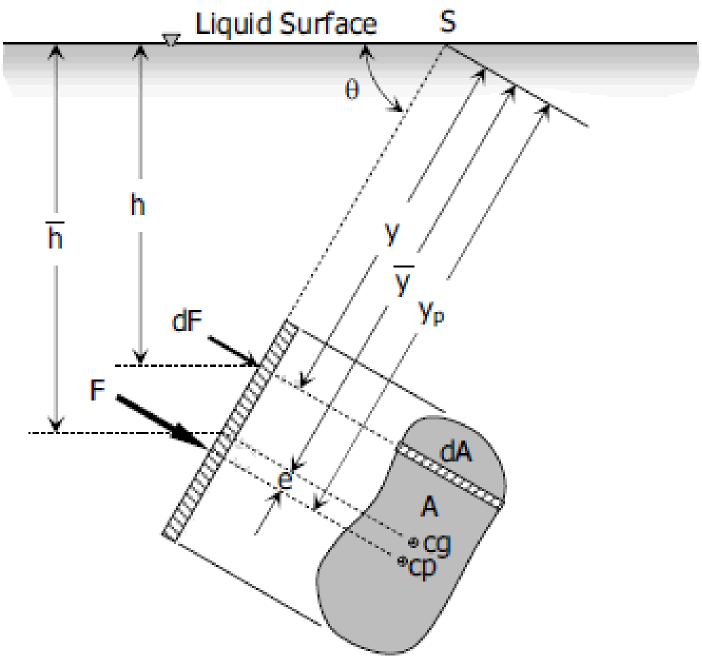
So that,

$$P_3 = P_A + \gamma_W \cdot (27\text{ cm}) = P_4 = P_B + \gamma_W \cdot (135\text{ cm}) + \gamma_M \cdot (15\text{ cm})$$

and,

$$\begin{aligned}\Delta P &= P_A - P_B = \gamma_W \cdot (135\text{ cm} - 27\text{ cm}) + \gamma_M \cdot (15\text{ cm}) \\ &= \gamma_W \cdot [108 + (13.6)(15)]\text{ cm} \\ &= (9800\text{ N/m}^3)(3.12\text{ m}) = 30{,}600\text{ N/m}^2 \text{ or } 30.6\text{ kN/m}^2\end{aligned}$$

2.7 Hydrostatic Force on a plane Surface



For a strip at depth h below the free surface:

$$P = \gamma h = \gamma\, y \sin\theta$$

The differential force dF acting on the element dA is:

$$dF = \gamma\, y \sin\theta . dA$$

Integrate both sides and note that γ and θ are constants,

$$F = \int_A dF = \gamma \sin\theta \int_A y\, dA = \gamma \sin\theta . A . \bar{y}$$ "first moment of area about point S"

$$\boxed{F = \gamma . A . \bar{h}}$$

Location of Total Hydrostatic Force (Centre of Pressure)

From the figure above, S is the intersection of the prolongation of the submerged area to the free liquid surface. Taking moment about point S.

$$F . y_P = \int y\, dF$$

Where,

$$dF = \gamma\, y \sin\theta . dA$$

$$F = \gamma\ .A.\bar{h}$$

$$[\gamma(\bar{y}\sin\theta)A]y_p = \int y\,[\gamma(y\sin\theta)\,dA]$$

$$(\gamma\sin\theta)A\bar{y}\,y_p = (\gamma\sin\theta)\int y^2\,dA$$

$$A\bar{y}\,y_p = \int y^2\,dA$$

Again from Calculus, $\int y^2\,dA$ is called moment of inertia denoted by I. Since our reference point is S,

$$A\bar{y}\,y_p = I_S$$

Thus,

$$\boxed{y_p = \frac{I_S}{A\bar{y}}}$$

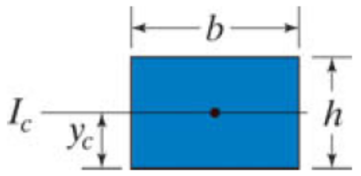
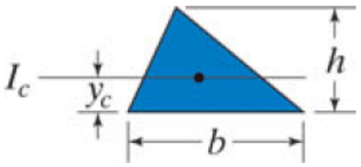
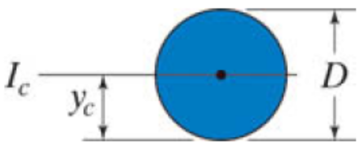
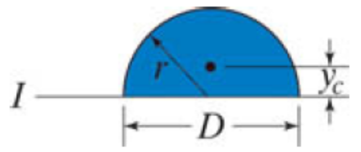
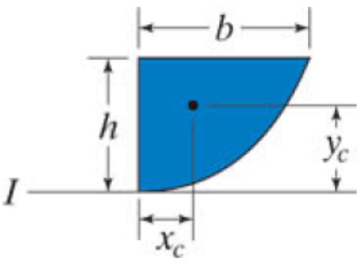
By transfer formula for moment of inertia $I_S = I_g + A\bar{y}^2$, the formula for y_p will become $y_p = \frac{I_g + A\bar{y}^2}{A\bar{y}}$ or

$$\boxed{y_p = \bar{y} + \frac{I_g}{A\bar{y}}}$$

From the figure $y_p = \bar{y} + e$ above, , thus, the distance between "cg" and "cp" is

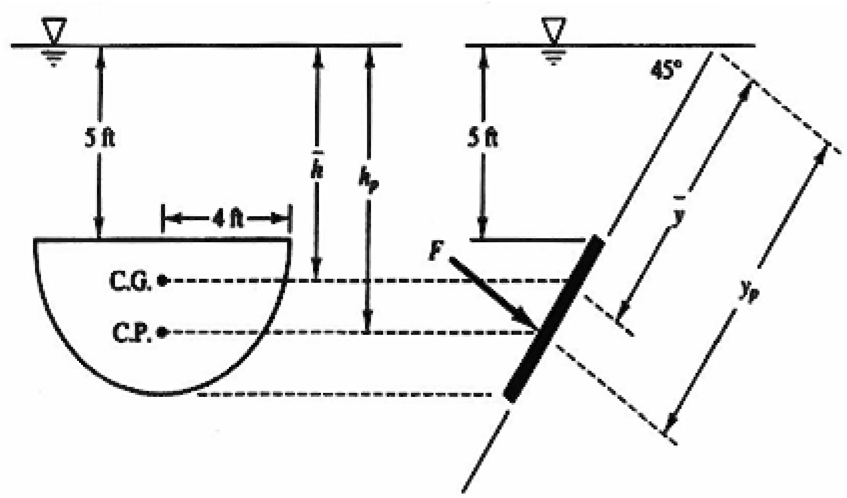
$$\boxed{\text{Eccentricity,}\quad e = \frac{I_g}{A\bar{y}}}$$

Properties of areas

Shape	Sketch	Area	Location of Centroid	Ic
		$b \times h$	$y_c = \frac{h}{2}$	$I_c = \frac{bh^3}{12}$
		$\frac{bh}{2}$	$y_c = \frac{h}{3}$	$I_c = \frac{bh^3}{36}$
		$\frac{\pi D^2}{4}$	$y_c = \frac{D}{2}$	$I_c = \frac{\pi D^4}{64}$
		$\frac{\pi D^2}{8}$	$y_c = \frac{4r}{3\pi}$	$I = \frac{\pi D^4}{128}$
		$\frac{2bh}{3}$	$x_c = \frac{3b}{8}$ $y_c = \frac{h}{2}$	$I = \frac{2bh^3}{7}$

Example 2-3

Determine F and y_P



Solution

$$F = \gamma \cdot \sin\theta \cdot \bar{y} \cdot A$$

where

$$A = \frac{1}{2}(\pi \cdot 4^2) = 25.1 \text{ ft}^2$$

and

$$\bar{y} = 5 \sec 45° + \frac{4 \cdot 4}{3\pi} = 8.77 \text{ ft}$$

Therefore,

$$F = 62.4 \cdot (\sin 45°)(8.77)(25.1) = 9710 \text{ lbs.}$$

$$Y_p = \frac{I_0}{A\bar{y}} + \bar{y}$$

From Table

$$I_0 = \frac{9\pi^2 - 64}{72\pi} r^4$$

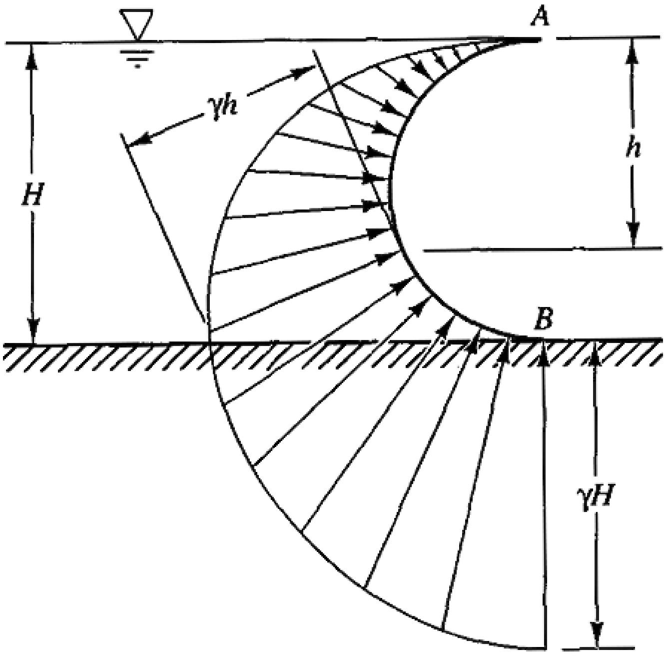
$$= 28.10 \text{ ft}^4$$

Therefore,

$$Y_p = \frac{28.10}{25.13(8.77)} + 8.77 = 8.90 \text{ ft}$$

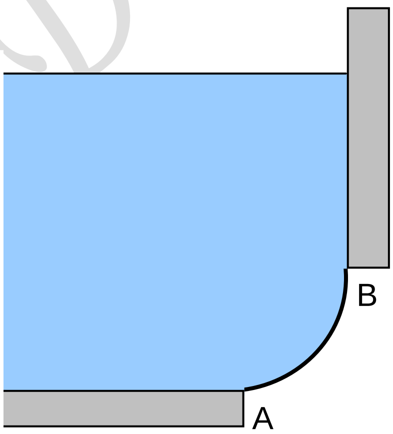
2.8 Hydrostatic Forces on Curved Surfaces

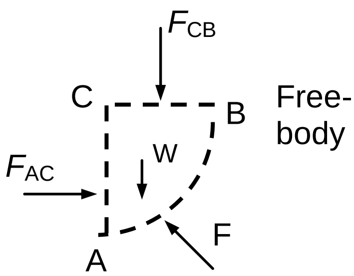
For a curved surface, the pressure forces, being normal to the local area element, vary in direction along the surface and thus cannot be added numerically. The figure shows pressure distribution on a semi-cylindrical gate.

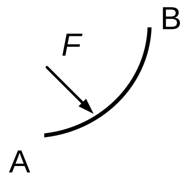


يؤثر الضغط عموديا على السطح، فإذا كان السطح دائريا مرت المحصلة "**F**" فى مركزه، ولمعرفة اتجاه المحصلة نفترض وجود ثقب فى الـ curved surface فيكون اتجاه خروج السائل هو اتجاه المحصلة. ويتم تحليل المحصلة الى مركبتين الأولى أفقية والأخرى رأسية.

The resultant hydrostatic force is computed by considering the free-body diagram of a body of fluid in contact with the curved surface, as illustrated below.







The steps involved in calculating the horizontal and vertical components of the hydrostatic force F are as follows:

- Summation of forces in the horizontal direction "$\mathbf{F_H}$" gives

$$F_H = F_{AC}$$

where F_{AC} is the hydrostatic force on plane surface AC. It acts through the center of pressure of side AC.

المركبة الأفقية "$\mathbf{F_H}$" هى عبارة عن القوى المؤثرة على اسقاط الـ curved surface على مستوى رأسى.

- Summation of forces in the vertical direction "$\mathbf{F_V}$" gives

$$F_V = W + F_{CB}$$

where W is the weight of the fluid (acting through the center of gravity) of the free-body diagram and F_{CB} is the hydrostatic force (acting through the centroid) on the surface CB.

المركبة الرأسية "$\mathbf{F_V}$" هى عبارة عن وزن السائل المحصور بين الـ curved surface ومسقطه على الـ free surface.

- The line of action of "F_V" is obtained by summing the moments about any convenient axis.
- The overall resultant force is found by combining the vertical and horizontal components vectorialy:

$$\boxed{F = \sqrt{F_H^2 + F_V^2}}$$

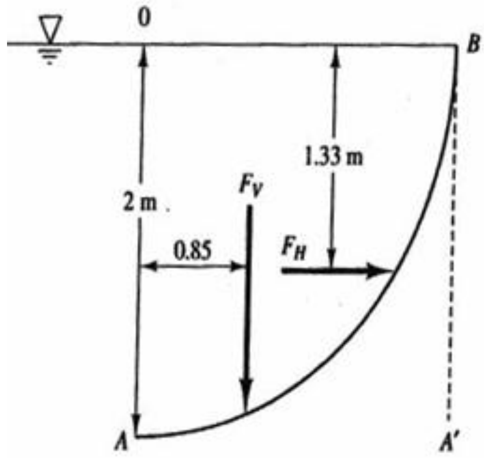
- The angle the resultant force makes to the horizontal is:

$$\boxed{\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)}$$

- The position of "**F**" is the point of intersection of the horizontal line of action of "$\mathbf{F_H}$" and the vertical line of action of "$\mathbf{F_V}$" .

Example 2-4

Determine the total hydrostatic pressure and the center of pressure on the 3 m long, 2m high quadrant gate in the shown figure.



Solution

The horizontal component is equal to the hydrostatic pressure force on the projection plane $A^{/}B$.

$$F_H = \gamma \cdot \bar{h} \cdot A = 9810\left(\frac{1}{2} \cdot 2\right)(2 \cdot 3) = 58{,}860 \text{ N}$$

The location of the horizontal component is $Y_p = I_0/A\bar{y} + \bar{y}$, where $A = 6\text{m}^2$ (projected area) and $I_0 = [(3)(2^3)]/12 = 2\text{m}^4$, $Y_p = 2/6(1) + 1 = 1.33$ m below the free surface. The vertical component is equal to the weight of water in the volume AOB. The direction of this pressure component is downward.

$$F_v = \gamma V = 9810\left[\frac{1}{4}\pi \cdot (2^2)\right](3) = 92{,}160 \text{ N}$$

The pressure center is located at $4(2)/3\pi = 0.85$ m. The resultant force of water on AB is

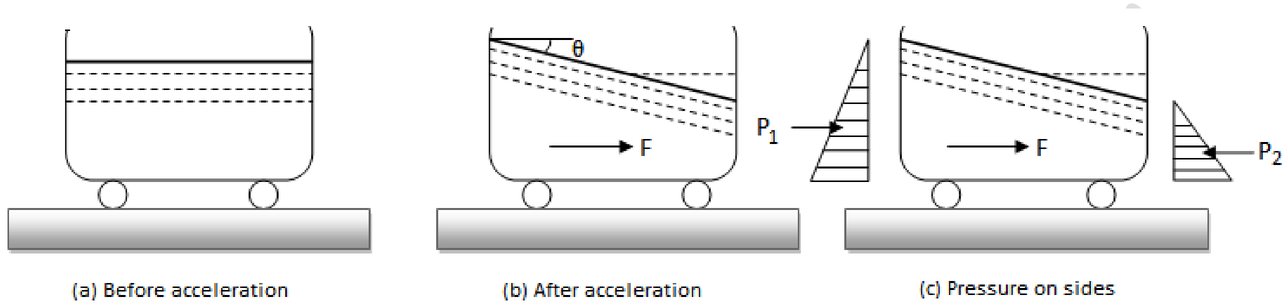
$$F = \sqrt{(58{,}860)^2 + (92{,}460)^2} = 109{,}600 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_v}{F_H}\right) = \tan^{-1}\frac{92{,}460}{58{,}860} = 57.5°$$

CHAPTER 3

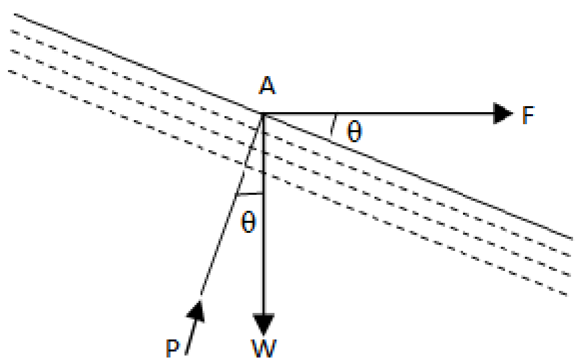
FLUID MASSES SUBJECTED TO ACCELERATION

3.1 Fluid Mass Subjected to Horizontal Acceleration:



- Consider a tank felled with liquid moves towards the right side with a uniform acceleration "**a**".
- As the tank stats moving under the action of acceleration force, the liquid does not remain in horizontal level.
- The liquid surface falls down on the direction of motion and rise up on the back side of the tank, the liquid surface makes angle "**θ**" with the horizontal.
- Consider the equilibrium of a fluid particle a lying on the inclined free surface as shown in the figure.

The force acting on the liquid particle are weight of the particle "**W = mg**" acting vertically downwards, accelerating force "**F**" acting toward right and normal pressure "**P**" exerted by the liquid.



Acceleration force $F = ma$

Resolving horizontally $P \sin\theta = F = ma$ (i)

Resolving vertically $P \cos\theta = W = mg$ (ii)

Dividing Equation (i) to (ii)

$$P \sin\theta / P \cos\theta = ma / mg = a/g$$

$$\boxed{\tan\theta = a/g}$$

فى حالة عدم وجود عجلة أفقية أو كان السائل يتحرك بسرعة منتظمة uniform velocity فإنه لا يوجد تغير فى الضغط فى الاتجاه الأفقى ويكون السطح أفقيا تماماً.

Now consider the equilibrium of the entire mass of the liquid. Let P_1 and P_2 are the hydrostatic pressure on the back side and front side of the tank.

Net force $P = P_1 - P_2$

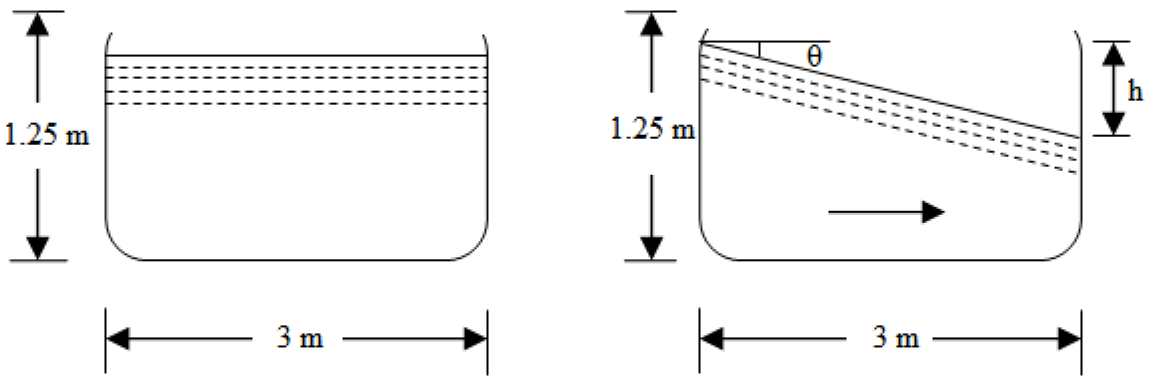
By Newton's second law of motion,

$P = ma$

$\mathbf{(P_1-P_2) = ma}$

Example 3-1

An open rectangular tank 3m long. 2.5m wide and 1.25m deep is completely filled with water. If the tank is moved with an acceleration of $1.5 m/s^2$, find the slope of the free surface of water and the quantity of water which will spill out of the tank.



(a) Before acceleration (b) After acceleration

Solution

Slope of the free surface of water

$$\tan\theta = \frac{a}{g} = \frac{1.5}{9.81} = 1.53 \qquad \Rightarrow \theta = 8.7^\circ$$

Quantity of water which will spill out of the tank

From the above figure we can see that the depth of water on the front side,

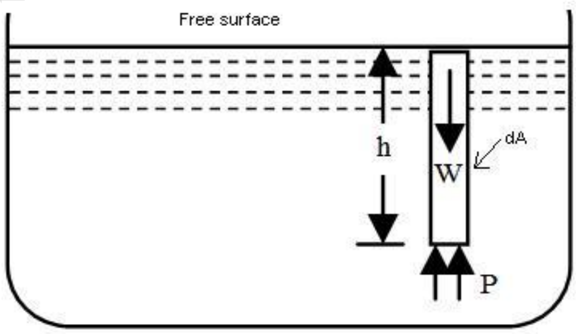
$$h = 3\tan\theta = 3 \times 0.153 = 0.459\ m$$

∴ Quantity of water which will spill out of the tank,

$$V = \frac{1}{2} \times 3 \times 2.5 \times 0.459 = 1.72\ m^3 = 1720\ litres$$

3.2 Fluid Mass Subjected to Vertical Acceleration :

Consider a tank containing liquid and moving vertically upwards with a uniform acceleration. Since the tank is subjected t an acceleration in the vertical direction only, therefore the liquid surface will remain horizontal. Now consider a small column of liquid of height h and area dA as shown in the figure.



The force acting on the liquid column s the weight of liquid column W acting vertically downwards, acceleration force F and pressure force exerted by the liquid on the column.

Applying Newton's second law of motion,

$\Sigma F_y = m\, a_y$

$P - W = m\, a_y$

$p.dA - \gamma hdA \;= \gamma hdA / g.\, a$

$p.dA = \gamma hdA / g\, .a + \gamma hdA$

$$\boxed{\mathbf{p = \gamma h\,(1 + a/g)}}$$

إذا تحرك خزان محتويا على سائل رأسيا إلى أعلى بعجلة منتظمة **a** ، فإن الضغط يزداد طرديا مع زيادة العجلة الرأسية ويظل سطح السائل أفقيا ما لم يتعرض لعجلة أفقية.

If the liquid is moving vertically downward with a uniform acceleration, the pressure will be,

$$\boxed{\mathbf{p = \gamma h\,(1 - a/g)}}$$

Example 3-2

An open rectangular tank 4m long and 2.5m wide contains an oil of specific gravity 0.85 up to a depth of 1.5m. Determine the total pressure on the bottom of the tank, when the tank is moving with an acceleration of "g/2" m/s^2 (i) <u>vertically upwards</u> (ii) <u>vertically downwards</u>.

Solution

<u>(i) Total pressure on the bottom of the tank, when it is vertically upwards:</u>

$\gamma_{oil} = 0.85 \times 9810 = 8.34\ KN/m^3$

$p = \gamma h\,(1 + a/g) = 8.34 \times 1.50\,(1 + g/2g) = 18.765\ KN/m^2$

Total pressure force on the bottom of the tank,

$P = 18.765 \times (4 \times 2.50) = 187.65\ KN$

<u>(ii) Total pressure on the bottom of the tank, when it is vertically downwards:</u>

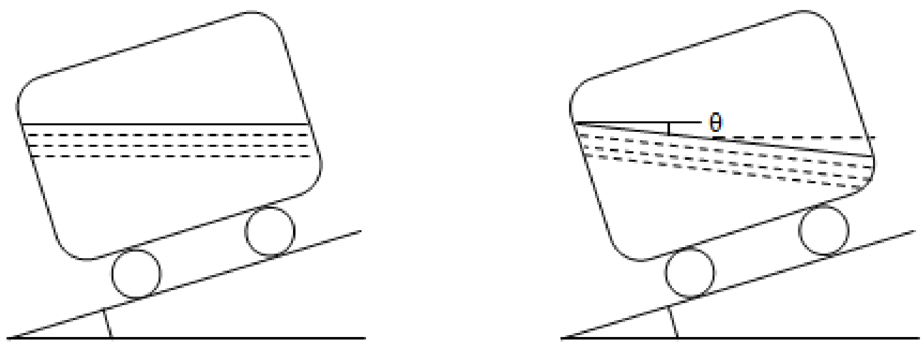
$p = \gamma h\ (1 - a/g) = 8.34 \times 1.50\ (1 - g/2g) = 6.255\ KN/m^2$

Total pressure force on the bottom of the tank,

$P = 6.255 \times (4 \times 2.50) = 62.55\ KN$

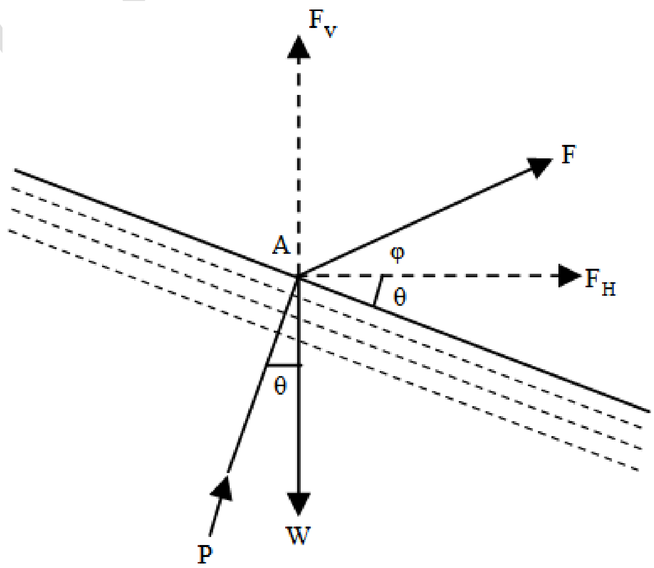
3.3 Accelerated along inclined plane:

Consider a tank open at top, containing a liquid and moving upwards along inclined plane with a uniform acceleration as shown in the figure.



(a) Before acceleration (b) After acceleration

We know that when the tank starts moving, the liquid surface falls down on the front side and rises up on the back side of the tank as shown in the figure.



Let,

ϕ = Inclination of the plane with the horizontal

θ = Angle, which the liquid surface makes with the horizontal, and

a = Acceleration of the tank

Acceleration force $F = ma$

Resolving horizontally $P \sin\theta = F_H = F \cos\phi = m\,a \cos\phi$ (i)

Resolving vertically $P \cos\theta = W + F_V = mg + m\,a \sin\phi$ (ii)

Dividing Equation (i) to (ii)

$$P \sin\theta / P \cos\theta = m\,a \cos\phi / mg + m\,a \sin\phi$$

$$\tan\theta = a \cos\phi / g + a \sin\phi$$

$$\boxed{\tan\theta = a_H/(g+a_V)}$$

If the liquid is moving downward,

$$\boxed{\tan\theta = a_H/(g-a_V)}$$

where,

a_H = horizontal component of the acceleration

a_V = vertical component of the acceleration

Example 3-3

A rectangular box containing water is accelerated at 3 m/s^2 upwards on an inclined plane 30 degree to the horizontal. Find the slope of the free liquid surface.

Solution

$a_H = 3 \cos 30 = 2.598\ m/s^2$

$a_V = 3 \sin 30 = 1.50\ m/s^2$

$$\boxed{\begin{array}{l}\tan\theta = 2.598/(9.81+1.50) = 0.2297 \\ \theta = 12.94^0\end{array}}$$

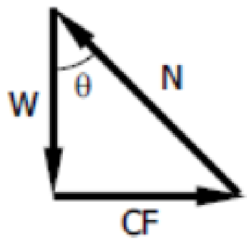
3.4 Uniform Rotation about Vertical Axis:

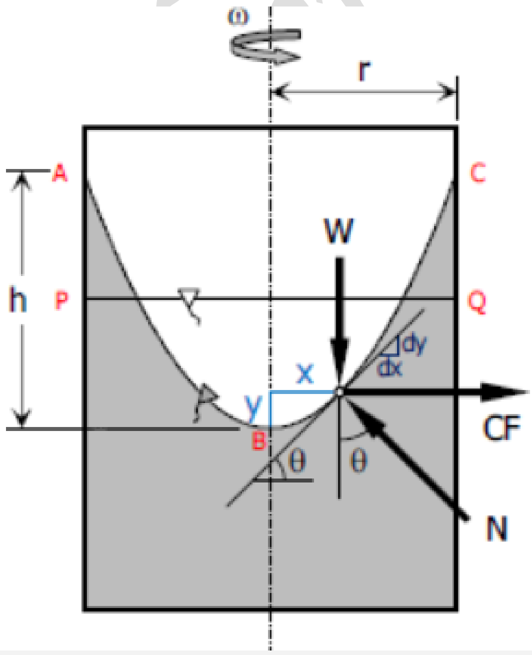
When at rest, the surface of mass of liquid is horizontal at **PQ** as shown in the figure. When this mass of liquid is rotated about a vertical axis at constant angular velocity **ω** radian per second, it will assume the surface **ABC** which is parabolic. Every particle is subjected to centripetal force or centrifugal force "قوة طاردة مركزية" **CF = mω²x** which produces centripetal acceleration towards the center of rotation. Other forces that acts are gravity force **W = mg** and normal force **N**.

$$\tan\theta = \frac{CF}{W}$$

$$\tan\theta = \frac{m\omega^2 x}{mg}$$

$$\boxed{\tan\theta = \frac{\omega^2 x}{g}}$$





Where tan θ is the slope at the surface of paraboloid at any distance x from the axis of rotation.

From Calculus, y' = slope, thus

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$dy = \frac{\omega^2}{g} x \, dx$$

$$\int dy = \frac{\omega^2}{g} \int x \, dx$$

$$\boxed{y = \frac{\omega^2 x^2}{2g}}$$

For cylindrical vessel of radius “**r**” revolved about its vertical axis, the height “**h**” of paraboloid is:

$$\boxed{h = \frac{\omega^2 r^2}{2g}}$$

حجم القطع المكافئ الدورانى = ½ مساحة القاعدة × الارتفاع

CHAPTER 4

EQUILIBRIUM OF FLOATING BODIES

4.1 Principle of Buoyancy

The general principle of buoyancy is expressed in the Archimedes' principle, which is stated as follows:

"For an object partially or completely submerged in a fluid, there is a net upward force (buoyant force) equal to the weight of the displaced fluid".

قاعدة ارشميدس تنص على أنّ الجسم المغمور كليا أو جزئيا في مائع يكون مدفوعا بقوه إلى أعلى تسمى "قوة الطفو" أو "قوة التعويم"، وهذه القوة تعادل وزن حجم المائع الذي يزيحه الجسم كليا أو جزئيا على الترتيب.

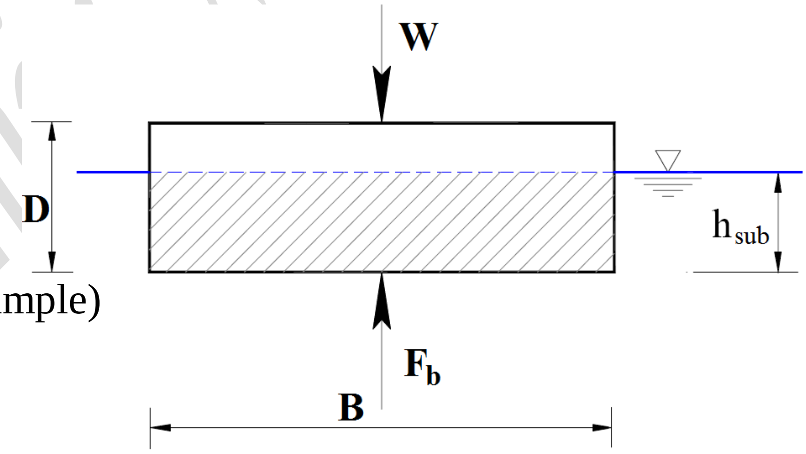
Weight of floating body (W) = Weight of liquid displaced (F_b)

$W = F_b$

$\gamma_b \,.\, V_b = \gamma_w \,.\, V_{sub}$

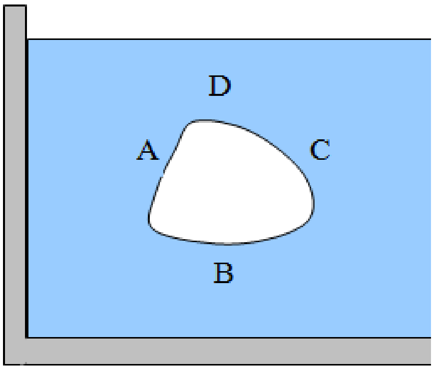
$S.G.\ \gamma_w \,.\, LBD = \gamma_w \,.\, LBh_{sub}$

$h_{sub} = S.G.\ D$ (in this example)

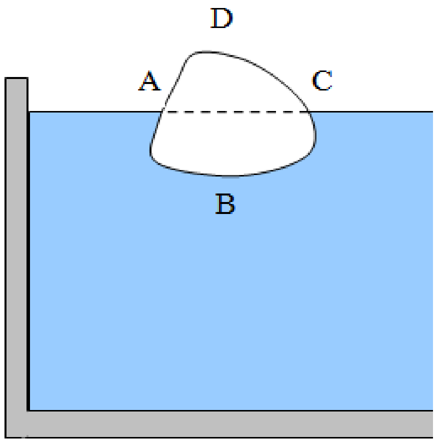


The length L is perpendicular to the drqwing

Where V_{sub} is the displaced volume (see the figures below).



Body is completely submerged in the fluid. The displaced volume is equal to the volume of the body, i.e. volume ABCDA.



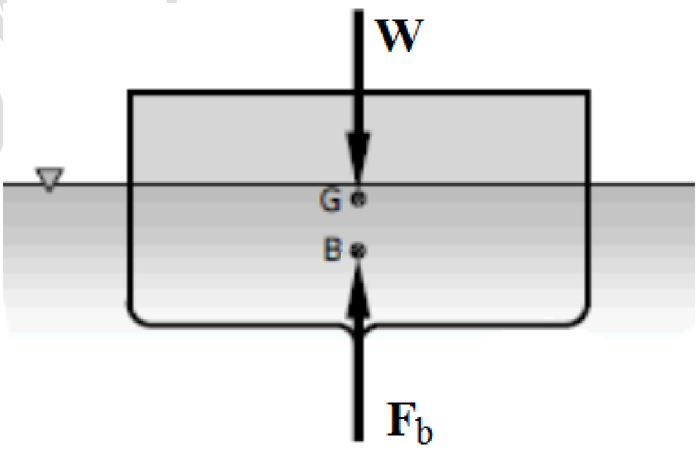
Body is partially submerged in the fluid. The displaced volume is equal to volume ABCA.

4.2 Stability of Floating Bodies

Any floating body is subjected by two opposing vertical forces. One is the body's weight "**W**" which is downward, and the other is the buoyant force "$\mathbf{F_b}$" which is upward. The weight is acting at the center of gravity "**G**" and the buoyant force is acting at the center of buoyancy "**B**".

يؤثر وزن الجسم "**W**" فى مركز ثقله "**G**" ، بينما تؤثر قوة التعويم "الطفو" "$\mathbf{F_b}$"فى المركز الهندسى للجسم المغمور "**B**"، وتسمى بمركز التعويم.

Center of Bouyancy "مركز الطفو "التعويم:

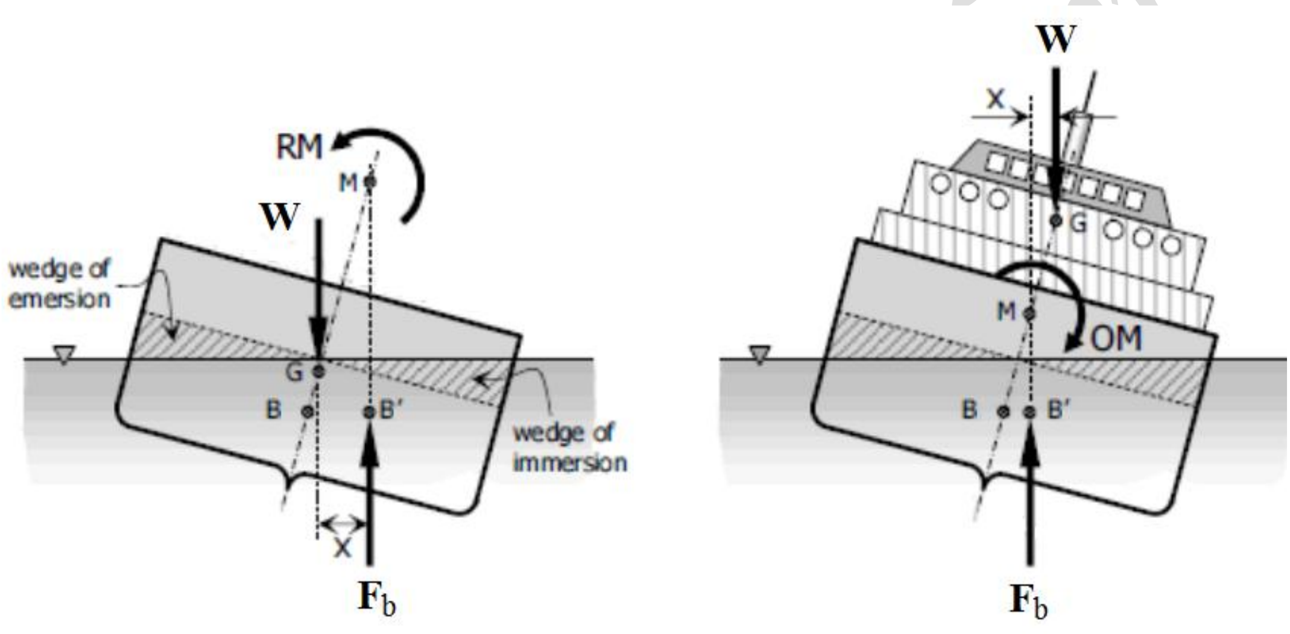


Upright Position

- It is the point of application of the force of buoyancy on the body.
- It is always the center of gravity of the volume of fluid displaced.

"**W**" and "$\mathbf{F_b}$" are always equal and if these forces are collinear, the body will be in upright position as shown below.

The body may tilt from many causes like wind or wave action causing the center of buoyancy to shift to a new position"**B/**" as shown below.



M is above G: Stable Position M is below G: Unstable Position

<u>Metacenter "M":</u> is the intersection point of the axis of the body and the line of action of the buoyant force.

- If "**M**" is <u>above</u> "**G**", "$\mathbf{F_b}$" and "**W**" will produce a righting moment **"RM"** which causes the body to return to its neutral position, thus the body is "**<u>stable</u>**". يكون الجسم فى حالة اتزان
- If "**M**" is <u>below</u> "**G**", the body becomes "**<u>unstable</u>**" because of the overturning moment **"OM"** made by "**W**" and "$\mathbf{F_b}$". يكون الجسم غير متزن
- If "**M**" <u>coincides</u> with "**G**", the body is said to be just stable which simply means critical "**<u>neutral</u>**". يكون الجسم متعادل

الخلاصة:

وضع الاتزان لأى جسم طاف يعتمد على وضع النقطة **M** بالنسبة للنقطة **G** ويكون الاتزان لذلك له ثلاث حالات كالتالى:

- **الاتزان مستقر:** إذا كانت النقطة **M** فوق النقطة **G**، حيث تولد القوتان **W** و $\mathbf{F_b}$ عزم ازدواج يعمل على اتزان المركب.
- **الاتزان غير مستقر:** إذا كانت النقطة **M** تحت النقطة **G**، حيث تولد القوتان **W** و $\mathbf{F_b}$ عزم ازدواج يعمل فى نفس اتجاه العزم الخارجى وبالتالى يؤدى الى انقلاب المركب.
- **الاتزان متعادل:** إذا كانت النقطة **M** منطبقة على النقطة **G**، حيث تولد القوتان **W** و $\mathbf{F_b}$ عزم ازدواج يجعل العزم المعاكس مساويا للصفر، وبالتالى يظل المركب على ما هو عليه ولا يعود لوضعه الأصلى مرة أخرى.

The value of righting moment or overturning moment is given by

$$\boxed{RM \text{ or } OM = Wx = W\,(MG\sin\theta)}$$

The distance "**MG**" is called "**metacentric height**".

<u>Metacentric hight "MG":</u> is the distance between the centre of gravity of a floating body ant the metacenter.

$$\boxed{\text{Metacentric height, } MG = MB \pm GB}$$

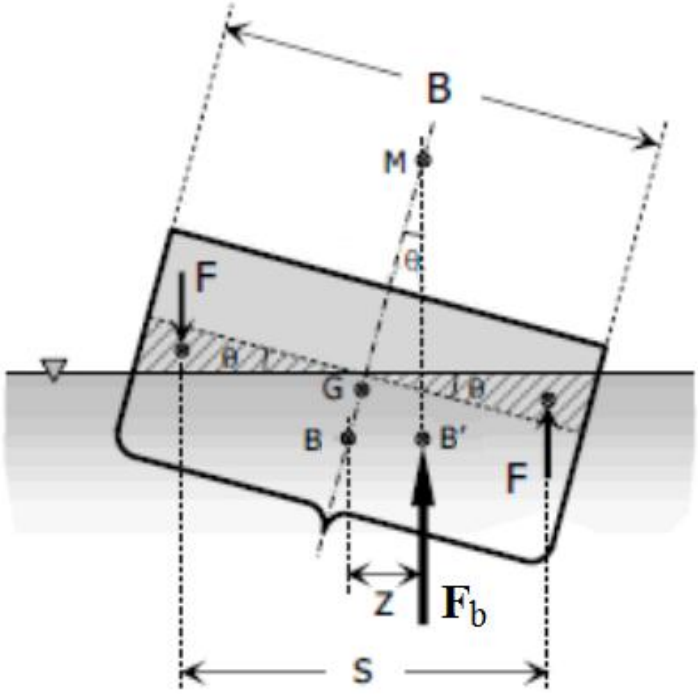
Use **(-)** if "**G**" is above "**B**" and **(+)** if "**G**" is below "**B**".

<u>Note that</u>: "**M**" is always above "**B**".

<u>Value of "MB":</u>

Assume that the body is rectangular at the top view and measures "**B**" by "**L**" at the waterline when in upright position.

The moment due to the shifting of the buoyant force is equal to the moment due to shifting of wedge



The length is L perpendicular to the drawing

$$F_b\, z = F\, s$$

$$\gamma V_{sub}\,(MB\sin\theta) = (\gamma v)s$$

$$V_{sub}\, MB\sin\theta = vs$$

$$\boxed{MB = \frac{vs}{V_{sub}\sin\theta}}$$

$$MB = \frac{[\,\frac{1}{2}(\frac{1}{2}B)(\frac{1}{2}B\tan\theta)L\,](\frac{2}{3}B)}{V_{sub}\,\sin\theta}$$

$$MB = \frac{\frac{1}{12}LB^3\tan\theta}{V_{sub}\,\sin\theta}$$

For small value of θ, **tan θ ≡ sin θ** and note that **1/12 LB^3 = I**, thus,

$$MB = \frac{I\sin\theta}{V_{sub}\,\sin\theta}$$

$$\boxed{MB = \frac{I}{V_{sub}}}$$

The formula above can be applied to any section.

Where

v = volume of the wedge either immersion or emersion

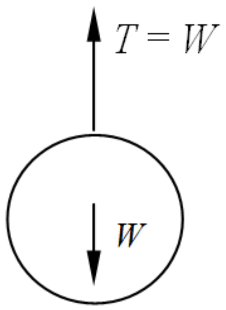
s = horizontal distance between the center of gravity of the wedges

θ = angle of tilting

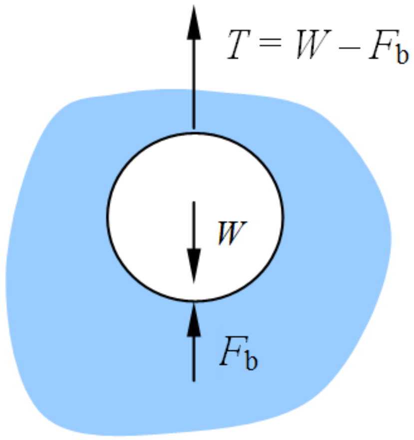
I = moment of inertia of the waterline section of the body

Some remarks about the weight of an object

- The weight of an object in a fluid medium refers to the tension in the spring when the object is attached to a spring balance.
- The weight (of the object) registered by the spring balance depends on the medium in which it is measured. [See the illustration below.]
- The weight commonly referred to in daily use is the weight in air.



Body in air



Body in water

CHAPTER 5

KINEMATICS OF FLUID FLOW

5.1 Kinematics :

Kinematics of fluid describes the fluid motion and its consequences without consideration of the nature of forces causing the motion.

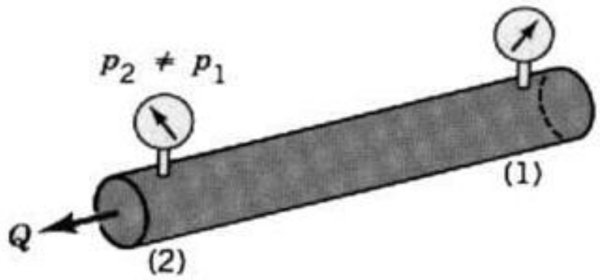
هو العلم الذى يهتم بدراسة الموائع من حيث السرعات وخطوط الانسياب دون الأخذ فى الاعتبار لأى قوى، ويمكن أن يطلق عليه (علم الحركة المجردة)

5.2 Difference between open-channel flow and the pipe flow:

Pipe flow السريان فى الأنابيب المغلقة :

- The pipe is completely filled with the fluid being transported.
- The main driving force is likely to be a pressure gradient along the pipe.

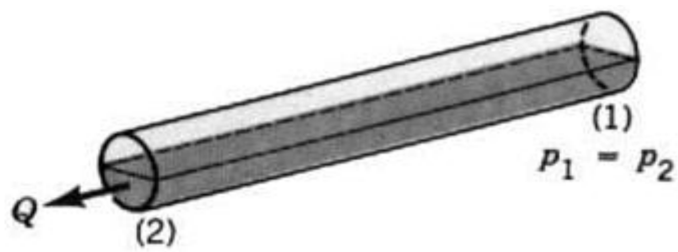
يحدث السريان عادة تحت ضغط ناتج من طلمبة أو خزان.



Open-channel flow السريان فى القنوات المفتوحة:

- Fluid flows without completely filling the pipe.
- Gravity alone is the driving force, the fluid flows down a hill.

يكون الضغط عند اى نقطة على السطح الحر مساويا للضغط الجوى، ويحدث هذا السريان تحت تأثير الجاذبية الأرضية.



5.3 Types of Flow أنواع السريان:

❑ Steady and Unsteady flow السريان المستقر وغير المستقر :

The flow parameters such as velocity (v), pressure (P) and density (r) of a fluid flow are <u>independent of time in a steady flow</u>. In unsteady flow they are independent.

For a steady flow $$\left(\partial v/\partial t\right)_{x_o,y_o,z_o} = 0$$

For an unsteady flow $$\left(\partial v/\partial t\right)_{x_o,y_o,z_o} \neq 0$$

❑ Uniform and non-uniform flow السريان المنتظم وغير المنتظم:

A flow is uniform if the flow characteristics at any given instant remain <u>the same at different points in the direction of flow</u>, otherwise it is termed as non-uniform flow.

For a uniform flow $$\left(\partial v/\partial s\right)_{t_o} = 0$$

For a non-uniform flow $$\left(\partial v/\partial s\right)_{t_o} \neq 0$$

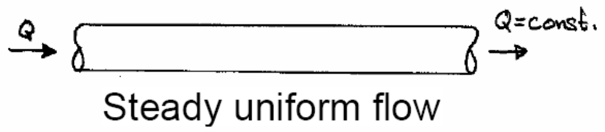
Steady = time independent
Uniform = constant section

فى السريان المستقر لا تتغير السرعة أو الضغط عند نقطة محددة مع الزمن.

فى السريان المنتظم لا تتغير السرعة و شكل السريان من نقطة إلى أخرى.

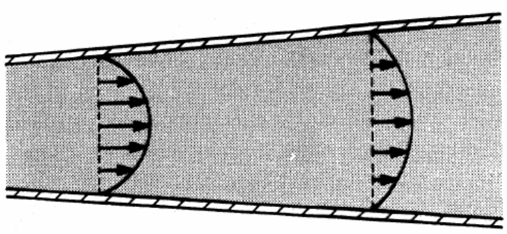
<u>Examples of flow types:</u>

<u>Steady uniform flow:</u>
flowrate (Q) and section area (A) are constant



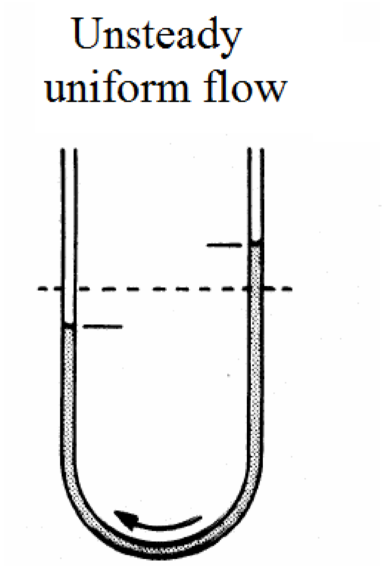
Steady uniform flow

<u>Steady non-uniform flow:</u>
Q = constant, A = A(x).


Steady non-uniform flow

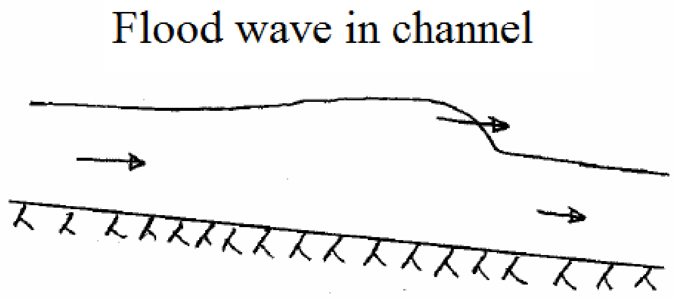
Unsteady uniform flow:

Q = Q(t), A = constant



Unsteady non-uniform flow:

Q = Q(t), A = A(x).



❑ Laminar and turbulent flow السريان الطبقى (اللزج) والسريان المضطرب:

Laminar flow السريان الطبقى (اللزج)

The fluid particles move along smooth well defined path or streamlines that are parallel, thus particles move in laminas or layers, smoothly gliding over each other

Turbulent flow السريان المضطرب

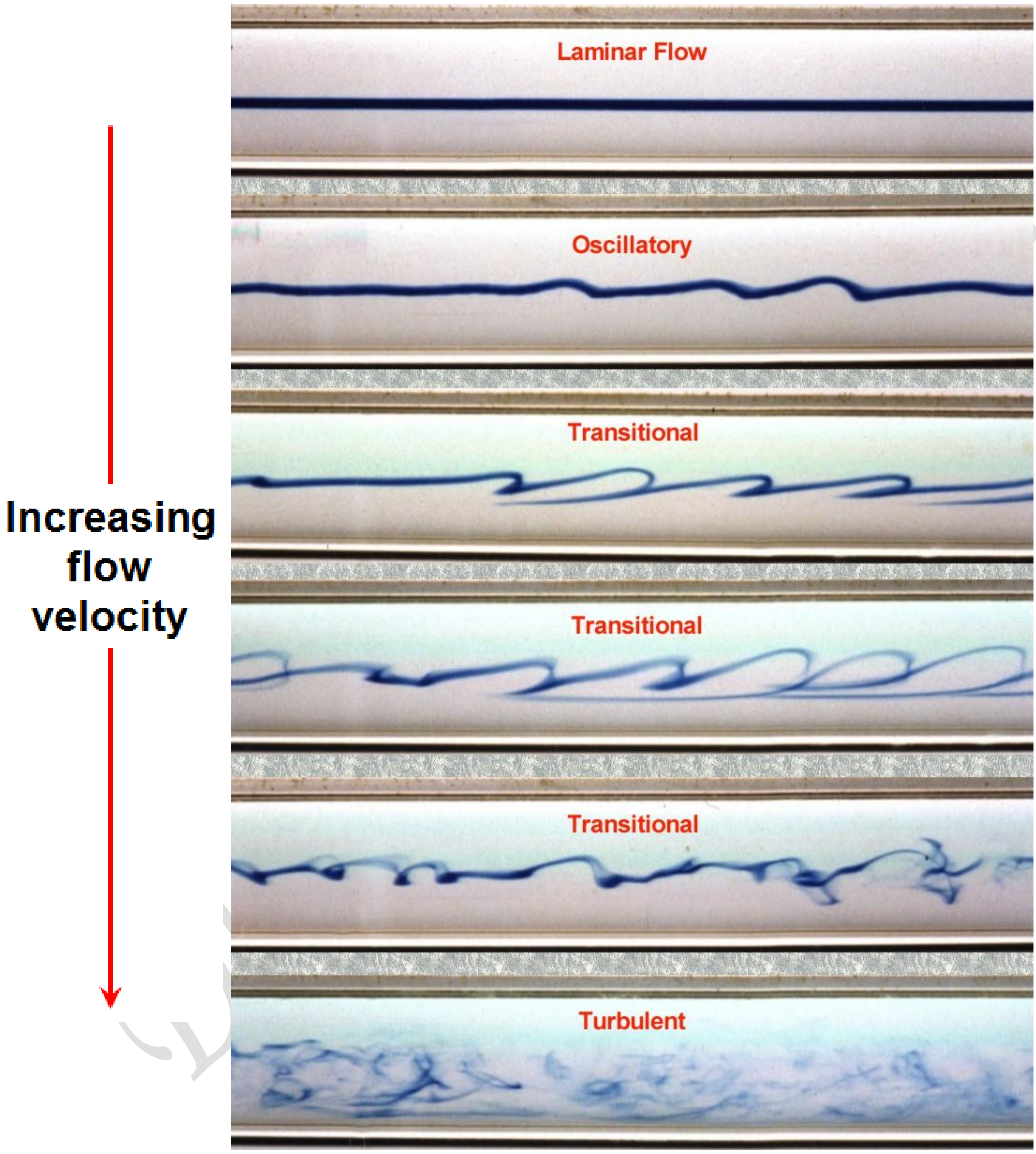
The fluid particles do not move in orderly manner and they occupy different relative positions in successive cross-sections.

There is a small fluctuation in magnitude and direction of the velocity of the fluid particles

Transitional flow السريان الانتقالى

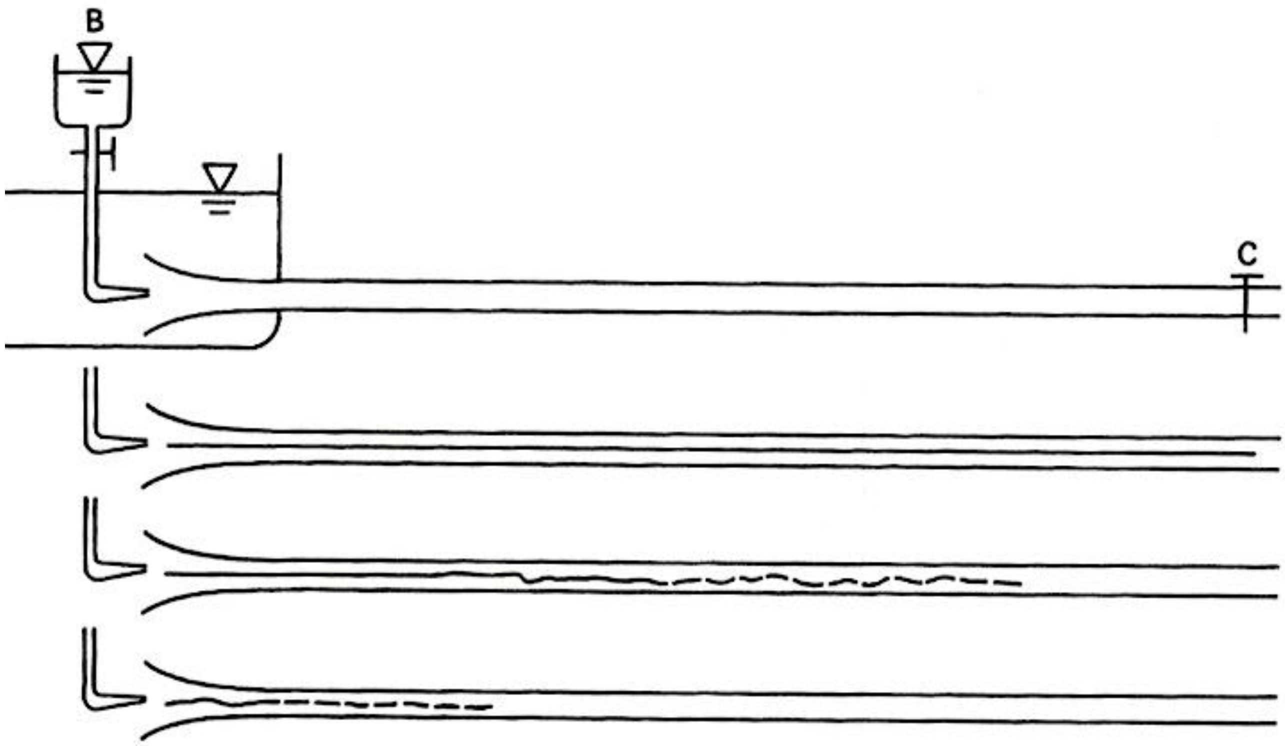
The flow occurs between laminar and turbulent flow

وقد وُجد من التجارب المعملية أن الفاقد فى الضاغط يعتمد على نوع السريان هل هو طبقى أم مضطرب، ويتحدد نوع السريان تبعا لسلوك الجزيئات أثناء حركتها.


Increasing flow velocity
Laminar Flow
Oscillatory
Transitional
Transitional
Transitional
Turbulent

❑ **Reynolds Experiment:**

Reynold performed a very carefully prepared pipe flow experiment. Reynold found that transition from laminar to turbulent flow in a pipe depends not only on the velocity, but only on the pipe diameter and the viscosity of the fluid.



Reynolds' apparatus.

Reynolds number is used to check whether the flow is laminar or turbulent. It is denoted by "$\mathbf{R_n}$". This number got by comparing inertial force with viscous force.

$$\boxed{R_n = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{Inertial\ Forces}{Viscous\ Forces}}$$

Where

V: mean velocity in the pipe السرعة المتوسطة [L/T]

D: pipe diameter قطر الماسورة [L]

ρ: density of flowing fluid كثافة السائل [M/L^3]

μ: dynamic viscosity اللزوجة الديناميكية [M/LT]

ν: kinematic viscosity اللزوجة الكينماتيكية [L^2/T]

جمع "رينولد" كل هذه العوامل والتى تؤثر على تحديد نوع السريان فى رقم واحد غير قياسى وسماه "رقم رينولد". وبالتالى يصبح"رقم رينولد" أكثر تعبيرا عن حالة السريان.

The Kind of flow depends on value of "$\mathbf{R_n}$"

If $\mathbf{R_n}$ < 2000 the flow is Laminar

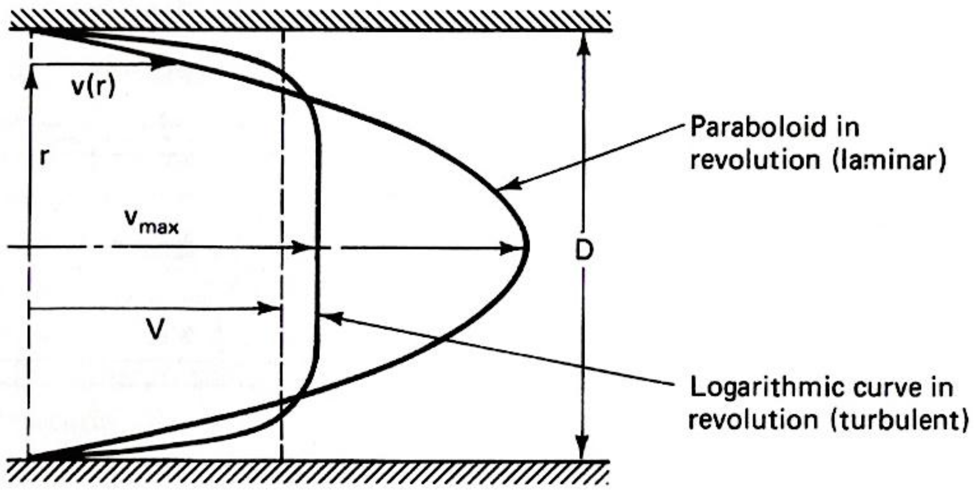
If $\mathbf{R_n}$ > 4000 the flow is turbulent

If 2000 < $\mathbf{R_n}$ < 4000 it is called transition flow.

Laminar Vs. Turbulent flows

Laminar flows characterized by:
- low velocities
- small length scales
- high kinematic viscosities
- R_n < Critical R_n
- Viscous forces are dominant

Turbulent flows characterized by
- high velocities
- large length scales
- low kinematic viscosities
- R_n > Critical R_n
- Inertial forces are dominant



Velocity profiles of laminar and turbulent flows in circular pipes

يتضح من منحنى توزيع السرعات أنه فى حالة السريان الطبقى تكون أقصى سرعة عند المنتصف وأقل سرعة عند جدار الماسورة وهى مساوية للصفر. أما فى حالة السريان المضطرب فيمكن تقسيم السريان الى منطقتين: منطقة الطبقة اللزجة ومنطقة الاضطراب. وكلما زاد"رقم رينولد" كلما اقترب توزيع السرعات من الشكل المستطيل (توزيع منتظم).

Example 5-1

40 mm diameter circular pipe carries water at 20°C. Calculate the largest flow rate (Q) which laminar flow can be expected.

Solution

$$D = 0.04m$$

$$\nu = 1\times10^{-6} \; at \; T = 20^o C$$

$$R_n = \frac{VD}{\nu} = 2000 \Rightarrow \frac{V(0.04)}{1\times10^{-6}} = 2000 \Rightarrow V = 0.05m/\sec$$

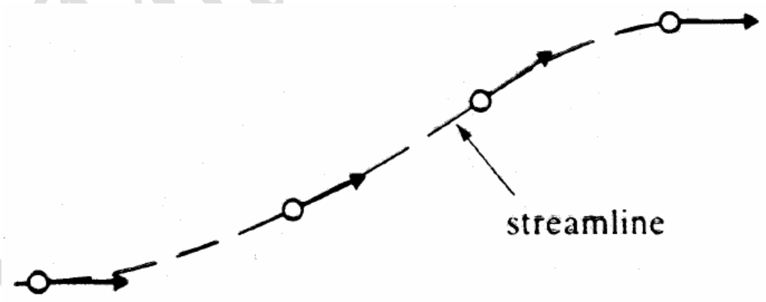
$$Q = V.A = 0.05\times\frac{\pi}{4}(0.04)^2 = 6.28\times10^{-5} m^3/\sec$$

❑ Streamlines and Streamtubes:

Streamline "التدفق" خط السريان:

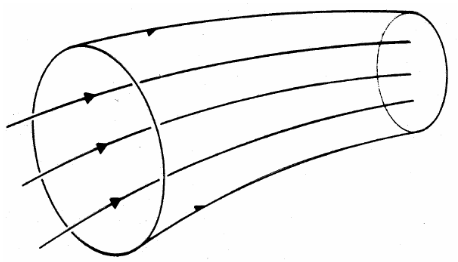
A curve that is drawn in such a way that it is tangential to the velocity vector at any point along the curve. A curve that is representing the direction of flow at a given time. No flow across a stream line.

هو ذلك الخط الذى يعبر عن اتجاه حركة الجزيئات الموجودة عليه فى لحظة زمنية معينة.



Streamtube"التدفق" أنبوب السريان :

A set of streamlines arranged to form an imaginary tube. Ex: The internal surface of a pipeline.



مجموعة من خطوط السريان تمثل فيما بينها أنبوب.

❑ Compressible And Incompressible Flows:

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field.

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field.

❑ Ideal and Real Fluids:

a- Ideal Fluids

- It is a fluid that has no viscosity, and incompressible
- Shear resistance is considered zero
- Ideal fluid does not exist in nature e.g. Water and air are assumed ideal

b- Real Fluids

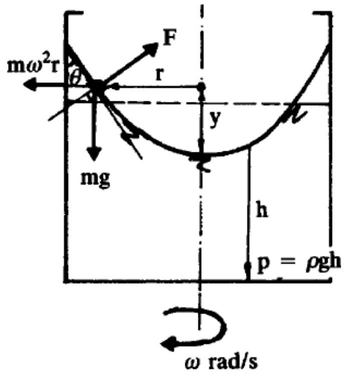
- It is a fluid that has viscosity, and compressible
- It offers resistance to its flow e.g. All fluids in nature

عند دراستنا لسائل نعتبر ان السائل غير قابل للانضغاط ومثالى وبذلك نهمل قوى الاحتكاك الداخلى للحصول على معادلات بسيطة، ثم يتم تصحيحها بعد ذلك لأخذ قوى الاحتاك فى الاعتبار.

❑ Rotational And Irrotational Flows:

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.



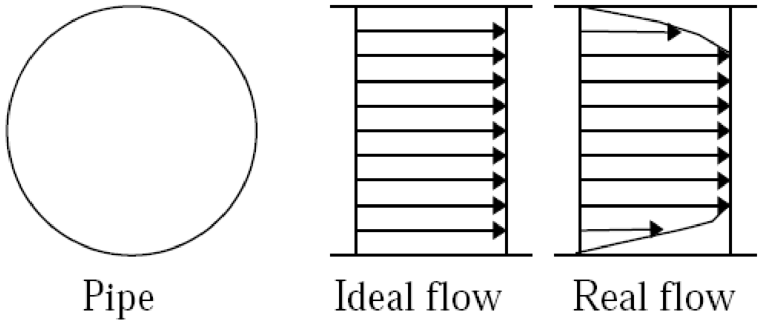
❑ One, Two And Three Dimensional Flows:

One-dimensional flow is the type of flow in which flow parameters (such as velocity, pressure, depth etc.) are a function of time and one space coordinate only.

$$\mathbf{v = f(x, t)}$$

e.g. Flow through a straight uniform diameter pipe

The flow is never truly 1 dimensional, because viscosity causes the fluid velocity to be zero at the boundaries.

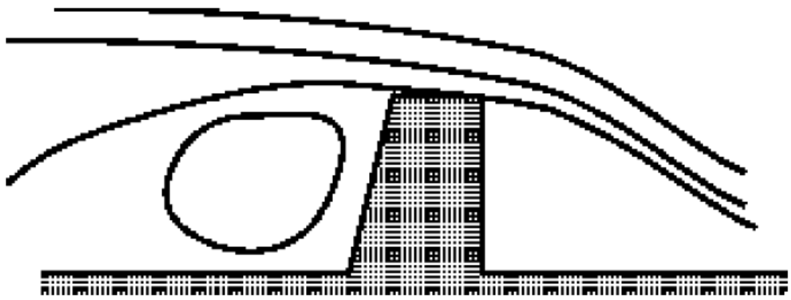


One dimensional flow in a pipe.

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time.

$$\mathbf{v = f(x, y, t)}$$

Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir which typical streamlines can be seen in the figure below.



Two-dimensional flow over a weir.

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time.

$$\mathbf{v = f(x, y, z, t)}$$

Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one.

5.4 Volume flow rate – Discharge :

The discharge is the volume of fluid passing a given cross-section per unit time.

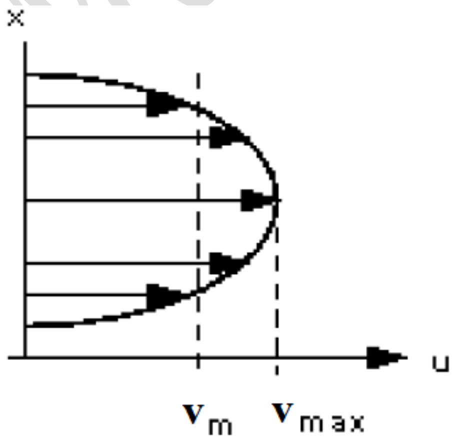
$$\text{discharge, } Q = \frac{\text{volume of fluid}}{\text{time}}$$

5.5 Mean Velocity:

It is the average velocity passing a given cection.

The velocity in the pipe is not constant across the cross section. Crossing the centreline of the pipe, the velocity is zero at the walls increasing to a maximum at the centre then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution.

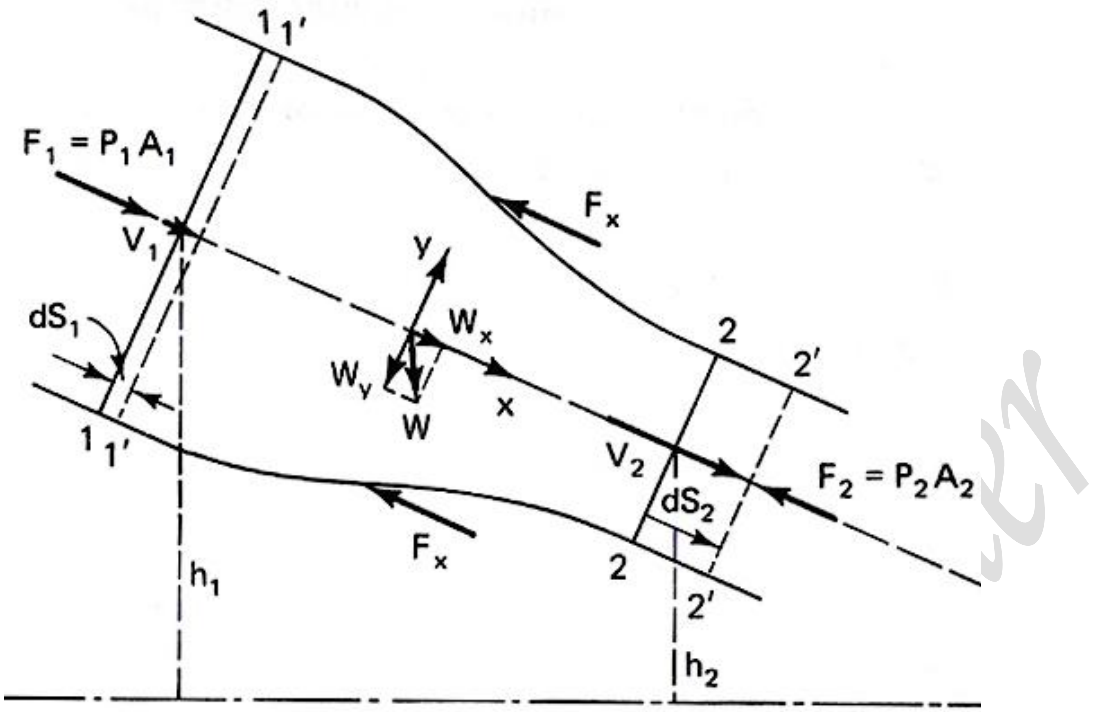
$$v_m = \frac{Q}{A}$$



A typical velocity profile across a pipe

5.6 Continuity equation for Incompressible Steady flow معادلة الاستمرار للسريان المستقر:

Cross section and elevation of the pipe are varied along the axial direction of the flow.



General description of flow in pipes

Conservation law of mass

$$\rho.dVol_{1-1'} = \rho.dVol_{2-2'} = mass\ \ flux(fluid\ \ mass)$$

Mass enters the control volume

Mass leaves the control volume

$$\rho.\frac{dVol_{1-1'}}{dt} = \rho.\frac{dVol_{2-2'}}{dt}$$

$$\rho.A_1\frac{dS_1}{dt} = \rho.A_2\frac{dS_2}{dt} \Rightarrow \rho.A_1.V_1 = \rho.A_2.V_2 = \rho.Q$$

Continuity equation for Incompressible Steady flow

$$\boxed{A_1.V_1 = A_2.V_2 = Q}$$

اذا استمر السائل فى تدفقه خلال مجرى مائى ما مثل قناة أو ماسورة منتظمة أو متغيرة المقطع فإن كمية السائل المارة خلال وحدة الزمن تكون متساوية عند جميع القطاعات خلال المجرى، وهذا هو ما يعرف بمعادلة الاستمرار للسريان المستقر.

Apply Newton's Second Law:

$$\sum \vec{F} = M\,\vec{a} = M\frac{d\vec{V}}{dt} = \frac{M\vec{V}_2 - M\vec{V}_1}{\Delta t}$$

$$\sum F_x = P_1 A_1 - P_2 A_2 - F_x + W_x$$

F_x is the axial direction force exerted on the control volume by the wall of the pipe.

$$but\ M/\Delta t = \rho.Q = mass\ \ flow\ rate$$

$$\sum F_x = \rho.Q(V_{x_2} - V_{x_1})$$

$$\sum F_y = \rho.Q(V_{y_2} - V_{y_1})$$

$$\sum F_z = \rho.Q(V_{z_2} - V_{z_1})$$

$$\boxed{\sum \vec{F} = \rho.Q(\vec{V}_2 - \vec{V}_1)}$$

Conservation of moment equation

5.7 Energy Head in Pipe Flow

Water flow in pipes may contain energy in three basic forms:

1- Kinetic energy طاقة الحركة,

2- potential energy طاقة الوضع,

3- pressure energy طاقة الضغط .

- **Consider the control volume:**
- **In time interval dt:**

➢ Water particles at sec.1-1 move to sec. 1\`-1\` with velocity V_1.

➢ Water particles at sec.2-2 move to sec. 2`-2` with velocity V_2.

To satisfy continuity equation:

$$A_1.V_1.dt = A_2.V_2.dt$$

<u>The work done by the pressure force:</u>

$$P_1.A_1.ds_1 = P_1.A_1.V_1.dt \qquad \text{....... on section 1-1}$$

$$-P_2.A_2.ds_2 = -P_2.A_2.V_2.dt \qquad \text{....... on section 2-2}$$

-ve sign because P_2 is in the opposite direction to distance traveled ds_2

<u>The work done by the gravity force:</u>

$$\rho g.A_1.V_1 dt.(z_1 - z_2)$$

<u>The kinetic energy:</u>

$$\frac{1}{2}M.V_2^2 - \frac{1}{2}M.V_1^2 = \frac{1}{2}\rho.A_1.V_1.dt(V_2^2 - V_1^2)$$

<u>The total work done by all forces is equal to the change in kinetic energy:</u>

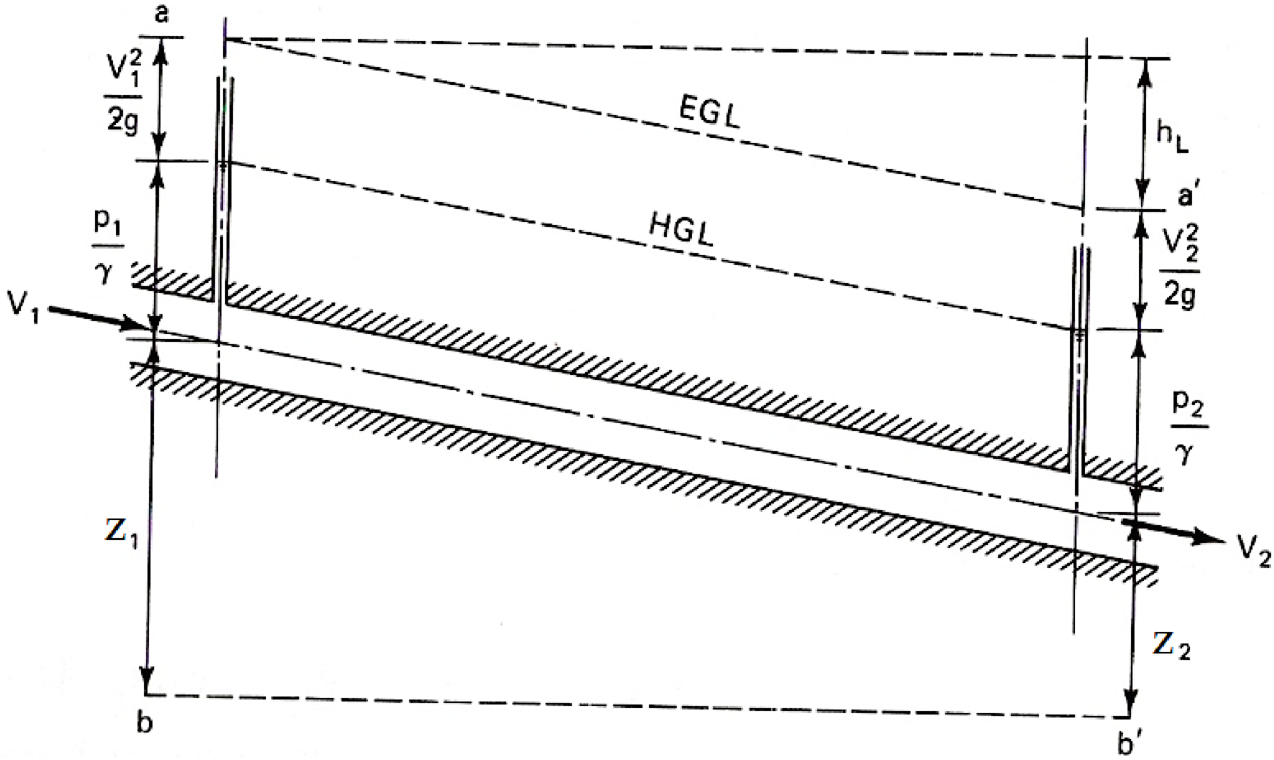
$$P_1.Q.dt - P_2.Q.dt + \rho g.Q.dt.(z_1 - z_2) = \frac{1}{2}\rho.Q.dt(V_2^2 - V_1^2)$$

Dividing both sides by ρgQdt

Bernoulli Equation

$$\boxed{\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2}$$

Energy per unit weight of water
OR: <u>Energy Head</u>



Energy head and Head loss in pipe flow

$$H_2 = \frac{V_2^{\;2}}{2g} + \frac{P_2}{\gamma} + z_2$$

Energy head = **Kinetic head** + **Pressure head** + **Elevation head**

$$H_1 = \frac{V_1^{\;2}}{2g} + \frac{P_1}{\gamma} + z_1$$

Notice that:

- In reality, certain amount of energy loss (h_L) occurs when the water mass flow from one section to another.
- The energy relationship between two sections can be written as:

$$\frac{V_1^{\;2}}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^{\;2}}{2g} + \frac{P_2}{\gamma} + z_2 + h_L$$

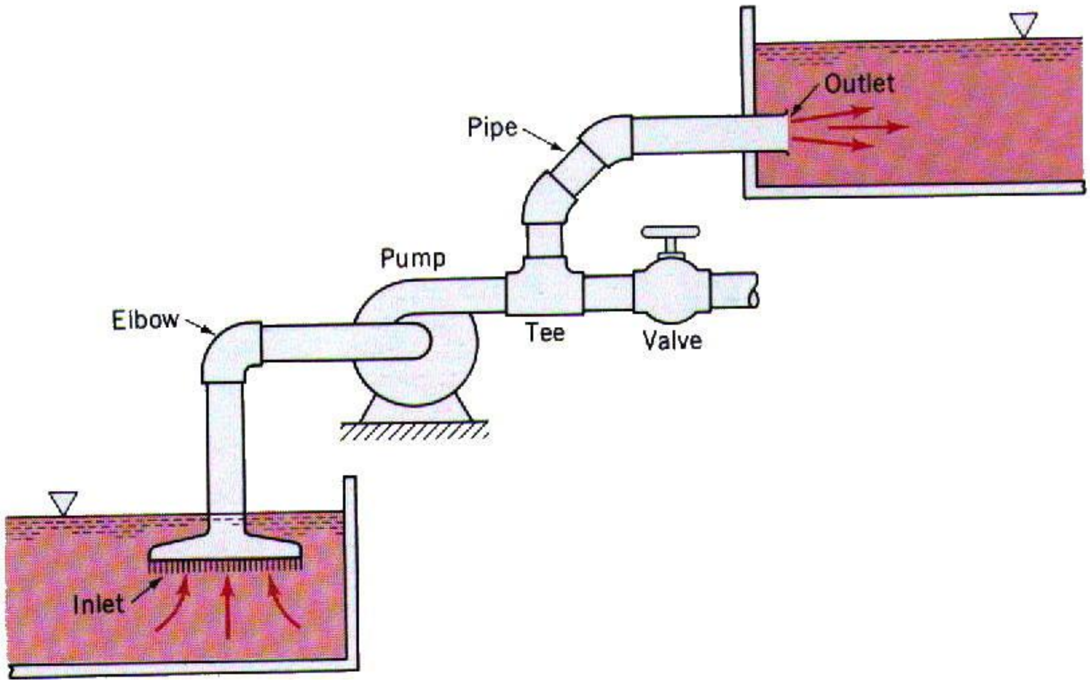
<u>Calculation of Head (Energy) Losses الطاقة المفقودة :</u>

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy (head) of fluid is lost.

Energy Losses (Head losses)

- **Major Losses**: loss of head <u>due to pipe friction</u> and to viscous dissipation in flowing water
- **Minor losses**: Loss due to the change of the velocity of the flowing fluid in the magnitude or in direction as <u>it moves through fitting</u> like Valves, Tees, Bends and Reducers.

يمكن تقسيم الفواقد فى الضاغط الى نوعين: الأولى وهى فواقد رئيسية ناتجة عن الاحتكاك على طول المجرى، والثانية فواقد ثانوية ناتجة عن وجود عائق فى المجرى أو نتيجة تغير شكل المجرى.

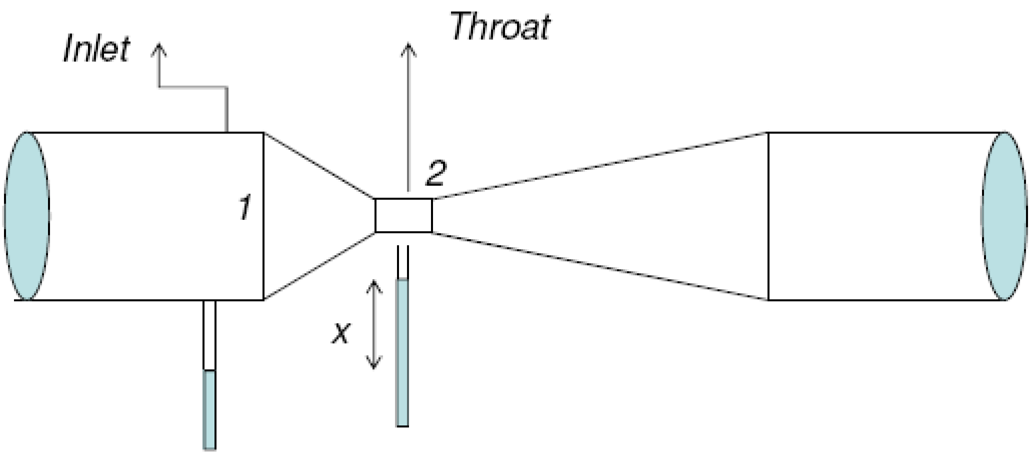


5.8 Applications of Bernoulli's Equation:

- Discharge through a Venturimeter مقياس فنشورى:

The Venturi meter is a device for measuring discharge in a pipe. It consists of a rapidly converging section which increases the velocity of flow and hence reduces the pressure. It then returns to the original dimensions of the pipe by a gently diverging "diffuser" section. By measuring the pressure differences the discharge can be calculated.

هو جهاز يستخدم لقياس التصرف المار فى الأنابيب عن طريق معرفة فرق الضغط بين القطاعين



Let d1 = diameter at the inlet (section 1)

p_1 = pressure at section 1

v_1 = velocity at section 1

A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at the throat (section 2)

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{v_1^{\,2}}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^{\,2}}{2g} + z_2$$

As pipe is horizontal z_1=z_2

$$\Rightarrow \frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g}$$

$$\Rightarrow h = \frac{{v_2}^2 - {v_1}^2}{2g}$$

Where "***h = (p_1-p_2)/ ρg***" difference of pressure heads at sections 1 and 2.

From the continuity equation at sections 1 and 2, we obtain

$$A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

Hence
$$h = \frac{{v_2}^2}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$\Rightarrow v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge
$$Q = A_1 v_1 = A_2 h$$

$$\Rightarrow Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Note that the above expression is for ideal condition and is known as theoretical discharge. Actual discharge will be less than theoretical discharge

$$\boxed{Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}}$$

C_d is the coefficient of venturimeter and its value is always less then 1

Expression of 'h' given by differential U-tube manometer:

Case 1: The liquid in the manometer is heavier than the liquid flowing through the pipe:

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

S_h: Specific gravity of the heavier liquid.
S_0: Specific gravity of the flowing liquid.

Case 2: The liquid in the manometer is lighter than the liquid flowing through the pipe:

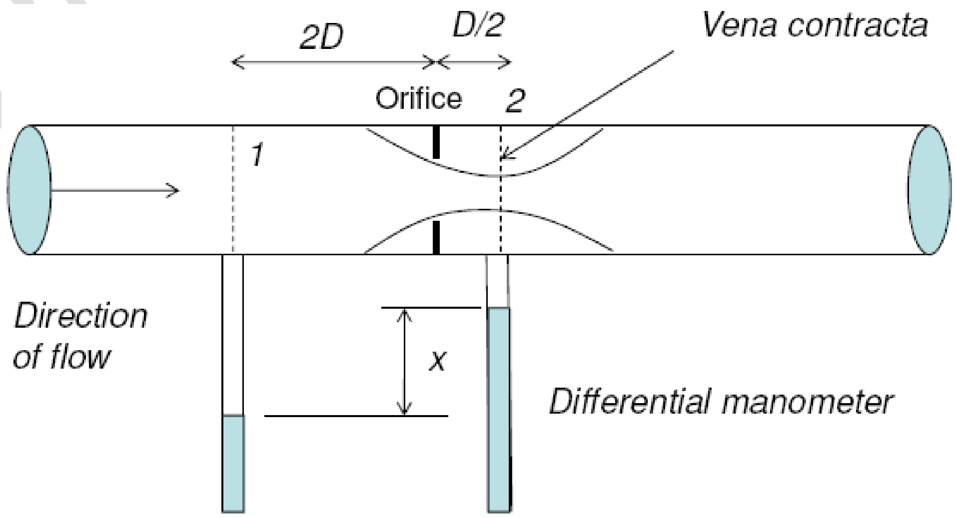
$$h = x\left[1 - \frac{S_L}{S_0}\right]$$

S_L: Specific gravity of the lighter liquid.
X: difference of the liquid columns in U-tube

- **Orifice meter مقياس الفتحة:**

Orifice meter: is a device used for measuring the rate of flow of a fluid flowing through a pipe.

- It is a cheaper device as compared to venturimeter. This also works on the same principle as that of venturimeter.
- It consists of flat circular plate which has a circular hole, in concentric with the pipe. This is called "**orifice**".
- The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.



Let d1 = diameter at the inlet (section 1)

p_1 = pressure at section 1

v_1 = velocity at section 1

A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at the throat (section 2)

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

$$\Rightarrow\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^2-v_1^2}{2g}$$

$$\Rightarrow h=\frac{v_2^2-v_1^2}{2g}$$

$$\Rightarrow v_2=\sqrt{2gh+v_1^2}$$

where h is the differential head.

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_c=\frac{A_2}{A_0}$

By continuity equation, w

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_1=\frac{A_0 C_c}{A_1}v_2$$

Hence, $$v_2=\sqrt{2gh+\frac{A_0^2 C_c^2 v_2^2}{A_1^2}}$$

$$\Rightarrow v_2=\frac{\sqrt{2gh}}{\sqrt{1-\frac{A_0^2}{A_1^2}C_c^2}}$$

Thus, discharge,

$$Q = A_2 v_2 = v_2 A_0 C_c = \frac{A_0 C_c \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

$$\Rightarrow C_c = C_d \frac{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}}$$

Hence,

$$\boxed{Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}}$$

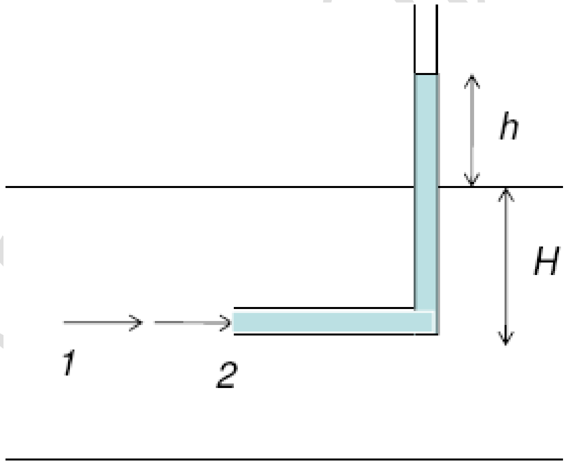
The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

- **Pitot tube :**

Orifice meter: is a device used for measuring the velocity of flow at any point in a pipe or a channel.

- Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.
- In simplest form, the pitot tube consists of a glass tube, bent at right angles.

أنبوب بيتوت يستخدم لقياس سرعة الانسياب عن اى نقطة.



Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

Let p_1 = pressure at section 1

p_2 = pressure at section 2

v_1 = velocity at section 1

v_2 = velocity at section 2

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0$.

$\frac{p_1}{\rho g} =$ Pressure head at 1=H

$\frac{p_2}{\rho g} =$ Pressure head at 2=h+H

Substituting these values, we get $H+\frac{v_1^2}{2g}=h+H$

$$\Rightarrow v_1=\sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$\boxed{(v_1)_{act} = C_v\sqrt{2gh}}$$

C_v is the coefficient of pitot-tube.

SOLVED PROBLEMS

CHAPTER 1

UNITS AND DIMENSIONS & FLUID PROPERTIES

Problem 1-1: Identify the dimensions and units for the following engineering quantities and terms (in British, and SI units): Density (ρ), specific weight (γ), specific gravity (S.G.), discharge (Q), surface tension (σ), shear stress (τ), pressure intensity (p), pressure head (p/γ), dynamic viscosity (μ), kinematic viscosity (ν), Linear momentum, angular velocity (ω),Reynolds number (R_n= ρVD/μ), Froude number (F_n= V/√gY).

Solution

Physical Quantity	Dimension	Units	
		British	SI
Density (ρ)	$[M\ L^{-3}]$	$Slug/ft^3$	Kg/m^3
Specific Weight (γ)	$[F\ L^{-3}]$ OR $[M\ L^{-2}\ T^{-2}]$	lb/ft^3	N/m^3 OR $Kg/m^2.sec^2$
Specific Gravity (S.G.)	$[M^0\ L^0\ T^0]$	Unitless	
Discharge (Q)	$[L^3\ T^{-1}]$	ft^3/sec	m^3/sec
Surface Tension (σ)	$[F\ L^{-1}]$ OR $[M\ T^{-2}]$	lb/ft OR $slug/sec^2$	N/m OR kg/sec^2
Shear Stress (τ)	$[F\ L^{-2}]$ OR $[M\ L^{-1}\ T^{-2}]$	lb/ft^2 OR $slug.ft/sec^2$	N/m^2 OR $kg/m.sec^2$
Pressure Intensity (P)	$[F\ L^{-2}]$ OR $[M\ L^{-1}\ T^{-2}]$	lb/ft^2 OR $slug.ft/sec^2$	N/m^2 OR $kg/m.sec^2$
Pressure Head (P/γ)	[L]	ft	m
Dynamic Viscosity (μ)	$[F\ T\ L^{-2}]$ OR $[M\ L^{-1}\ T^{-1}]$	$lb.sec/ft^2$ OR slug/ft.sec	$N.sec/m^2$ OR kg/m.sec
Kinematic Viscosity (ν)	$[L^2\ T^{-1}]$	ft^2/sec	m^2/sec
Linear Momentum (M)	$[M\ L\ T^{-1}]$	Slug.ft/sec	Kg.m/sec
Angular Velocity (ω)	$[T^{-1}]$	1/sec	1/sec
Reynold's Number (R_n)	$[M^0\ L^0\ T^0]$	Unitless	
Froude's Number (F_n)	$[M^0\ L^0\ T^0]$	Unitless	

Problem 1-2: Convert the following:

a) A discharge of 20 ft^3/min. to lit/sec.

b) A force of 10 poundals to dynes.

c) A pressure of 30 lb / $inch^2$ to gm/cm^2.

d) A specific weight of 62.4 lb / ft^3 to kg/lit.

e) A density of 7 gm / cm^3 to slug / ft^3.

f) A dynamic viscosity of 25 dyne.sec/ cm^2 to lb.sec/ ft^2 .

g) A dynamic viscosity of 10 gm / cm.sec to slug / ft.sec

Solution

a) $20\text{ft}^3/\text{min} = 20 * \frac{\frac{(30.48)^3}{1000}}{60} \times \frac{lit}{sec} = 9.44 \frac{lit}{sec}$

b) 10 Poundals $= 10 \frac{pound\ .\ ft}{sec^2} = 10 \times \frac{453.6 * 30.48}{1} \frac{gm\ .\ cm}{sec^2} = 138257.28 dynes$

c) $30 \frac{lb}{in^2} = 30 * \frac{453.6}{(2.54)^2} \frac{gm}{cm^2} = 2109.24 \frac{gm}{cm^2}$

d) $62.4 \frac{lb}{ft^3} = 62.4 * \frac{453.6/1000}{(\frac{30.48}{1000})^3} \frac{kg}{lit} = 1 \frac{kg}{lit}$

e) $7 \frac{gm}{cm^3} = 7 * \frac{\left(\frac{1}{453.6}\right)*(\frac{1}{32.2})}{(\frac{1}{30.48})^3} \frac{slug}{ft^3} = 13.57 \frac{slug}{ft^3}$

f) $25 \frac{dyne\ .\ sec}{cm^2} = 25 * \frac{\left(\frac{1}{981}\right)*(\frac{1}{453.6})}{(\frac{1}{30.48})^2} \frac{lb\ .\ sec}{ft^2} = 0.052 \frac{lb\ .sec}{ft^2}$

g) $10 \frac{gm}{cm\ .\ sec} = 10 * \frac{\left(\frac{1}{453.6}\right)*(\frac{1}{32.2})}{(\frac{1}{30.48})} \frac{slug}{ft\ .\ sec} = 0.0209 \frac{slug}{ft\ .\ sec}$

Problem 1-3: Determine the specific weight, density and specific gravity of a liquid that occupies a volume of 200 lit., and weighs 178 kg. Will this liquid float on the surface of an oil of specific gravity (0.8)? Provide results in SI units.

Solution

W =178 kg_f
=178 x 9.81
=1746.18 N

V =200 lit
=200/1000
=0.20 m^3

Specific Weight $\gamma = \frac{W}{V} = \frac{1746.18}{0.20} = 8730.9\ N/m^3$

Density $\rho = \frac{\gamma}{g} = \frac{8730.9}{9.81} = 890\ kg/m^3$

S.G. $= \frac{\gamma\ fluid}{\gamma\ water} = \frac{\rho\ fluid}{\rho\ water}$
=8730.9/9810 or 890/1000
=0.89

Since S.G of fluid (0.89) is higher than SG of oil (0.80) so the fluid will not float over the oil (Higher S.G. means denser fluid)

Problem 1-4: Calculate the capillary rise in mm in a glass tube of 6 mm diameter when immersed in (a) water, and (b) mercury, both liquids being at C^0 20. Assume σ to be 73×10^{-3} N/m for water and 0.5 N/m for mercury. The contact angle for water and mercury are zero and 130^0 respectively.

Solution

a) Water:

$$H_w = \frac{4\,\sigma_w \cos\theta_w}{\gamma_w\, D} = \frac{4*73*10^{-3}*\cos 0}{9810*6\times10^{-3}} * 1000 = 4.96\ mm$$

b) Mercury:

$$H_{Hg} = \frac{4\sigma_{Hg} \cos\theta_{Hg}}{\gamma_{Hg}\, D} = \frac{4*0.5*\cos 130}{(13.6*9810)*6\times10^{-3}} * 1000 = -1.61mm$$

Problem 1-5: Calculate the internal pressure of a 25 mm diameter soap bubble if the surface tension in the soap film is 0.5 N/m.

Solution

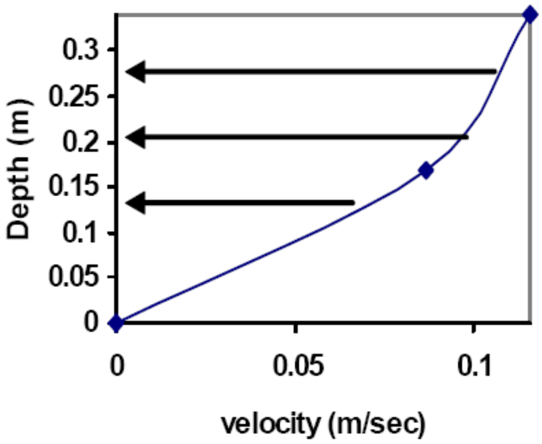
$$P = \frac{8\sigma}{D} = \frac{8*0.5}{25\times10^{-3}} = 160N/m^2$$

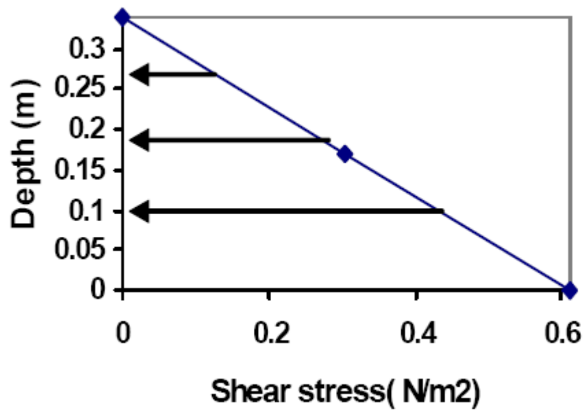
Problem 1-6: The velocity distribution of a viscous liquid (μ = 0.9 N.s /m^2) over a fixed boundary is approximately given by: v = 0.68y - y2 in which y is the vertical distance in meters, measured from the boundary and v is the velocity in m/s. Determine the shear stress at the boundary and at the surface (at y = 0.34m), sketch the velocity and shear stress profiles for the given flow.

Solution

μ=0.9 N.s/m^2
V=0.68y-y^2
τ=μ dv/dy= $0.9\times(0.68-2y)$
At surface y=0 , $\tau = 0.9\times(0.68-2\times 0) = 0.612$ N/m^2
At y=0.34 , $\tau = 0.9\times(0.68-2\times 0.34) = 0$ N/m^2

y	0	0.17	0.34
V(y)	0	0.0867	0.1156
τ(y)	0.612	0.306	0





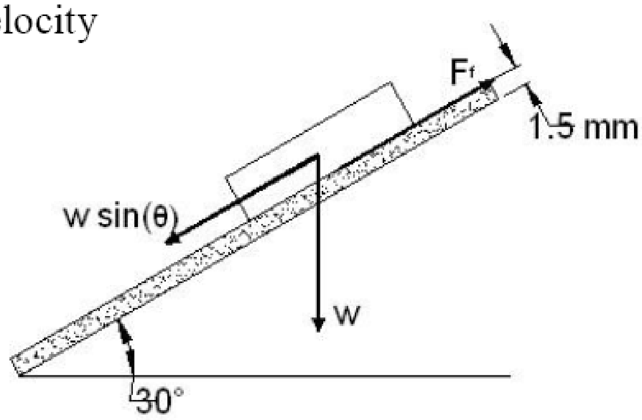
Problem 1-7: A square plate of 60 cm side slides down a plane inclined at 30^0 to the horizontal at a uniform speed of 10 cm/s. The plate weighs 3 kg and a thin oil film of thickness 1.5 mm fills the spacing between the plate and the plane. Find the viscosity of the filling oil and the shear at both surfaces.

Solution

For a body moving with a uniform velocity

$$\frac{dv}{dt} = 0$$

Then $\sum F = ma = 0$



Driving Force – Friction Force = 0

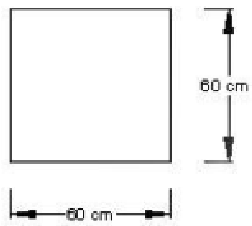
W sin(θ) - F_f = 0

W sin(θ) = F_f

W sin(θ) = τ x Area of plate

W sin(θ) = $\mu \frac{dv}{dy}$ x Area of plate

3 x 9.81 x sin(30) = $\mu \frac{dv}{dy}$ x (0.6x0.6)



Assuming linear velocity distribution

$$\frac{dv}{dy} = \frac{V2-V1}{Y2-Y1} = \frac{10-0}{\frac{1.5-0}{10}}$$

$= 66.67\ \text{sec}^{-1}$

μ = (3x9.81xsin(30))/(66.67x0.6x0.6)

= 0.613125 $N.sec/m^2$

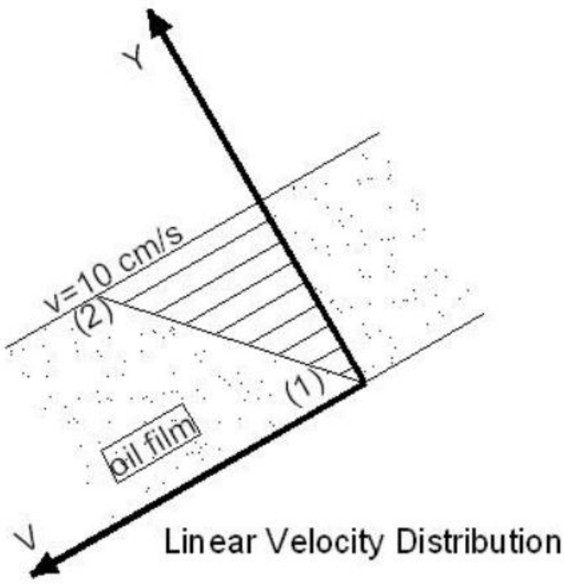
Shear stress is uniform as long as the velocity is linearly distributed

Shear stresses is equal at both surface of the oil

$$\tau = \mu \frac{dv}{dy}$$

= 1.631 x 66.67

= 40.875 N/m^2



Problem 1-8: A 25 mm diameter steel cylinder falls under its own weight at a uniform rate of 0.1 m/s inside a tube of slightly larger diameter. Castor oil of viscosity $0.25 N.s/m^2$ fills the spacing between the tube and the cylinder. Find the diameter of the tube. (specific weight of steel = $7.8 t/m^3$).

Solution

D=25mm

$\mu=0.25 \ N.S/m^2$

V=0.1m/sec

$\gamma_{steel} = 7.8 \ t/m^3$

$\Sigma F = m \ a$

V =const. a=0

$\Sigma F = 0$ $W = F_f$

Assume linear velocity distribution

$$\gamma * V = \mu * \frac{dv}{dy} * A$$

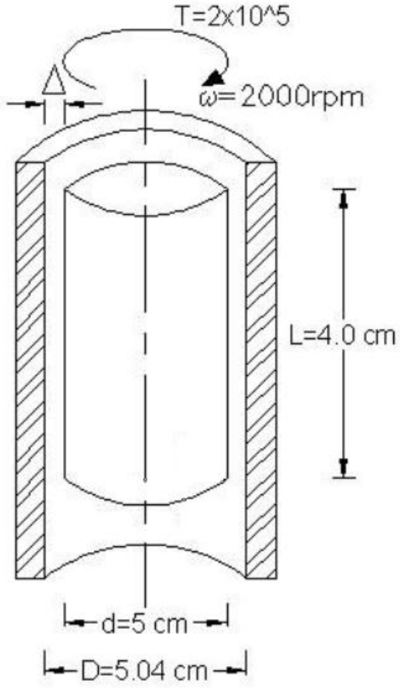
$$(7.8 * 1000) * 9.81 * \frac{\pi(\frac{25}{1000})^2}{4} * L = 0.25 * \frac{0.1}{y} * \pi * \left(\frac{25}{1000}\right) * L$$

y=0.052mm

$D_{tube} = D + 2y = 25 + 2*0.052 = 25.105$ mm

Problem 1-9: A rotating-cylinder viscometer consists of two concentric cylinders of diameters 5.0 cm and 5.04 cm, respectively. Find the viscosity of the tested oil which fills the gap between both cylinders to a height of 4.0 cm when a torque of 2×10^5 dyne.cm is required to rotate the inner cylinder at 2000 rpm.

Solution



r = 2.5 cm
T = $2\text{x}10^5$ dyne.cm
ω = 2000 rpm
Δ = (5.04-5.0)/2 = 0.02 cm

$$\omega(\text{rpm})\ \text{x}\frac{2\pi}{60} = \omega(\text{rad/sec})$$
$$= 2000 \text{ x } 2\pi/60$$

V(velocity) $= \omega r$
$= 2000 \text{ x } 2\pi/60 \text{ x } 2.5$
$= 523.6$ cm/sec

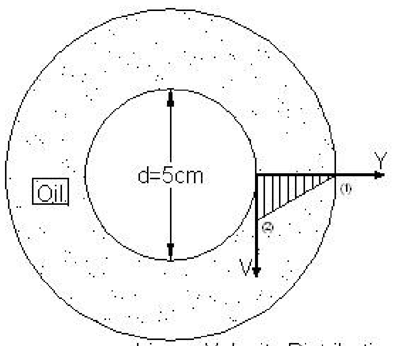
Torque = Force x Displacement

$\Sigma F = ma = 0$
Driving Force – Friction Force = 0

$$\frac{T}{r} - \mu\frac{dv}{dy}\ .\text{Area} = 0 \qquad (1)$$

Assuming linear velocity distribution

$$\frac{dv}{dy} = \frac{V2-V1}{Y2-Y1} = \frac{523.6-0}{0.02-0} = 26180 \text{ sec}^{-1} \qquad (2)$$



Linear Velocity Distribution

Area = Surface area under friction
$= 2\pi r.L$
$= 2\pi\ (2.5)\ (4.0)$
$= 62.832 \text{ cm}^2$ (3)

Substituting with (2) and (3) in the equation (1)
(2x105/2.5) – μ x 26180 x 62.832 = 0

μ =0.0486 poise

Problem 1-10: A flat circular disk of radius 1.0 m is rotated at an angular velocity of 0.65 rad/s over a fixed surface. Find the power required to rotate the disk if an oil film of viscosity 0.15 poise and thickness 0.5 mm separates the disk from the surface. Sketch (with values) the shear stress distribution along a radius of the disk.

Solution

$$power = \frac{work}{time} = F_{f*v}$$

ω =0.65 rad. /sec
μ = 0.15 poise
dy =0.5 mm

Assume linear velocity distribution

$$dp = dF_f.v = dF_f.\omega r = \mu \frac{dv}{dy} .dA.\omega r$$

$$dP = \mu.\frac{\omega r}{dy}.2\pi r dr.\omega r$$

$$dP = \mu \frac{2\pi\omega^2}{dy}(r^3.dr)$$

$$Power = \mu \frac{2\pi\omega^2}{dy}\int_0^{100} r^3\,dr$$

$$= \mu \frac{2\pi\omega^2}{dy}\left[\frac{r^4}{4}\right]_0^{100}$$

$$Power = 0.15\frac{2\pi(0.65)^2}{0.05}\left[\frac{(100)^4}{4}\right] = 199098434.4 \text{ dyne . cm/sec}$$

Shear stress distribution along the radius of the disk:

$$\tau = \mu .\frac{dv}{dy} = \mu \frac{\omega r}{dy}$$

$$= \mu \frac{\omega}{dy}.r = 0.15 * \frac{0.65}{0.05} r = 1.95\,r$$

R (cm)	0	20	40	60	80	100
τ (dyne/cm^2)	0	39	78	117	156	195

Problem 1-11: The pressure exerted on a liquid increases from 500 to 1000 kpa. The volume decreases by 1%. Determine the bulk modulus of the liquid.

Solution

P1=500 KPa

P2=1000 KPa

$\frac{\Delta V}{V}$ = -0.01 (decrease in volume)

$$K = -\left[\frac{\Delta P}{\frac{\Delta V}{V}}\right]$$

$$= -\left[(1000-500)/(-0.01)\right]$$

$$= 50{,}000 \text{ KPa } (KN/m^2)$$

CHAPTER 2

FLUID PRESSURE AND HYDROSTATIC FORCE

Problem 2-1: For the tank shown in Fig. (1), calculate the pressure in KPa at points A, B, C, and D.

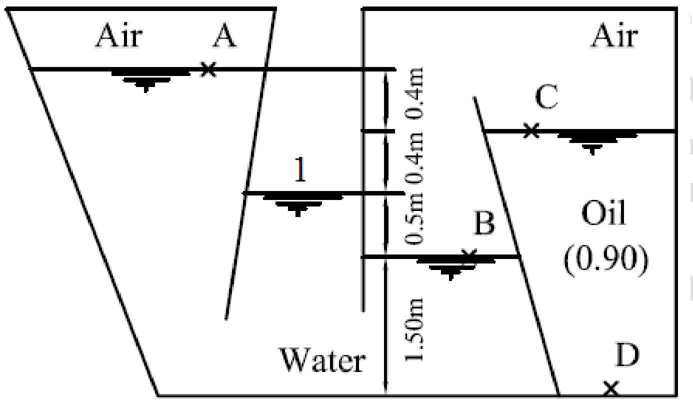


Figure (1)

Solution

$P_1=0$

$P_A=P_1-0.8*\gamma_w=0-0.8*9810=-7848\ N/m^2=-7.848$ Kpa

$P_B=P_1+0.5*\gamma_w=0+0.5*9810=4905\ N/m^2=4.905$ Kpa

$P_B=P_C$

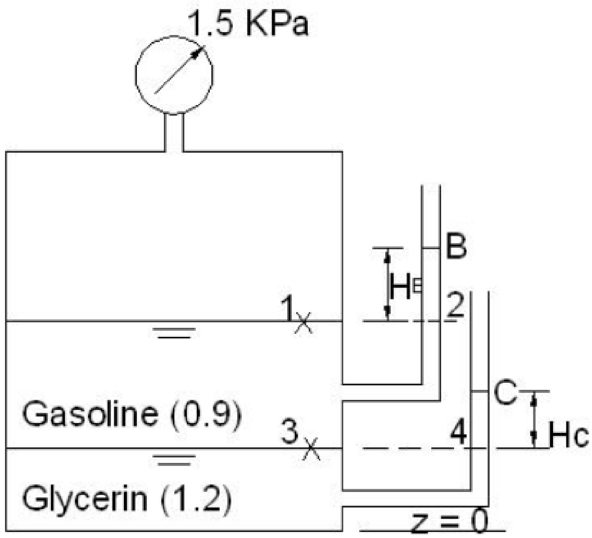
(Neglecting height of air)

$P_D=P_c+2.4*\gamma_{oil}=4905+2.4*(0.9*9810)=26094.6\ N/m^2$

$=26.0946$ KPa

Problem 2-2: The pressure gage in the tank shown in Fig. reads 1.5 KPa. Determine the elevations of the liquid levels in the open piezometer tubes B and C.

Solution



$P_1 = P_2$

$1.5 = \gamma_{gas} H_B$

$= (0.95 \times 9.81)\, H_B$

$H_B = 0.161$ m

Elevation (B) = (1+1.5) + 0.161 = 2.661

$P_3 = P_4$

$P_{air} + \gamma_{gas}(1.5) = \gamma_{Gly} H_C$

$1.5 + (0.95 \times 9.81) \times 1.5 = (1.2 \times 9.81)\, H_C$

$H_C = 1.315$ m

Elevation (C) = 1 + 1.315 = 2.315

Problem 2-3: For the pipes shown in Fig. (3), determine the absolute and gage pressures at the center of each pipe (A) in N /m^2. Find the pressure head in each the pipe expressed in terms of meters of water.

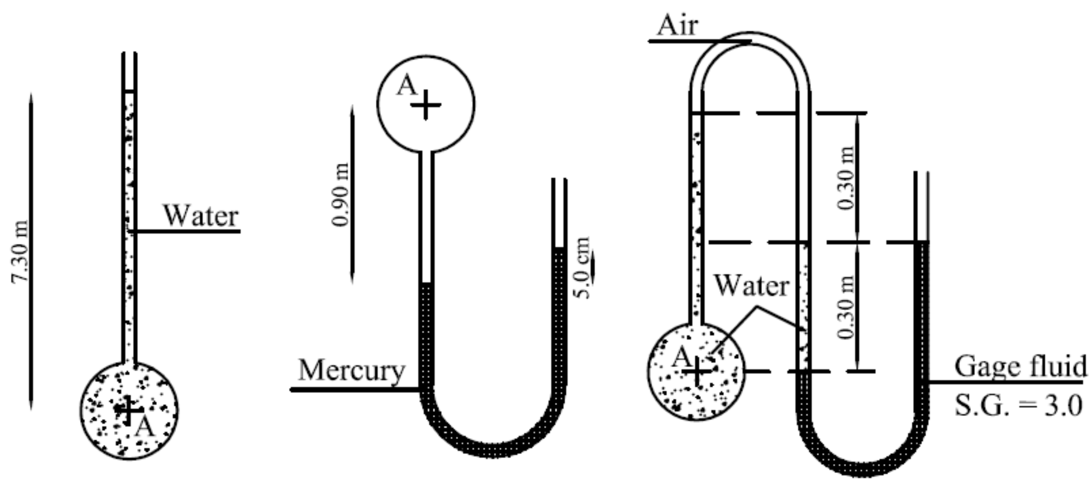


Figure (3-a) Figure (3-b) Figure (3-c)

Solution

a) $P_{A(abs)}=\gamma_w H_w+P_{atm}$

$$=9810*7.3+10^5=1.716\times10^5 \text{ N/m}^2$$

$P_{A(gage)}=\gamma_w H_w=9810*7.3=71613 \text{ N/m}^2$

$$H=\frac{P_{A(gage)}}{\gamma_w}=7.3 \text{ m of water}$$

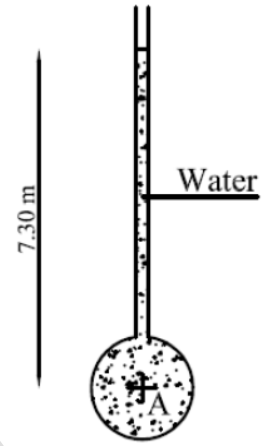


Figure (3-a)

b) $P_1=P_2$

$P_{A(gage)}+\gamma_w(0.9)=\gamma_{Hg}(0.05)$

$P_{A(gage)}+9810\ (0.9)=13.6*9810*0.05$

$P_{A(gage)}=-\ 2158.2 \text{N/m}^2$

$P_{A(abs)}=P_{A(gage)}+P_{atm}=-2185.2+10^5=97841.8 \text{N/m}^2$

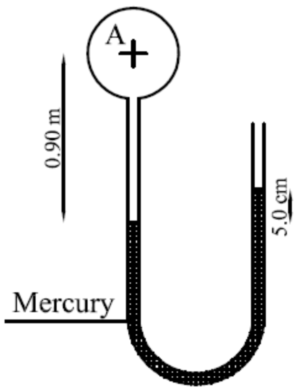


Figure (3-b)

c) $P_1 = P_2$

$\gamma\ (0.3) = P_2$

$P_2=3*9810*(0.3)=8829\ \text{N/m}^2$

$P_3=P_2-\gamma_w(0.3)=8829-9810*0.3=5886 \text{ N/m}^2$

$P_3=P_4 \qquad \text{(air)}$

$$P_{A(gage)} = P_4 + \gamma_w(0.6) = 5886 + 9810 * 0.6$$
$$= 11772 \text{ N/m}^2$$

$$H = \frac{P_{A(gage)}}{\gamma_w} = \frac{11772}{9810} = 1.2 \text{ m of water}$$

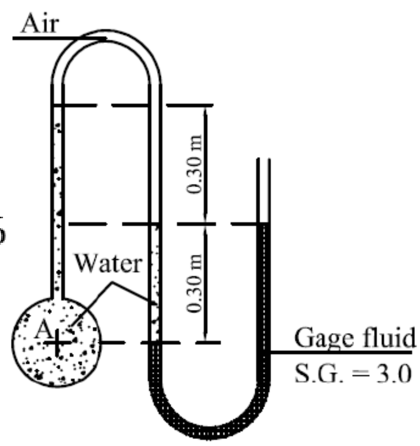
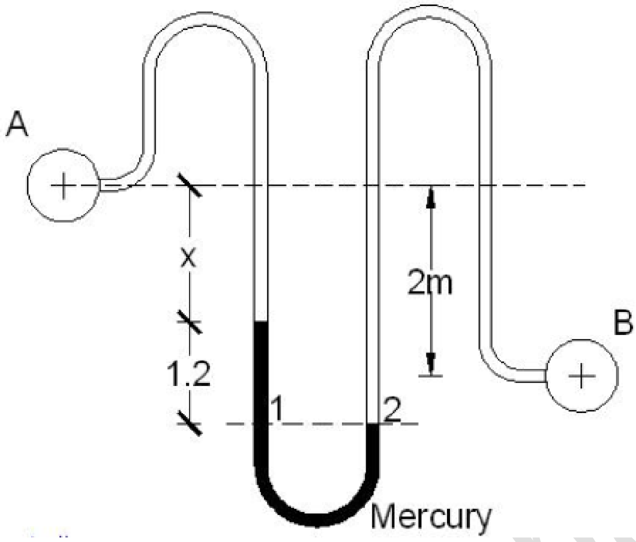


Figure (3-c)

Problem 2-4: Determine the pressure difference between pipes A and B in the setup shown in the Fig.



Solution

$P_1 = P_2$

$P_A + \gamma_w(x) + 1.2\ \gamma_{Hg} = P_B - 2\ \gamma_w + \gamma_w\ (x + 1.2)$

$P_A + 1.2\ \gamma_{Hg} = P_B - 0.8\ \gamma_w$

$\Delta P \quad = P_B - P_A$

$= 1.2\gamma_{Hg} + 0.8\gamma_w$

$= 1.2 \text{ x } 9810 \text{ x } 13.6 + 0.8 \text{ x } 9810$

$= 167947.2\ N/m^2$

Problem 2-5: Gate AB in Fig. (5) is 6 m wide and weighs 2 tons. It is hinged at B and rests against a smooth wall at A. Determine the water level h at which the gate will start to open.

Solution

Use pressure distribution method since the gate is rectangular shape.

$$F_1=(5\gamma_w*10)*6=(5*1*10)*6=300\ t$$

$$F_2=\left(\frac{8\gamma_w*10}{2}\right)*6=\left(\frac{8*1*10}{2}\right)*6=240t$$

$$F_3=(\gamma_w\ h*10)*6=(1*h*10)*6=60h\ t$$

$$F_4=\left(\frac{8\gamma_w*10}{2}\right)*6=\left(\frac{8*1*10}{2}\right)*6=240\ t$$

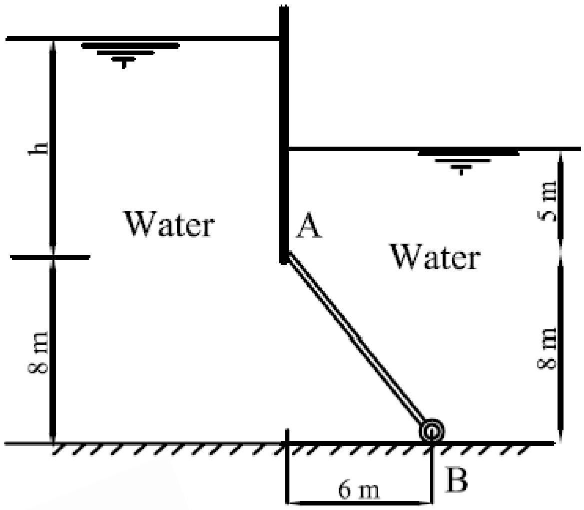
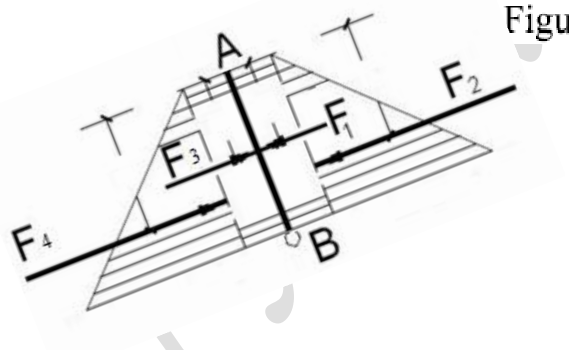


Figure (5)

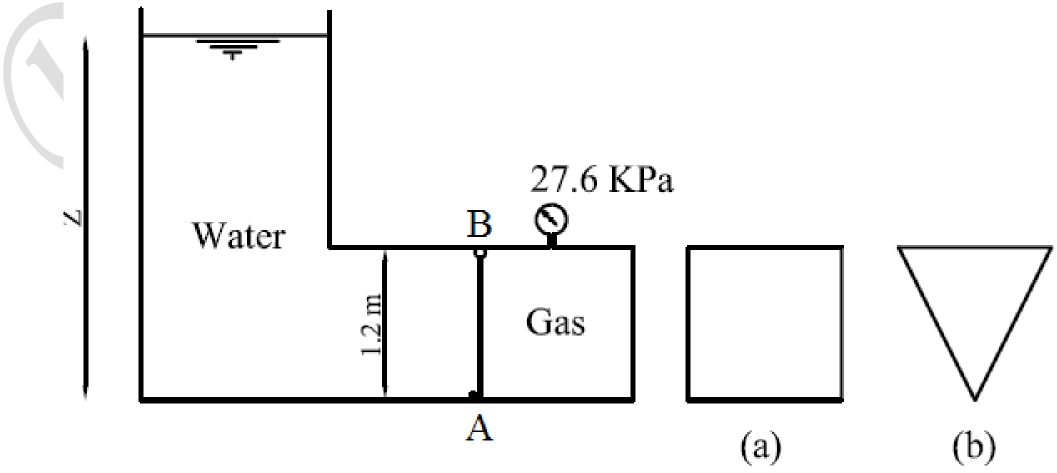
$$(F_1-F_3)*5+w*3=0$$

$$(300-60h)*5+2*3=0$$

$$h=5.02\ m$$



Problem 2-6: Find the minimum value of Z for which the gate shown in Fig. will rotates counterclockwise if the gate is (a) rectangular, 1.2 by 1.2 m; (b) triangular, 1.2 m base as axis of motion, height 1.2 m.

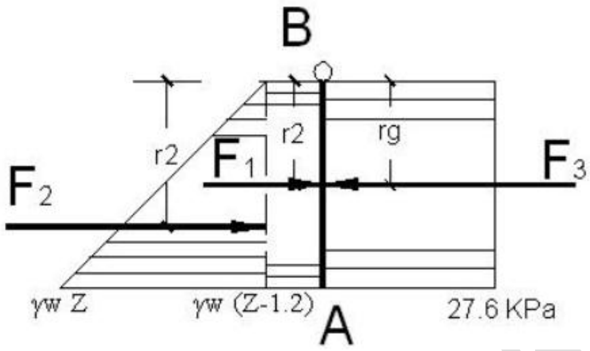


Solution

a. The gate is Rectangular

Use pressure distribution method since the gate is of a planar rectangular shape.

For the gate to rotate at B, then the reaction at A is equal to zero



Force (KN)	Arm (m)
$F_1=\gamma_w(Z-1.2) \times 1.2 \times 1.2$ $=9.81(Z-1.2) \times 1.2 \times 1.2$ $=14.1264\ (Z-1.2)$	$r_1=1.2/2=0.60m$
$F_2=1/2\ (1.2\gamma_w) \times 1.2 \times 1.2$ $=1/2\ (1.2 \times 9.81) \times 1.2 \times 1.2$ $=8.4758$	$r_2=1.2 \times 2/3=0.80m$
$F_{gas}=27.6 \times 1.2 \times 1.2$ $=39.744$	$r_g=1.2/2=0.60m$

$\Sigma M_B=0$

$F_1 \times r_1 + F_2 \times r_2 - F_{gas} \times r_g = 0$

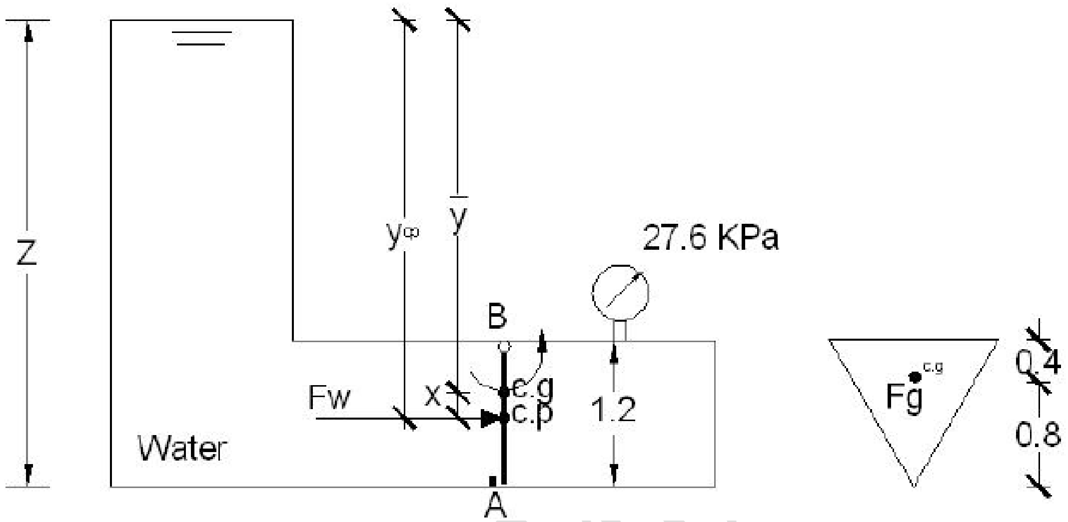
$14.1264(Z-1.2) \times 0.60 + 8.4758 \times 0.80 - 39.744 \times 0.60 = 0$

$Z = 3.213$ m

b. <u>The gate is Traiangular</u>

Use general law method.

For the gate to rotate at B, then the reaction at A is equal to zero



$$x = Y_{cp} - \bar{Y} = \frac{I}{A\bar{Y}}$$

$A = \frac{1}{2} \times 1.2 \times 1.2 = 0.72\ m^2$

$\bar{I} = bh^3/36 = 1.2 \times 1.2^3/36 = 0.0576\ m^4$

$\bar{Y} = Z - 0.8$

$$x = \frac{0.0576}{0.72\,(Z-0.8)}$$

Force (KN)	Arm (m)
Fw = 9.81 x 0.72 x (Z-0.8) = 7.0632 (Z-0.8)	r = x + 0.4
Fgas = ½ x 1.2 x 1.2 x 27.6 = 19.872 KN	r_{gas} = 0.40

$\Sigma M_B = 0$

$F_w \times r - F_{gas} \times r_g = 0$

$$7.0632(Z-0.8) \times \left(\frac{0.0576}{0.72\,(Z-0.8)} + 0.4\right) - 19.872 \times 0.4 = 0$$

$Z = 3.4135$ m

Problem 2-7: (a) A circular butterfly gate pivoted about a horizontal axis passing through its centroid is subjected to hydrostatic thrust and counterbalanced by a force F, applied at the bottom as shown in Fig. (7). If the diameter of the gate is 4 m and the water depth is 1 m above the gate. Determine the force F, required to keep the gate in position.

(b) If the gate is to retain water to its top level on the other side also, determine the net hydrostatic thrust on the gate and suggest the new condition for the gate to be in equilibrium.

Solution

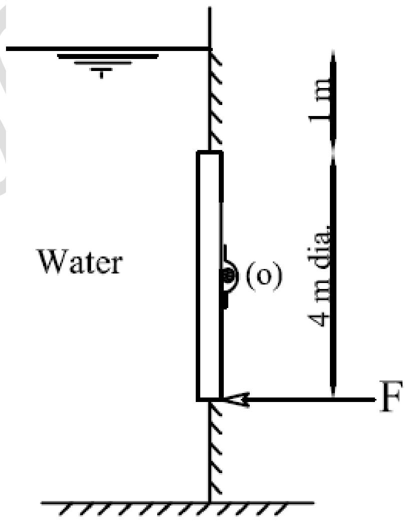


Figure (7)

$F_p = \gamma\, A\, \hbar = 1 * \frac{\pi}{4}(4)^2 * 3 = 37.7$ ton

$$X = \frac{I_{c.g}}{A.\, y_c} = \frac{\frac{\pi}{64}(4)^4}{\frac{\pi}{4}(4)^2 * 3} = 0.333\ m$$

$\sum M_{pivot} = 0$

$F_p * X - F * 2 = 0$

$37.7 * 0.33 - F * 2 = 0$ $\quad F = 6.283$ ton

<u>If water from two sides</u>

$F_{p1} = 37.7$ t , $X_1 = 0.33$m

$F_{p2} = \gamma\, A\, \hbar_2 = 1 * \frac{\pi}{4}(4)^2 * 2 = 25.13$ t

$$X_2 = \frac{I_{c.g}}{A\, y_c} = \frac{\frac{\pi}{64}(4)^4}{\frac{\pi}{4}(4)^2 * 2} = 0.5\ m$$

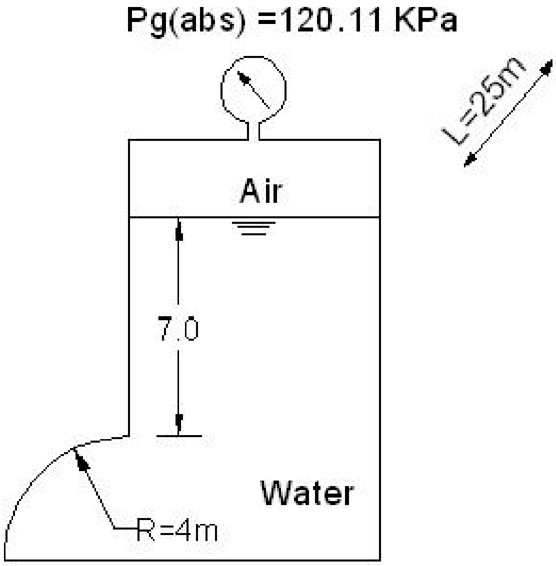
$F_{net} = F_{p1} - F_{p2} = 12.17$ ton

$F_{p1} * X_1 - F_{p2} * X_2 = F_{net} * X$

$X = 0$

The resultant force passing through the pivot and the gate is in equilibrium.

Problem 2-8: Find the magnitude and location of the resultant force on the cylindrical arc shown in the Fig. if Pg = 120.11 KPa (abs) and the arc is 25m long (normal to the paper).



Solution

a) Pg=120.11 KPa (abs)

Pg_{gauge} $= 120.11-101.325$
$=18.785$ KPa

Hw eq. $= \dfrac{\text{Pg gauge}}{\gamma w}$
$= 18.785/9.81 = 1.915$ m

F_H $= \gamma A\bar{h} = 9.81 \times (25\text{x}4) \times (1.915+7+2)$
$=10707.615$ KN

Fv $=\gamma V = \gamma AL$

For calculating area

$A = 12.915\text{x}4 - \frac{1}{4}\pi (4)^2 = 39.094 \text{ m}^2$

Fv $=9.81 \times (39.094) \times 25$
$=9587.7$ KN

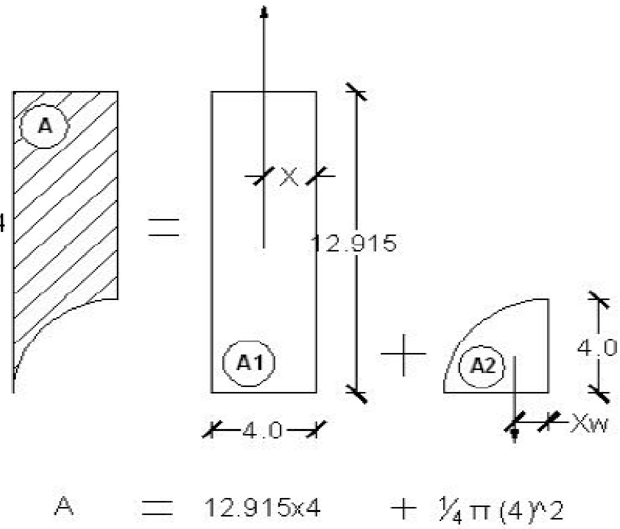
$$Y_{cp} = \bar{Y} + \frac{I}{A\bar{Y}}$$

$A = 25 \times 4 = 100 \text{ m}^2$

$\bar{I} = bh^3/36 = 25 \times 4^3/12 = 400/3 \text{ m}^4$

$\bar{Y} = 10.915$ m

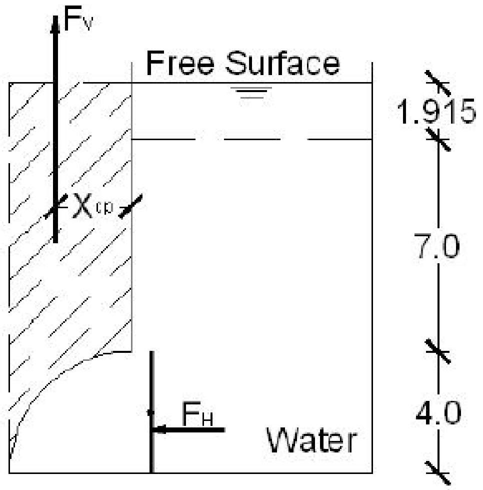
$Y_{cp} = 11.037$ m



Using first moment of Area, then

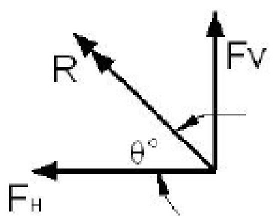
$A_1 X - A_2 X_w = A X_{cp}$
$(12.915 \times 4) \times 2 - \frac{1}{4}\pi(4)^2 \times 1.698 = 39.094 \times X_{cp}$

$X_{cp} = 2.097$ m



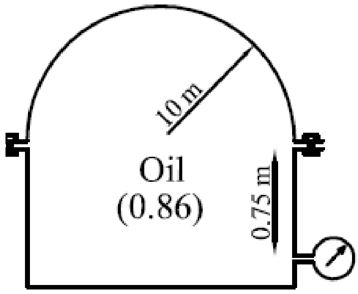
R $= \sqrt{Fh^2 + Fv^2}$
$= \text{sqrt}(10707.615^2 + 9587.7^2) = 14372.79$ KN

θ $= \tan^{-1} F_V/F_H$
$= \tan^{-1} (9587.7/10707.615)$
$= 41.84°$



Problem 2-9: The cylindrical dome shown in the Fig. is 10 m long and is secured to the top of a tank by bolts. If the pressure gauge reads 2.42 bar, and the tank is full of oil (0.86), calculate the total tension force carried by the bolts. Neglect own weight of dome.

Solution



$P=\gamma_{oil}*H$

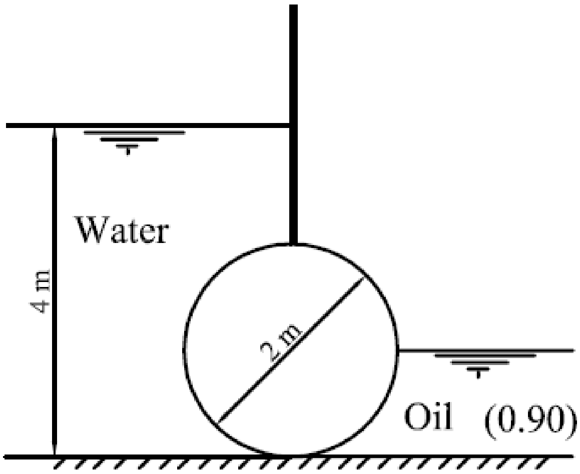
$2.42\times10^5=(0.86*9810)*H$

$H=28.685$ m

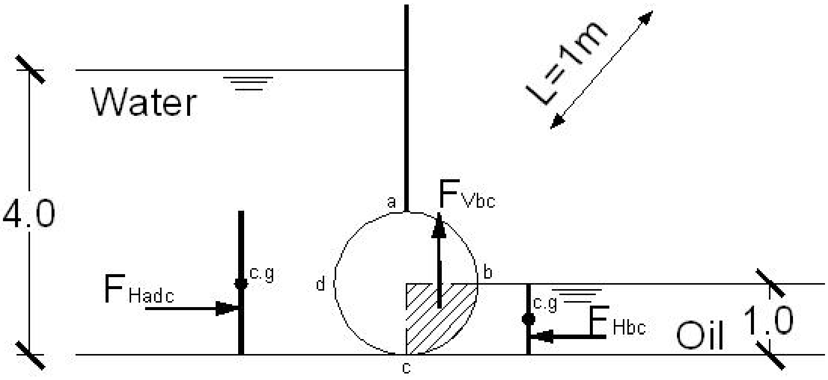
$\uparrow F_v=\gamma_{oil}*V=(0.86*1)\left[(H-0.75)*20-\frac{\pi}{2}(10)^2\right]*10=3453.94$ t

The tension force in bolts =3453.94 ton

Problem 2-10: The cylinder shown in the Fig. retains oil from one side and water from the other side. Determine the resultant hydrostatic thrust on the cylinder per unit length and its inclination to the vertical.



Solution



<u>Part ab</u>

$F_{Hab} = 0$
$F_{Vab} = 0$

<u>Part bc</u>

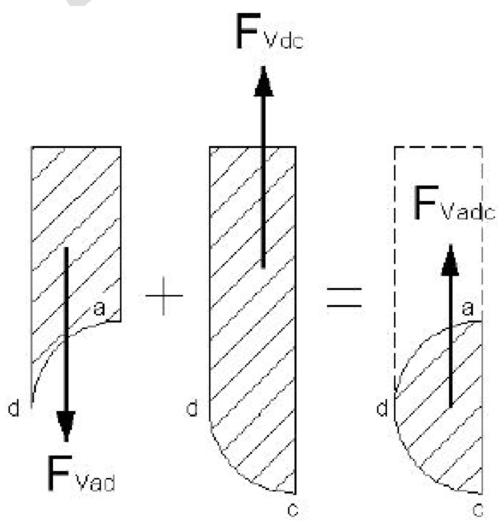
$F_{Hbc} = \gamma A \bar{h}$
$= 0.9 \times (1 \times 1) \times 0.5$
$= 0.45$ t

$F_{Vbc} = \gamma V = \gamma A L$
$= 0.9 \times \frac{1}{4} \pi (1)^2 \times 1$
$= 0.707$ t

<u>Part adc</u>

$F_{Hadc} = \gamma A \bar{h}$
$= 1 \times (2 \times 1) \times 3$
$= 6$ t

$F_{Vadc} = \gamma V = \gamma AL$
$F_{Vadc} = 1 \times 1/2\, \pi(1)^2 \times 1$
$= 1.5708$ t



For calculating area

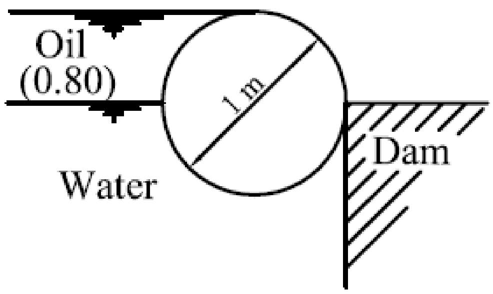
$\rightarrow F_{Htotal} = \rightarrow F_{Hadc} - F_{Hbc} \leftarrow$
$= 6 - 0.45 = 5.55$ t

$\uparrow F_{Vtotal} = \uparrow F_{V\,adc} + \uparrow F_{Vbc}$
$= 1.5708 + 0.707 = 2.278$ t

R $= \sqrt{Fh^2 + Fv^2}$
$= \text{sqrt}(5.55^2 + 2.278^2) = 6$ t

θ $= \tan^{-1} F_H/F_V$ (angle with vertical)
$= \tan^{-1} (2.278/5.55)$
$= 22.3°$

Problem 2-11: The log shown in the Fig. retains oil and water. Determine the resultant hydrostatic thrust pushing the log against the wall per unit length. Also find the specific gravity of the log material, assuming the reaction of the wall is normal to the log surface.



Solution

$F_{\overrightarrow{Hab}}=\gamma_{oil}\ A\ \hbar=0.8*(0.5*1)*0.25=0.1\ t$

$F_{Hbcd}=0$

$F_{\overrightarrow{H}}=F_{Hab}=0.1\ t$ (the force pushing the log against the wall)

$W=F_v$

$\downarrow F_{vab}=\gamma_{oil}\ .V=0.8*\left[0.5*0.5-\frac{\pi}{4}*(0.5)^2\right]*1=0.043\ t$

$\uparrow F_{vbcd}=\gamma_{oil}\ .V_{oil}+\gamma_w\ .V_w=\left[0.8*(1*0.5)+1*(\frac{\pi}{2}(0.5)^2)\right]*1=0.792\ t$

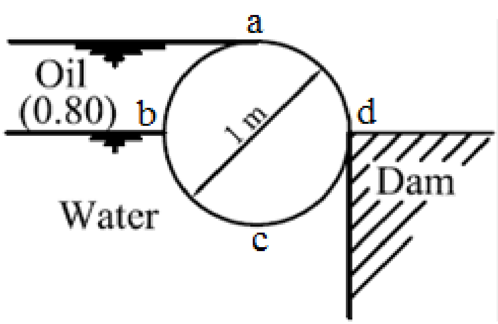
$$\uparrow F_v=F_{vbcd}-F_{vab}=0.75\ ton$$

$W=0.75\ t$

$\gamma_{log}=\frac{W}{V}=\frac{0.75}{\pi(0.5)^2}=0.955\ t/m^3$

$S.G_{log}=\frac{\gamma_{log}}{\gamma_w}=\frac{0.995}{1}=0.955$

.



CHAPTER 3

FLUID MASSES SUBJECTED TO ACCELERATION

Problem 3-1: An open rectangular tank is 2m long, 1m wide, and 1.5m deep. If the tank contains gasoline to a depth of 1m, calculate the maximum horizontal acceleration with which the tank can move without spilling any gasoline. Find the amount of gasoline retained in the tank when it is accelerated at double the previous value.

Solution

For an open tank and no spilling

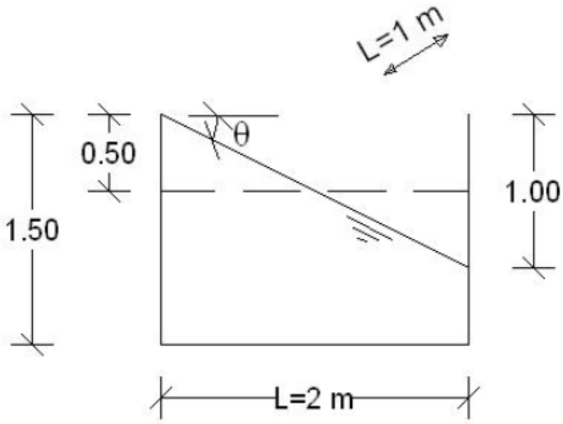
X = 1 m

Tan θ = X / L

= a_x / g

1/2 = a_x / 9.81

a_x = 0.5 x 9.81 = 4.905 m/sec^2



If the acceleration is doubled

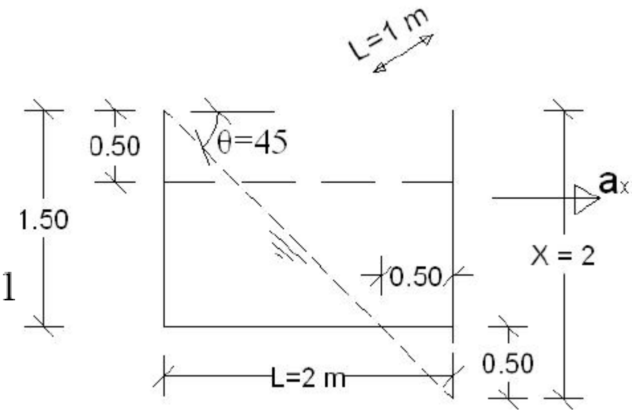
X = 2 m

Tan θ = X / L = 2/2 = 1 m

θ = 45

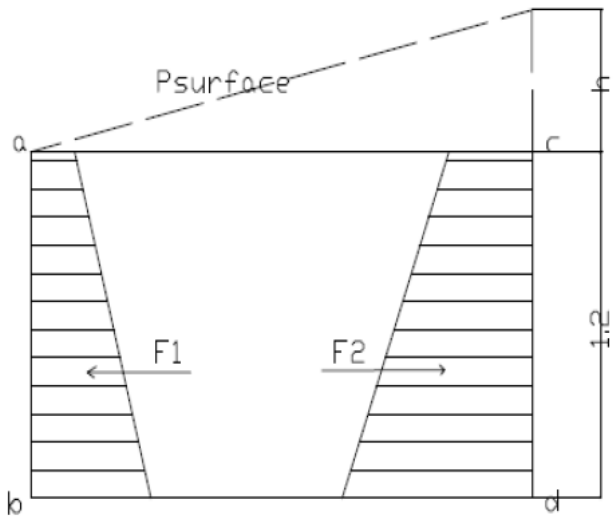
Amount of gasoline = 0.5 x 1.5 x 1.5 x 1

= 1.125 m^3



Problem 3-2: A closed rectangular tank 1.2 m high, 2.5 m long, and 1.5 wide is filled with water. The pressure at the top of the tank is raised to 60 KN/m^2. Calculate the pressure at the corners of this tank when it is decelerated horizontally along the direction of its length at 5 m/s^2. Calculate the forces on both ends of the tank and check their difference by Newton's law.

Solution



$\tan\theta = ax/g = 5/9.81 = h/2.5$

$h = 1.274$ m

$P = P_{surface} + \gamma_w * H$

$P_a = 60 + 9.81*0 = 60\ KN/m^2$

$P_b = 60 + 9.81*1.2 = 71.77\ KN/m^2$

$P_c = 60 + 9.81*1.274 = 72.50\ KN/m^2$

$P_d = 60 + 9.81*2.474 = 84.27\ KN/m^2$

Force on the rear side $= 1.2*(60+71.77)/2 *1.5$

$F_1 = 118.593$ KN

Force on the front side $= 1.2*(72.50+84.27)/2 *1.5$

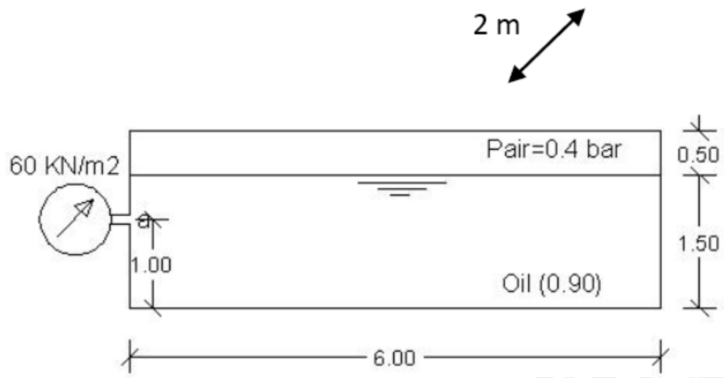
$F_2 = 141.093$ KN

$\Sigma F = m.\, ax$

$m.\, ax = 1*1.2*2.5*1.5*5 = 22.5$ KN

$F_2 - F_1 = 141.093 - 118.593 = 22.5$ KN

Problem 3-3: A closed rectangular tank 6m long, 2m wide and 2m deep is partially filled with oil (0.9) to a depth of 1.5 m under a pressure of 0.4 bar. Determine the maximum acceleration with which the tank can move in the direction of its length if a safety valve that withstands a maximum pressure of 60 KN/m^2 is fitted at the center of its rear end.



Solution

Pa max = 60 KN/m^2
$= \gamma_{oil}\, H + 0.4\text{x}10^5$
$= 0.9\text{x } H + 40$

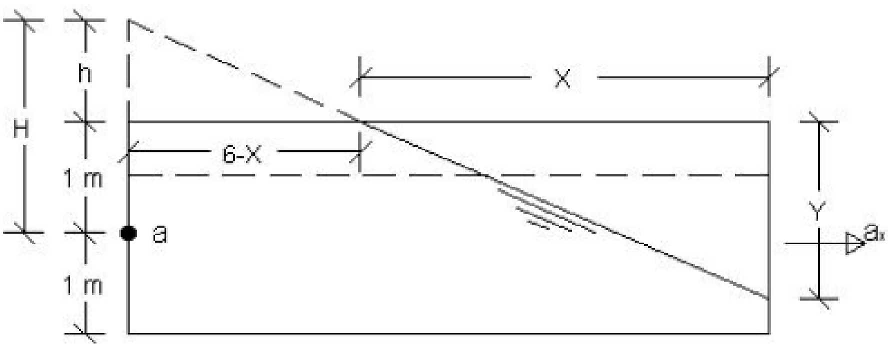
H = 2.265 m

$$\text{Tan } \theta = \frac{h}{L\text{-}X} = \frac{H\text{-}1}{6\text{-}X} = \frac{1.265}{6\text{-}X} \qquad (1)$$

Volume of air before motion =
Volume of air after motion

B x L x 2= (X.Y / 2) x 2
0.5 x 6 x 2 = X.Y x 2 (2)

Using equation (1) and (2)
X = 3.46 m
Y = 1.73 m



Tan θ = a_x / g
1.73 / 3.46 = $a_{x\,max}$ / 9.81

$a_{x\,max}$ = 4.905 m/sec^2

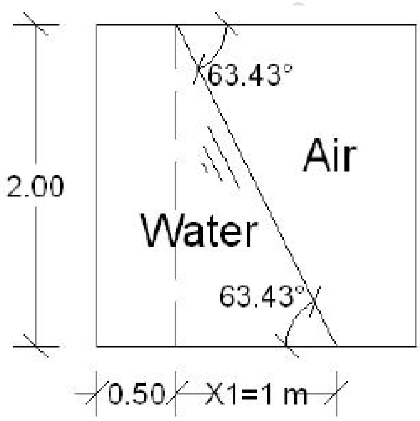
Problem 3-4: A closed cubic box of side 2.0 m contains water to a depth of 1.0 in. calculate the pressure force on the top and the bottom sides of the tank if it moves with horizontal acceleration of 19.6 m/s^2. Sketch the water surface and the pressure distributions on the four sides.

Solution

$\tan\theta = X / L = ax / g = 19.6 / 9.81 = 2$

$\theta = 63.43\,°$

$X = 2\,L = 2 \times 2 = 4\text{m}$



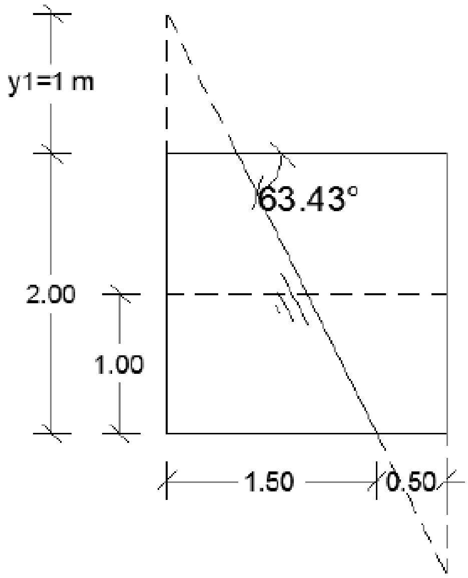
Dimensions After Motion

Before motion

Volume of Water = 2x1x2 = 4 m^3

Volume of Air = 2x1x2 = 4 m^3

Volume of water = Volume of Air



After Motion

$\tan\theta = 2 / X1 = 2$

$X1 = 1.0$ m

Then dimensions are as the opposite figure.

$Y1 = 1.0$ m

For the top side of the tank

$P1 = \gamma\, y1 = 1.0 * 1 = 1\ \text{t/m}^2$

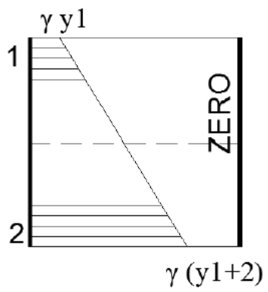
$F = ½ * 1 * 0.5 * 2\text{m} = 0.5\ \text{t}$

For the bottom side

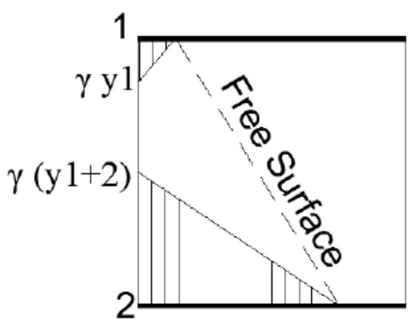
$P2 = \gamma\,(y1+2\text{m}) = 1.0 * (1+2) = 3\ \text{t/m}^2$

$F = ½ * 3 * 1.5 * 2\text{m} = 4.5\ \text{t}$

Left and Right Sides



Top and Bottom sides



Problem 3-5: Find the force on the bottom side of the tank in problem 3-4, when the tank moves with the same acceleration, but in the vertical direction: 1) upwards, 2) downwards.

Solution

ay= 19.6 m/sec^2

Case (1) moves up ward:

$$P=\gamma H\left(1+\frac{ay}{g}\right)=9.81*1\left(1+\frac{19.62}{9.81}\right)=29.43 KN/m^2$$

Force =PA =29.43*(2*2) =117.72 KN

Case (2) moves down ward:

$$P=\gamma H\left(1-\frac{ay}{g}\right)=9.81*1\left(1-\frac{19.62}{9.81}\right)=-9.81 KN/m^2$$

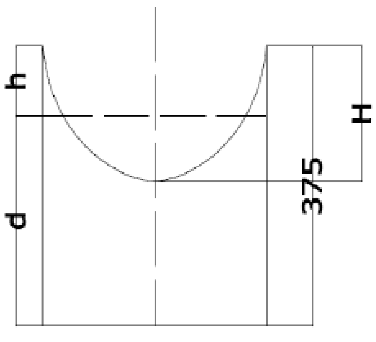
Force =PA =-9.81*(2*2) = - 39.24 KN

Problem 3-6: A 375 mm high opened cylinder, 150 mm diameter is filled with water and rotated about its vertical axis at an angular speed of 33.5 rad/s. Determine the depth of water in the cylinder when brought to rest. Find the volume of water that remains in the cylinder if the speed is doubled.

Solution

a) ω=33.5 rad./sec

$$H=\frac{\omega^2 R^2}{2g}=\frac{(33.5)^2*(\frac{0.15}{2})^2}{2*9.81}=0.321\ m$$



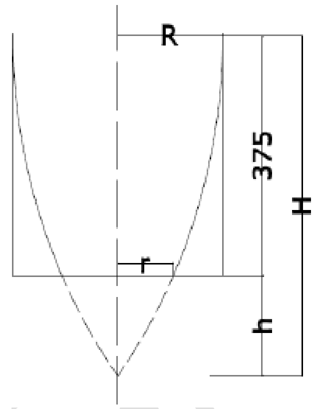
The volume of air is const

$$\frac{1}{2}*\frac{\pi}{4}*(0.15)^2*0.321=h*\frac{\pi}{4}*(0.15)^2$$

h=0.1605 m

At rest the depth =0.375 – 0.1605 =0.2145 m

b) ω=33.5*2 =67 rad./sec



$$H = \frac{\omega^2 R^2}{2g} = \frac{(67)^2 * (\frac{0.15}{2})^2}{2 * 9.81} = 1.287\, m > 0.375\, m$$

h= 1.287 – 0.375 =0.912 m

$$h = \frac{\omega^2 r^2}{2g} = \frac{(67)^2 * r^2}{2 * 9.81} = 0.912m$$

r=0.0631 m

The volume of air

$= \frac{1}{2} * \frac{\pi}{4} * (0.15)^2 * 1.287 - \frac{1}{2} * \pi (0.0631)^2 * 0.912 = 5.668\times10^{-3}\ m^3$

The volume retained $= \frac{\pi}{4} * (0.15)^2 * 0.375 - 5.668\times10^{-3}$

$= 9.59\times10^{-4}\ m^3$

Problem 3-7: A cylindrical tank of 1.0 m diameter, 2.0 m height contains water to a depth of 1.5 in. if the tank is to rotate around its vertical axis, what is the range of the angular velocity (ω) would cause spilling of water but would not uncover the base.

Solution

D = 1.0 m

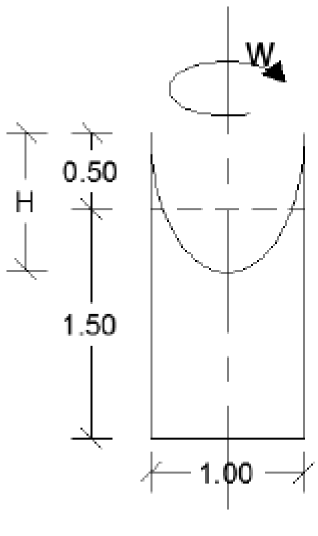
No spilling

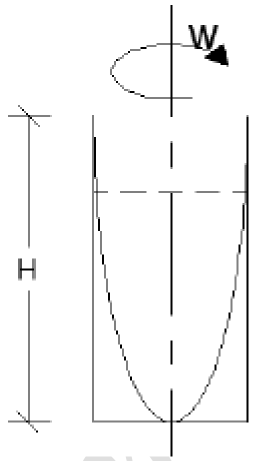
H = 2*0.5 = 1.0 m

$$H = \frac{\omega^2 R^2}{2g}$$

$1.0 = \omega^2 * 0.5^2 / (2*9.81)$

ω = 8.86 rad /sec





Spilling but not uncover the base

$$H = \frac{\omega^2 R^2}{2g}$$

$2.0 = \omega^2 * 0.5^2 / (2*9.81)$

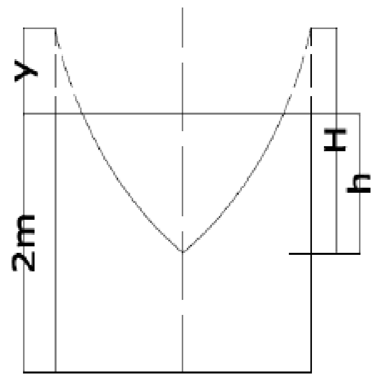
$\omega = 12.53$ rad /sec

The range of the angular speed is

8.86 rad/sec $\leq \omega \leq$ 12.53 rad/sec

Problem 3-8: A closed cylindrical tank 2-m long, 1-m diameter is partially filled with water to 1.5 m depth. The pressure at the water surface is 40 KN/m^2. If the tank is rotated about its vertical axis with an angular velocity of 12 rad/s, determine the pressure at the bottom of the tank; at its center, and edge. Also calculate the force on the top of the tank.

Solution



ω= 12 rad. /sec

$H=\omega^2 R^2 /2g = (12)^2 (0.5)^2 / (2*9.81) =1.835$ m

$h=\omega^2 r^2 /2g$ ………… (1)

The volume of air is const.

$\frac{1}{2} * \pi r^2 * h = 0.5 * \pi * (0.5)^2$ ………. (2)

By solving 1 & 2

h=1.355 m r=0.43 m

y=H – h =1.835 – 1.355 =0.48 m

$P_a = P_{surface} + \gamma_w *(2 - h) =40+9.81*(2-1.355) =46.33$KN

$P_b = P_{surface} + \gamma_w *(2+y) =40+9.81*(2+0.48) =64.33$ KN

The force on the top of the tank

$= P_{surface} * \pi r^2 + (P_{surface} + \gamma_w *(y/2))*(\pi R^2 - \pi r^2)$

$=40* \pi (0.43)^2 + (40+9.81*(0.48/2))*(\pi (0.5)^2 - \pi (0.43)^2) =31.9$ KN

Problem 3-9: A closed cylindrical tank 2m high and 1 m diameter is filled with glycerin (1.25) under pressure of 2 t/m^2. The tank is rotated about its vertical axis at a rate of 2000 rpm. If the top plate of the tank is connected to the cylinder by means of rivets that withstand a force of 50 KN, find the number of rivers required to hold the top plate to the cylinder.

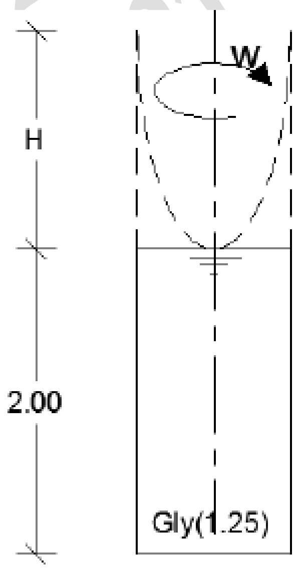
Solution

$N = 2000$ rpm

$\omega = 2000 * 2\pi/60 = 209.4$ rad/sec

$P = 2$ t/m^2 = 19.62 KN/m^2

$T_{rivets} = 50$ KN



$$H = \frac{\omega^2 R^2}{2g} = 209.4^2 * 0.5^2 / (2*9.81) = 558.72 \text{ m}$$

Force on top plate of the tank $= (P + \gamma_{gly}\, H/2) * \pi R^2$

$= (19.62 + 1.25*9.81*558.72/2) * \pi(0.5)^2$

$= 2705.91$ KN

No. of Rivets = Force / T_{rivets} = 2705.91 / 50 = 54.11 → 55 Rivets

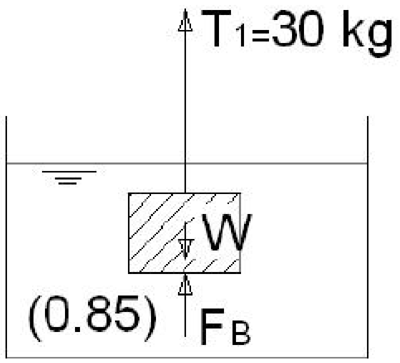
CHAPTER 4

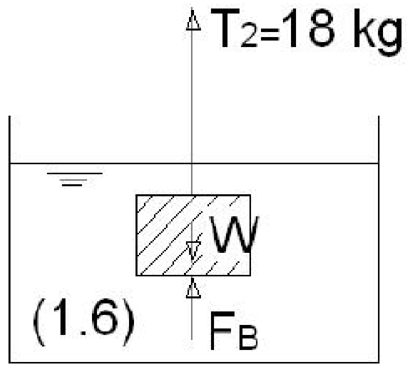
EQUILIBRIUM OF FLOATING BODIES

Problem 4-1: A body is weighed twice while suspended in two different fluids of specific gravities 0.85 and 1.6. If the balance readings are 30 and 18 kg. Determine the density of the body.

Solution

$W = F_B + T$





$$W = F_B + T_1$$
$$W = \gamma\, V + 30$$
$$= (0.85x1000)V + 30$$
$$= 850V + 30 \quad (1)$$

$$W = F_B + T_2$$
$$W = \gamma\, V + 18$$
$$= (1.6x1000)V + 18$$
$$= 1600V + 18 \quad (2)$$

Equating both (1) and (2)

$$850\ V + 30 = 1600\ V + 18$$
$$V = 0.016\ m^3$$

$$W = 850*0.016 + 30$$
$$=43.6\ Kg_f$$

$$\gamma = \frac{W}{V} = 43.6 / 0.016 = 2725\ Kg_f/m^3$$

$$\rho = 2725\ Kg/m^3$$

Problem 4-2: A cylinder of specific gravity 0.95 and length 80 cm floats with its axis vertical in a vessel containing oil of specific gravity 0.84 and water. If it is required that the cylinder become totally immersed, what should he the minimum depth of oil required to satisfy this condition.

Solution

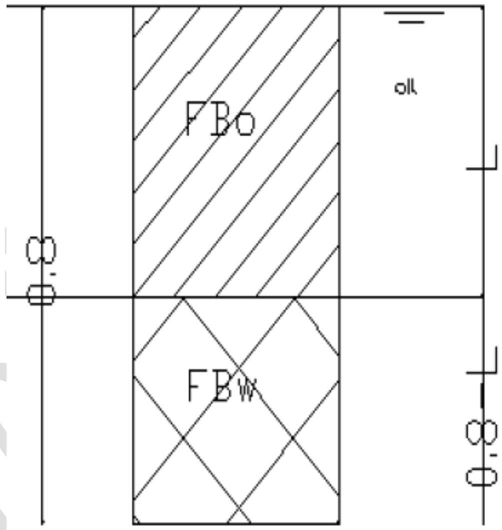
$W = F_{Bo} + F_{Bw}$

$\gamma * V = \gamma_o * V_o + \gamma_w * V_w$

$0.95 * A * 0.8 = 0.84 * A * L + 1 * A * (0.8 - L)$

$0.76 = 0.84L + 0.8 - L$

$L = 0.25 \text{ m}$



Problem 4-3: A rectangular tank has dimensions 120 x 66 x 60-cm and weighs 1410 N. If the tank is allowed to float in fresh water, will the equilibrium be stable when the 60 cm edge is vertical?

Solution

$W = 1410 \text{ N}$

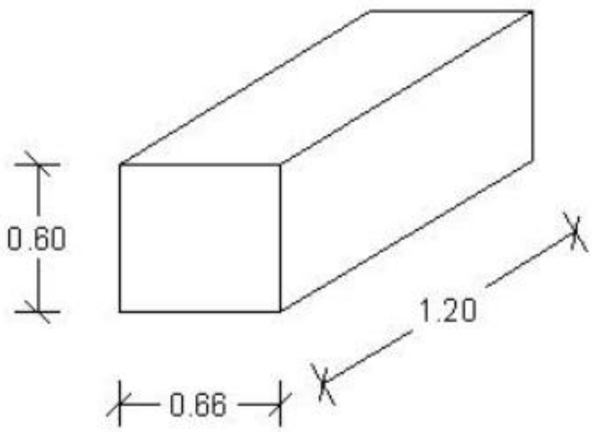
For a floating body in equilibrium

$W = F_B$

$1410 = \gamma V_{imm}$

$= 9810 \times (0.66 \times 1.2) \times H$

$H = 0.18 \text{ m}$

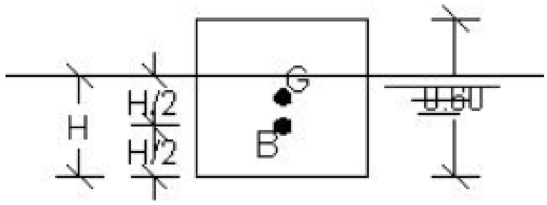


$GB = 0.60/2 - 0.18/2 = 0.21$ m

$V_{imm} = 0.66 \times 1.2 \times 0.18 = 0.14256\ m^3$

$I_{min} = bh^3/12$

$=1.2 \times 0.66^3/12 = 0.02875\ m^4$



$$MG = \frac{Imin}{Vimm} - GB$$

$=0.02875/0.14256 - 0.21 = -0.0083$ (-ve)

Unstable

Problem 4-4: A hollow wooden cylinder of specific gravity 0.60 has an outer diameter 0.80 m. an inner diameter 0.40 m, and opened at its ends. It is required to float in oil of specific gravity 0.85, calculate the maximum height of the cylinder keep it floating with its axis vertical.

Solution

$W=F_B$

$$0.6*\frac{\pi}{4}*((0.8)^2-(0.4)^2)*H=0.85*\frac{\pi}{4}*((0.8)^2-(0.4)^2)*h$$

$0.6H=0.85h$ ……… (1)

$$MG=\frac{I_{min}}{v_{imm}}-BG$$

$$I_{min}=\frac{\pi*(0.8)^4-(0.4)^4}{64}=0.01885\ m^2$$

$$v_{imm}=\frac{\pi}{4}*((0.8)^2-(0.4)^2)$$

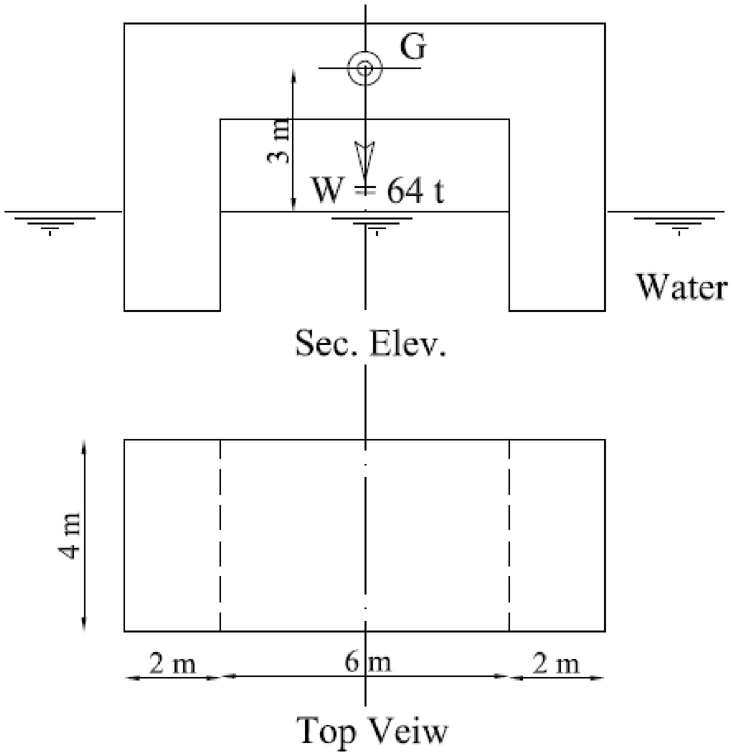
$$MG=\frac{0.05}{h}-\left(\frac{H}{2}-\frac{h}{2}\right) \text{ ……… (2)}$$

Solving 1&2

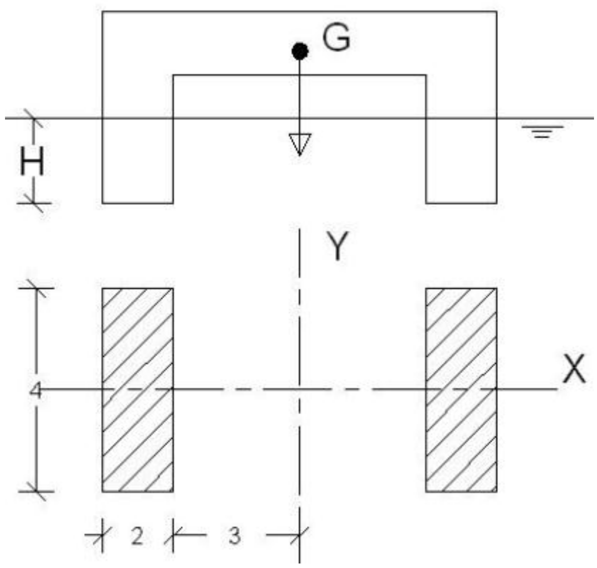
$H=0.69$ m

$h=0.49$ m

Problem 4-5: Check the stability of the floating unit in the shown fig.



Solution



W = 64 ton

For a floating body in equilibrium

$W = F_B$

$64 = \gamma V$

$64 = 1 \text{ x } 2 \text{ x } (2\text{x}4) H$

$H = 4$ m

$V_{disp} = 2 \text{ x } (2\text{x}4) \text{ x } 4 = 64\text{m}3$

$GB = 2+3 = 5\text{m}$

$$I_y = 2\left(\frac{4 \times 2^3}{12} + (4x2)x(4)^2\right) = 261.33\ m^4$$

$$I_x = 2\left(\frac{2 \times 4^3}{12}\right) = 21.33\ m^4$$

$$I_{min} = 21.33\ m^4$$

$$MG = \frac{Imin}{Vimm} - GB$$

$= 21.33/64 – 5 = -4.667$ (-ve) Unstable

Problem 4-6: A rectangular pontoon 10.3m long, 7.3m wide, and 2.4 m deep has a displacement weight of 74 tons. If an empty cylindrical boiler of 5.5m diameter and 10.3m long is mounted on its deck, check the stability of the system while floating in sea water (1.03), when the boiler's axis is: a) horizontal (parallel to pontoon axis), b) vertical. The boiler weighs 70 Ions.

Solution

Case (1) "the boiler axis // to pontoon axis"

$W_1 + W_2 = F_B$

74+70=1.03*10.3*7.3*h

h=1.86 m

74*1.2+70*5.15=144*OG

OG =3.12 m

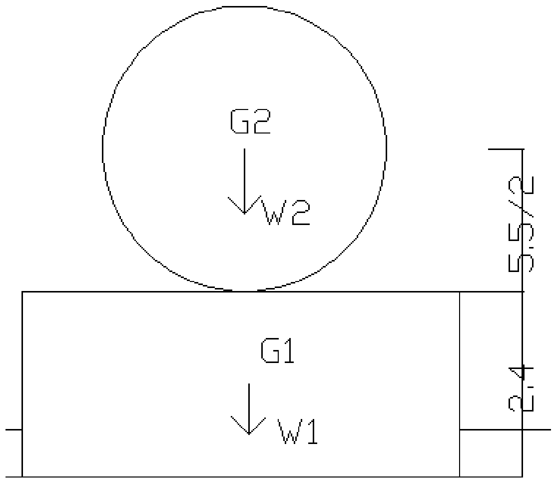
BG =OG –OB =2.19 m

$I_{min} = 10.3(7.3)^3/12 = 333.91\ m^4$

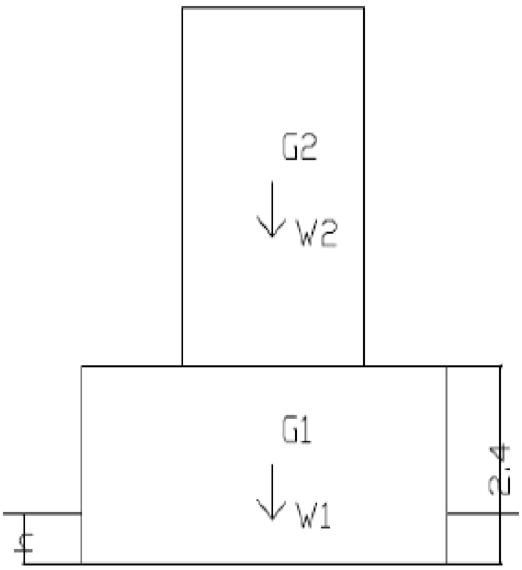
$V_{immersed} = 10.3*7.3*1.86 = 139.85\ m^3$

$MG = I_{min} / V_{immersed} - BG = 0.197$ m

MG (+) ve ………… Stable



Case (2) "the boiler axis is vertical"



$W_1 + W_2 = F_B$

$74+70=1.03*10.3*7.3*h$

$h=1.86$ m

$W_1 *OG_1 + W_2 *OG_2 = (W_1 + W_2)*OG$

$74*1.2+70*7.55=144*OG$

$OG = 4.29$ m

$OB = OG - OB = 3.36$ m

$I_{min} = 10.3\ (7.3)^3 /12 = 333.91\ m^4$

$V_{immersed} = 10.3*7.3*1.86 = 139.85\ m^3$

$MG = I_{min} / V_{immersed} - BG = -0.99$ m

MG (-) ve …………. Unstable

CHAPTER 5

KINEMATICS OF FLUID FLOW

Problem 5-1: Determine the type of flow in a 305 mm diameter pipe when
a) Water flows at average velocity of 1.0 m/sec and $\nu = 1.13 \times 10^{-6}$ m^2/sec.
b) Glycerin flows at a rate of 1.8 lit/min having $\rho = 1260$ kg/m^3 and $\mu = 0.9$ N.s/m^2.

Solution

a) D=305mm & v=1m/sec & $\upsilon=1.13*10^{-6}$ m^2/sec

$R\ n= v\ D / \upsilon=1*0.305/1.13*10^{-6}=269911.5 >4000$

The flow is turbulent

b) D=305mm & Q=1.8 lit/sec & ρ=1260 Kg/m^3 &

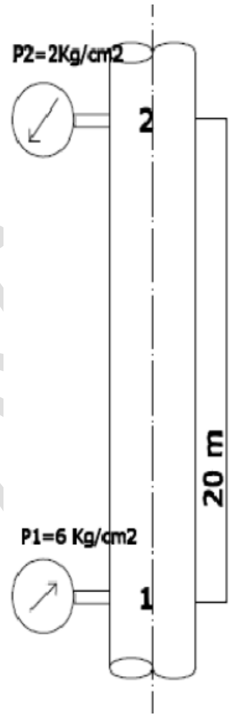
μ=0.9N.s/m^2

$V=Q/A=1.8*10^{-3}/ (\pi/4(0.305)2) =2.46*10^{-2}$ m/sec

$R\ n=\rho\ v\ D/\mu=1260*2.46*10^{-2}*0.305/0.9 =10.5 \quad <2000$

The flow is laminar

Problem 5-2: Oil of specific gravity 0.9 flows through a vertical pipe of diameter 5 cm. A pressure gauge reads 6 kg/cm^2 while another gauge 20 m higher reads 2 kg/cm^2. Find the direction of flow and the head losses.



Solution

$T.E_1 = 0 + 6 * 104 / 900 + v^2/2g = 66.67 + v^2/2g$

$T.E_2 = 20 + 2 * 104 / 900 + v^2/2g = 42.22 + v^2/2g$

$T.E_1 > T.E_2$

the flow direction is upwards (from 1 to 2)

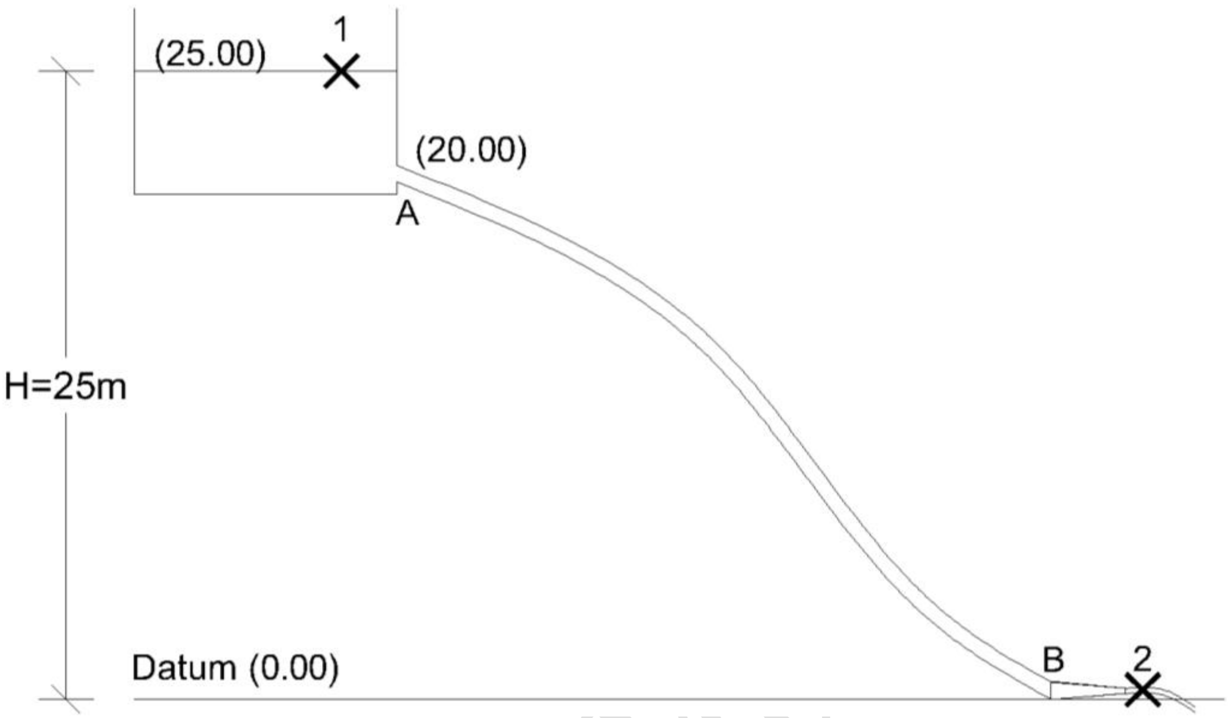
$h_f = T.E_1 - T.E_2 = 24.44$ m

Problem 5-3: A tank where the water level is 25.0 m above an arbitrary datum feeds a pipeline AB ending at B with a nozzle 4.0 cm diameter. Pipe AB is 15.0 cm diameter with point A being 20.0 m above datum and point B at datum. Find:

i) The discharge through the pipeline, the pressures and water velocities at A & B.

ii) If friction losses in the nozzle are 0.5 m, and between A & B are 5.0 m, resolve (i) and plot the hydraulic gradient and total energy lines.

Solution



i) Ideal flow

Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$25 + 0 + 0 = 0 + 0 + v_2^2 / 2g \rightarrow v_2 = 22.147$ m/sec

$Q = A_2 v_2 = \pi/4\ (0.04)^2 * 22.147 = 0.0278\ m^3/sec$

$Q = A_A v_A = A_B v_B = \pi/4\ (0.15)^2 * v_A = 0.0278\ m^3/sec$

$v_A = v_B = 1.575$ m/sec

Apply B.E bet. 1 & B

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_B + v_B^2 / 2g + P_B / \gamma$

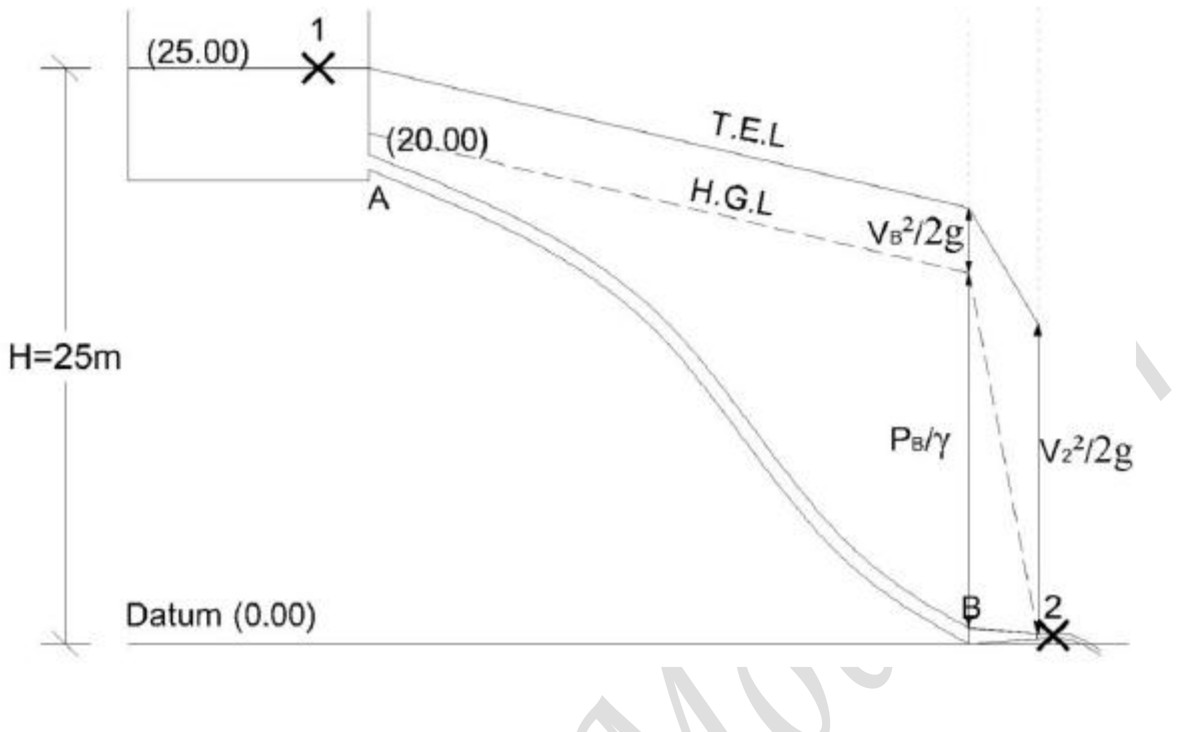
$25 + 0 + 0 = 0 + v_B^2 / 2g + P_B / \gamma \rightarrow P_B = 24.873$ m of water

Apply B.E bet. 1 & A

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_A + v_A^2 / 2g + P_A / \gamma$

$25 + 0 + 0 = 20 + v_A^2 / 2g + P_A / \gamma \rightarrow P_A = 4.873$ m of water

ii) Real flow



$hl_{nozzle} = 0.5$ m

$hl_{AB} = 5$ m

Apply B.E bet. 1 & 2

$$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_2 + v_2^2 / 2g + P_2/\gamma + hl$$

$$25 + 0 + 0 = 0 + 0 + v_2^2 / 2g + 5.5 \rightarrow v_2 = 19.56 \text{ m/sec}$$

$$Q = A_2 v_2 = \pi/4\,(0.04)^2 * 19.56 = 0.02458 \text{ m}^3/\text{sec}$$

$$Q = A_A v_A = A_B v_B = \pi/4\,(0.15)^2 * v_A = 0.02458 \text{ m}^3/\text{sec}$$

$$v_A = v_B = 1.391 \text{ m/sec}$$

Apply B.E bet. 1 & B

$$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_B + v_B^2 / 2g + P_B/\gamma + hl$$

$$25 + 0 + 0 = 0 + v_B^2 / 2g + P_B/\gamma + 5 \rightarrow P_B = 19.9 \text{ m of water}$$

Apply B.E bet. 1 & A

$$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_A + v_A^2 / 2g + P_A/\gamma$$

$$25 + 0 + 0 = 20 + v_A^2 / 2g + P_A/\gamma \rightarrow P_A = 4.901 \text{ m of water}$$

Problem 5-4: Water (assumed frictionless and incompressible) flows steadily from a large tank and exits through a vertical constant diameter pipe as shown in Fig. (1). Air in the tank is pressurized to 50 kN/m^2. Determine: (i) the height (h) to which water rises, (ii) the water velocity in the pipe, and (iii) the pressure in the horizontal part of the pipe.

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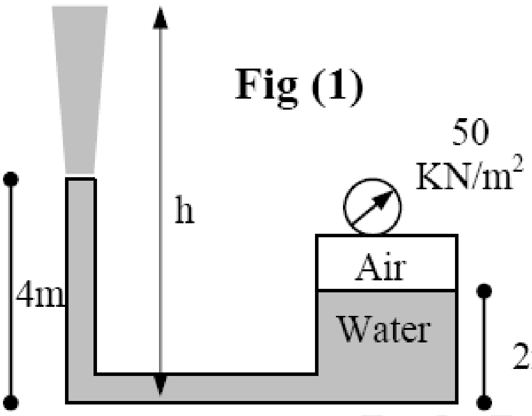
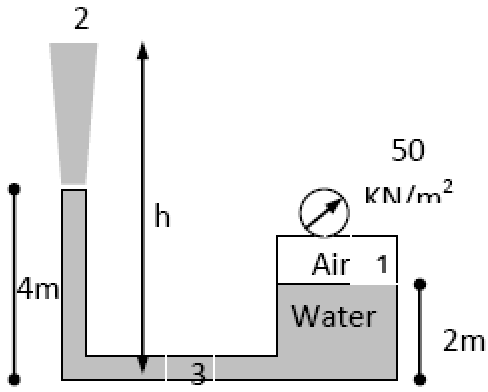


Solution



Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$2 + 50/9.81 + 0 = h + 0 + 0$

$h = 7.097\text{m}$

Apply B.E bet. 1 & 3

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_3 + v_3^2 / 2g + P_3 / \gamma$

$2 + 50/9.81 + 0 = 4 + 0 + v_3^2 / 2g$

$v_3 = 7.795$ m/sec

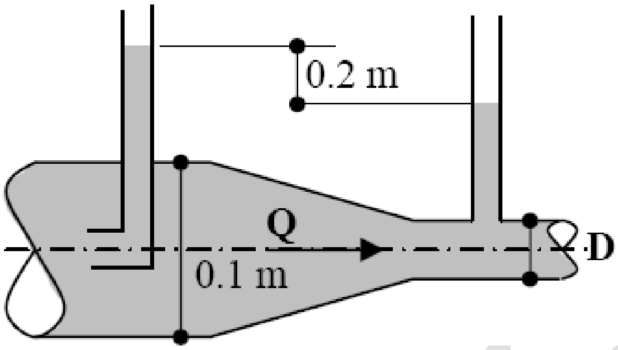
Apply B.E bet. 1 & 4

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_4 + v_4^2 / 2g + P_4 / \gamma$

$2 + 50/9.81 + 0 = 0 + (7.795)^2 / 2g + P_4 / \gamma$

$P_4 = 4$ m of water

Problem 5-5: Water flows through the pipe contraction shown in the Fig. For the given difference in peizometer levels, determine the flow rate as a function of the diameter of the small pipe, D.



Solution

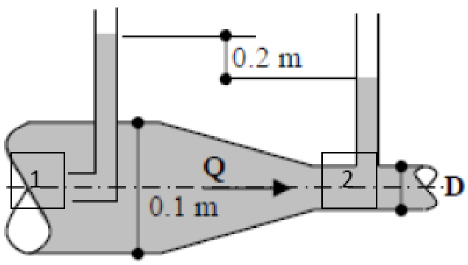
Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$v_2^2 / 2g - v_1^2 / 2g = (P_1 - P_2) / \gamma = 0.2$

$Q^2 / 2(\pi/4\, D^2)^2 g - Q^2 / 2(\pi/4\,(0.1)^2)^2 g = 0.2$

$Q = 1.556 / \sqrt{(1/D^4 - 10000)}$



Problem 5-6: A siphon filled with oil of specific gravity 0.8 discharges 220 lit/s to the atmosphere at an elevation of 3.0 m below oil level. The siphon is 0.2 m in diameter and its invert is 5.0 m above oil level. Find the losses in the siphon in terms of the velocity head. Find the pressure at the invert if two thirds of the losses occur in the first leg.

Solution

$v = Q / A = 0.22 / (\pi/4\ (0.2)2) = 7$ m/sec

Apply B.E bet. 1 & 2

$Z1 + v12 / 2\,g + P1 / \gamma = Z2 + v22 / 2\,g + P2 / \gamma + hl$

$5 + 0 + 0 = 0 + 0 + (7)2/2g + hl$

$hl = 0.50$ m

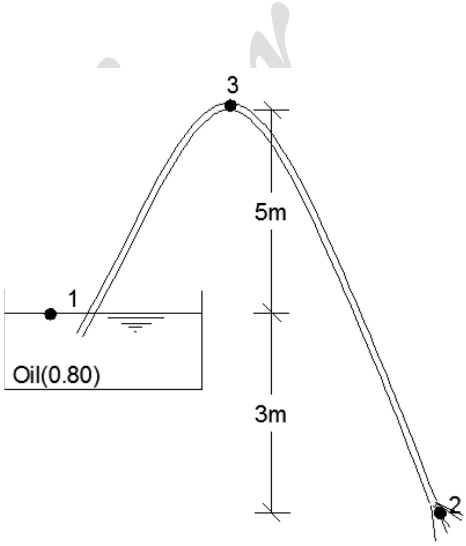
Apply B.E bet. 1 & 3

$Z1 + v12 / 2\,g + P1 / \gamma = Z3 + v32 / 2\,g + P3 / \gamma + hl$

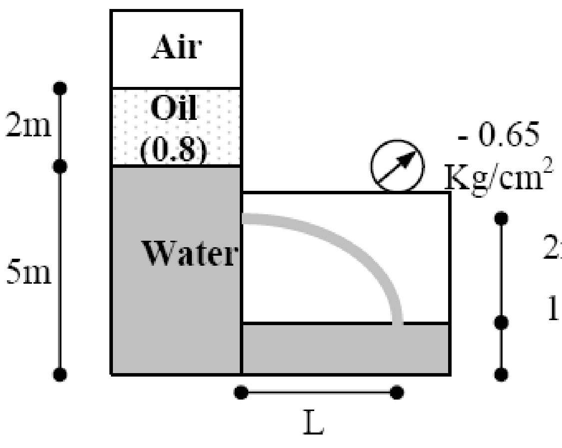
$5 + 0 + 0 = 8 + (7)2/2g + P3 / \gamma + 2/3 * 0.5$

$P3 / \gamma = -\ 7.833$ m

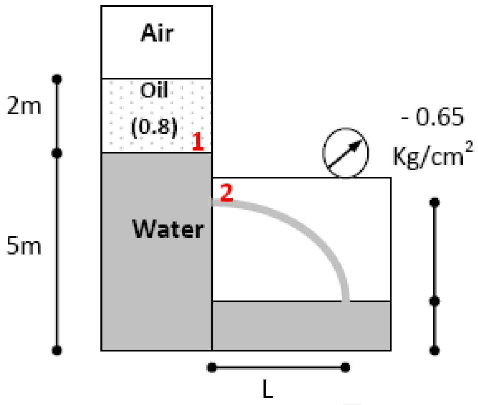
$P3 = -\ 7.833$ m of oil $= -61474.6\ N/m^2$



Problem 5-7: A 5.0-cm diameter orifice ($C_d = 0.6$) discharges water from tank A to tank B as shown in the Fig. The vacuum gauge in tank B reads 0.65 Kg/cm^2 below atmospheric pressure, while the air pressure above oil in tank A is 70 KN/m^2, find the discharge from the orifice and the distance (L).



Solution



$P_1 = P_{surface} + \gamma_O h_O$

$= 70 + 0.8 * 9.81 * 2$

$= 85.696\ KN/m^2$

$P_2 = -\ 0.65\ kg/cm^2 = -\ 0.65 * 10^4\ kg/m^2$

<u>Apply B.E bet. 1 & 2 (Ideal)</u>

$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_2 + v_2^2 / 2g + P_2/\gamma$

$5 + 0 + 85.696 / 9.81 = 3 + v_2^2 / 2g + (-\ 0.65 * 10^4)/1000$

$v_{2th} = 18.35\ m/sec$

$Q_{act} = Cd\ A_{orifice}\ v_{2th} = 0.6 * \pi/4\ (0.05)^2 * 18.35 = 0.0216\ m^3/sec$

$d = v_o\, t + ½\, a\, t^2$

<u>In Y – dir.</u> $2 = 0 + ½\, g\, t^2 \rightarrow t = 0.639\ sec$

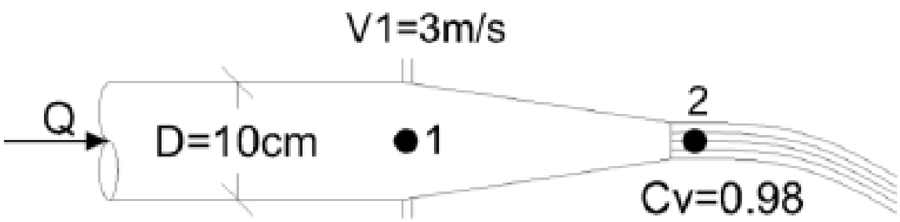
<u>In X – dir.</u> $L = v_{2act}\, t + 0$

Assume $Cv = 0.95 \rightarrow v_{2act} = Cv * v_{2th}$

$L = 0.95 * 18.35 * 0.639 = 11.13\ m$

<u>Problem 5-8:</u> The velocity of water in a pipe 10.0 m diameter is 3.0 m/s. At the end of the pipe there is a nozzle the velocity coefficient of which is 0.98. If the pressure in the pipe is 0.7 kg/cm^2, find the jet diameter, the rate of flow, and the power lost due to friction in the nozzle.

Solution



$P_1 = 0.7$ kg/cm$^2 = 0.7 * 10^4$ kg/m^2

<u>Apply B.E bet. 1 & 2 (Ideal)</u>

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$0 + (3)^2/2g + 0.7 * 10^4 /1000 = 0 + v_2^2 / 2g + 0$

$v_{2th} = 12.1$ m/sec

$v_{2act} = Cv * v_{2th} = 11.86$ m/sec

$Q = A_1 v_1 = \pi/4\ (0.1)^2 * 3 = 0.024$ m^3/sec

$Q = A_2 v_2 = \pi/4\ d^2 * 11.86 = 0.024$ m^3/sec

$d = 0.05$m

<u>Apply B.E bet. 1 & 2 (Real)</u>

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$0 + (3)^2/2g + 0.7 * 10^4 /1000 = 0 + 0 + (11.86)^2 / 2g + hl$

$hl = 0.3$ m

Power Lost due to friction $= \gamma\ Q\ hl = 9810 * 0.024 * 0.3 = 70.632$ watt

<u>Problem 5-9:</u> Water issues from a 10.0-cm diameter circular sharp edged orifice under a head of 12.0 m. If a volume of 13.6 m^3 is collected in 3 minutes, what is the coefficient of discharge? If the diameter of the jet at the venacontracta is 8.0-cm what are the values for Cv & Cc?

<u>Solution</u>

Qact = V/ t = 13.6 / (3 * 60) = 0.07756 m3/sec

Qth= vth* aorifice

= π/4 (0.1)2 √ 2 * 9.81 * 12 22.147 = 0.121m3/sec

Qact= Cd * Qth → Cd = 0.627

aact= Cc * ath

π/4 (0.08)2 = Cc π/4 (0.1)2 → Cc = 0.64

Cd = Cc * Cv → Cv = 0.98

Problem 5-10: A horizontal venturi-meter carries a liquid (S.G.= 0.8) and has an inlet to throat diameters of 15 / 7.5 cm. If the actual discharge is 40 l/s, and Cd = 0.96, find:
i)the pressure difference between the inlet and the throat, ii) the deflection in the mercury U-tube manometer connecting these points, and iii) the energy lost between the inlet and the throat. Also sketch the H.G.L. and the T.E.L. for the system.

Solution

A1 = π/4 (0.15)2 = 0.01767 m2

A2 = π/4 (0.075)2 = 4.418 * 10-3 m2

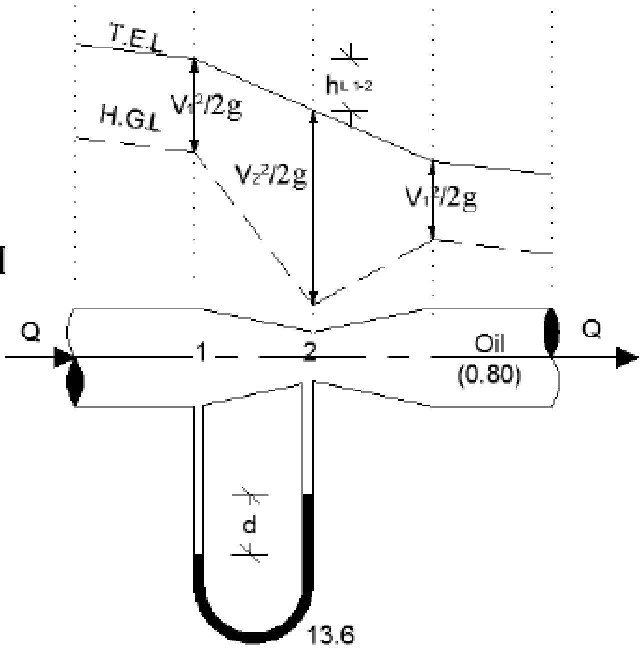
Cd = 0.96

Q = 0.04 m3/sec

Q = Cd (A1 A2) / √ (A12 – A22) √ 2g H

= 0.04 m3/sec

H = 4.25 m



i) H = (Z1 + P1 / γ) - (Z2 + P2 / γ)

H = (0 + P1 / γ) - (0 + P2 / γ) = 4.25 m

ΔP = 4.25 m of oil

H = d (γ’ / γ – 1) = d(13.6 / 0.8 – 1) = 4.25

ii) d = 0.266 m

iii) v1 = Q / A1 = 2.264 m/sec & v2 = Q / A2 = 9.054 m/sec

Apply B.E bet. 1 & 2

Z1 + v12 / 2 g + P1 / γ = Z2 + v22 / 2 g + P2 / γ + hl

hl = (Z1 + P1 / γ) - (Z2 + P2 / γ) + (v12 - v22) / 2g

= 4.25 + (2.264)2 – (9.054)2) /2g = 0.333 m

Problem 5-11: A venturi-meter is installed in a 30 cm diameter vertical pipe conveying oil of S.G. = 0.9. The throat diameter is 15.0 cm and the flow being upward. The difference in elevation between the throat and inlet is 30.0 cm. A mercury manometer connected to the venturi registers a deflection of 20.0 cm. If C_d = 0.96, calculate the discharge and the pressure difference between the inlet and throat. If the discharge remains constant while the pipe is shifted to the horizontal position, how will the reading of the manometer be affected?

Solution

$A_1 = \pi/4\ (0.3)^2 = 0.07069\ m^2$

$A_2 = \pi/4\ (0.15)^2 = 0.01767\ m^2$

$Cd = 0.96$

$H = d\ (\gamma' / \gamma - 1) = 0.2\ (13.6 / 0.9 - 1)$

$= 2.82\ m$

$Q = Cd\ (A_1 A_2) / \sqrt{(A_1^2 - A_2^2)}\ \sqrt{2g\ H}$

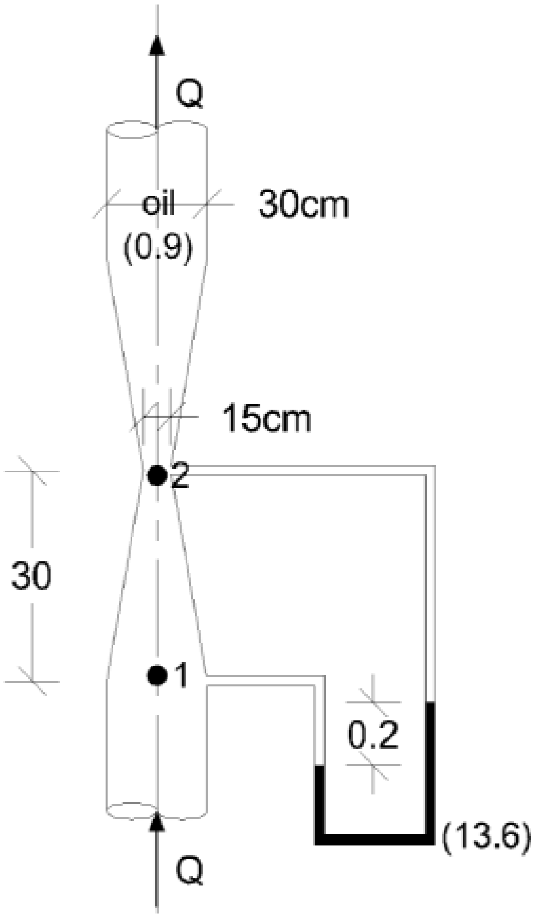
$= 0.1358\ m^3/sec$

$(Z_1 + P_1 / \gamma) - (Z_2 + P_2 / \gamma) = H$

$(0 + P_1 / \gamma) - (0.30 + P_2 / \gamma) = 2.82$

$(P_1 - P_2) / \gamma = 2.82 + 0.30 = 3.12\ m$

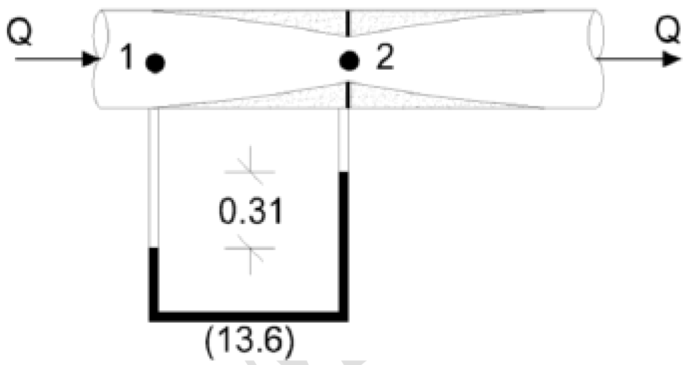
$\Delta P = 3.12\ m$ of oil



When the pipe is shifted to the horizontal position with the same discharge the reading of the manometer will not be affected.

Problem 5-12: An orifice-meter having a 0.15/0.075 m pipe to orifice diameters and $C_d = 0.65$ is used in measuring the flow of water in a pipe. If a U-tube mercury differential manometer gives 0.31 m deflection when connected to the meter, find the rate of flow. What is the maximum possible rate of flow of water if the pressure at inlet to the meter is maintained at 3.6 KN/m^2 and its vapor pressure is 1.8 KN/m^2 abs.

Solution



$P_1 = 3.6 \text{ KN/m}^2$

$A_1 = \pi/4\ (0.15)^2 = 0.01767 \text{ m}^2$

$A_2 = \pi/4\ (0.075)^2 = 4.418 * 10^{-3} \text{ m}^2$

$H = d\ (\gamma' / \gamma - 1)$

$= 0.31\ (13.6 / 1 - 1) = 3.906 \text{ m}$

$Q = Cd\ (A_1 A_2) / \sqrt{(A_1^2 - A_2^2)}\ \sqrt{2g H} = 0.026 \text{ m}^3\text{/sec}$

Q_{max} results when P_2 becomes minimum ($P_2 = P_{vapor}$)

$P_2 = P_{vapor} = 1.8 \text{ KN/m}^2$ (abs)

$P_1 = P_{Atm} + 3.6 = 101.3 + 3.6 = 104.9 \text{ KN/m}^2$ (abs)

$H_{max} = (Z_1 + P_1 / \gamma) - (Z_2 + P_2 / \gamma)$

$= (0 + 104.9/9.81) - (0 + 1.8/9.81) = 10.51 \text{ m}$

$Q_{max} = Cd\ (A_1 A_2) / \sqrt{(A_1^2 - A_2^2)}\ \sqrt{2g H_{max}} = 0.043 \text{ m}^3\text{/sec}$

Problem 5-13: A pitot-tube is placed in a pipe carrying water. A mercury differential pressure gauge reads 10.0 cm. Assuming $C_v = 0.99$, what is the velocity of water in the pipe? If the problem is reversed and mercury is flowing in the pipe and water is used in the inverted differential gauge with the same reading, what would be the velocity of mercury in the pipe?

Solution

(a) water is flowing in the pipe

Apply B.E bet. 1 & 2

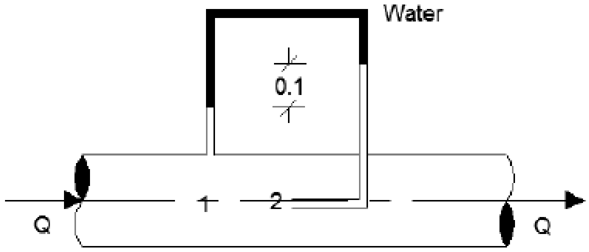
$Z_1 + v_1^2 / 2\,g + P_1 / \gamma = Z_2 + v_2^2 / 2\,g + P_2 / \gamma$

$v_1^2 / 2\,g = (P_1 - P_2) / \gamma = d\,(\gamma / \gamma - 1)$

$= 0.1\,(13.6 / 1 - 1) = 1.26$

$v_{th} = \sqrt{2 * 9.81 * 1.26} = 4.97$ m/sec

$v_{act} = Cv * v_{th} = 0.99 * 4.97 = 4.92$ m/sec



a) mercury is flowing in the pipe

Apply B.E bet. 1 & 2

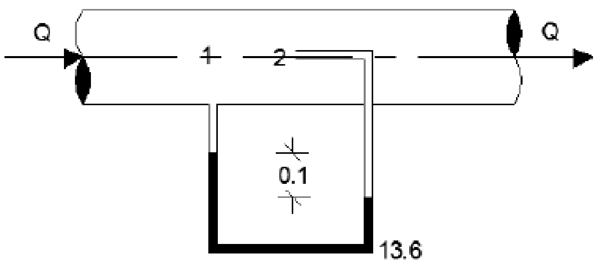
$Z_1 + v_1^2 / 2\,g + P_1 / \gamma = Z_2 + v_2^2 / 2\,g + P_2 / \gamma$

$v_1^2 / 2\,g = (P_1 - P_2) / \gamma = d\,(1 - \gamma / \gamma)$

$= 0.1\,(1 - 1/13.6) = 0.0926$

$v_{th} = \sqrt{2 * 9.81 * 0.0926} = 1.348$ m/sec

$v_{act} = Cv * v_{th} = 0.99 * 1.348 = 1.335$ m/sec



ASSIGNMENTS

ASSIGNMENT 1

UNITS AND DIMENSIONS & FLUID PROPERTIES

Q1: Using dimensional analysis, put down the dimensions and units in the engineering systems {pound (Ib), foot (ft), second (s)} and {kilogram (kg), meter (m), second (s)} for the following engineering quantities:

- Density (ρ), specific weight (γ), surface tension (σ), pressure intensity (p), dynamic viscosity (μ), kinematic viscosity (υ), energy per unit weight, power, liner momentum, angular momentum, shear stress (τ).

Q2: Show that the following terms are dimensionless:

$$\frac{v.y}{\upsilon}, \frac{\rho.v.y}{\mu}, \frac{v}{\sqrt{g.y}}, \frac{p}{\rho.v^2}, \frac{L.v^2}{h.g.d}$$

Q3: Find the dimensions for the following terms:

$$\frac{v^2}{g}, \frac{p}{\gamma}, \rho.v^2, \gamma.y, \frac{dp}{dx}, \frac{\tau}{y}, \rho.Q.v, \gamma.Q.L$$

Q4: Convert the following terms:

- $\gamma = 1000\ kg/m^3$ to Ib/ft^3
- $g = 9.81\ m/sec^2$ to ft/sec^2
- $p = 7\ kg/cm^2$ to N/m^2
- $\gamma = 710\ dyne/cm^3$ to Ib/ft^3, N/m^3
- $\mu = 4640.84$ poise to $Ib.sec/ft^2$, $Pa.sec$

Q5: What is the diameter of a spherical water drop if the inside pressure is $15\ N/m^2$ and the surface tension is $0.074\ N/m$.

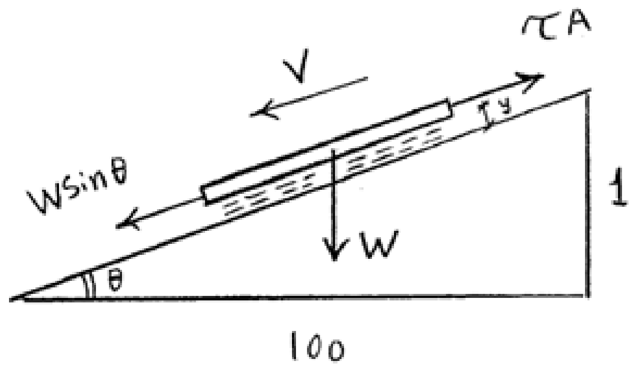
Q6: The pressure within a bubble of soapy water of 0.05 cm diameter is 5.75 gm/cm^2 greater than that of the atmosphere. Calculate the surface tension in the soapy water in S.I. units.

Q7: Calculate the capillary effect in millimeters in a glass tube of 4 mm diam., when immersed in (i) water and (ii) in mercury. The temperature of liquid is 20^o C and the values of surface tension of water and mercury at this temperature in contact with air are 0.0075 kg/m and 0.052 kg/m respectively. The contact angle for water = 0 and for mercury = 130°.

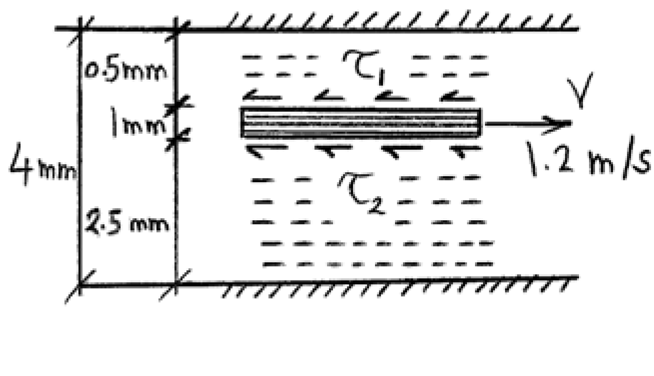
Q8: To what height will water rise in a glass tube if its diameter is (σ = 0.072 N/m)

a) 1.50 cm b) 2.0 mm

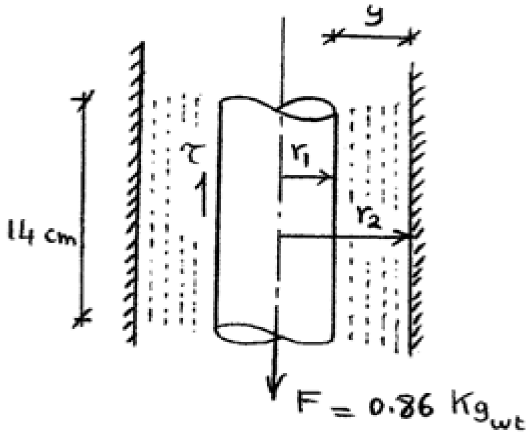
Q9: The space between a square smooth flat plate (50 x 50) cm^2, and a smooth inclined plane (1:100) is filled with an oil film (S.G. = 0.9) of 0.01 cm thickness. Determine the kinematic viscosity in stokes if the plate is 2.3 kg. The velocity of the plate = 9 cm/sec.



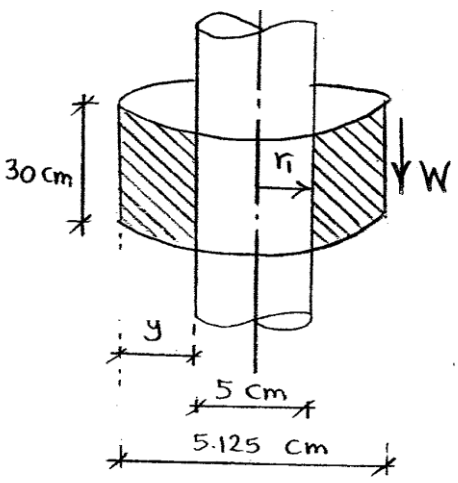
Q10: For the shown figure, Calculate the friction force if the plate area is (2m x3m) and the viscosity is 0.07 poise.



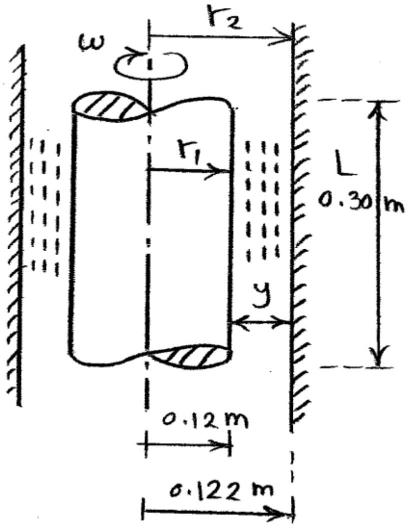
Q11: A piston 11.96 cm diameter and 14 cm long works in a cylinder 12 cm diameter. A lubricating oil which fills the space between them has a viscosity 0.65 poise. Calculate the speed at which the piston will move through the cylinder when an axial load of 0.86 kg is applied. Neglect the inertia of the piston.



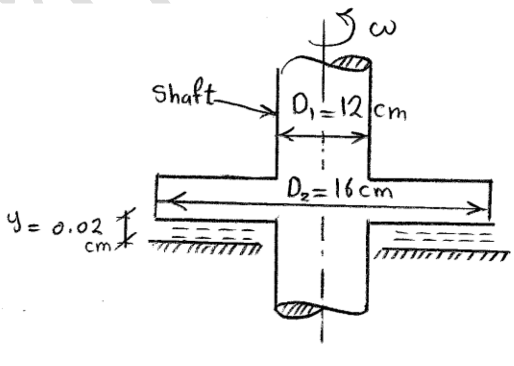
Q12: A piece of pipe 30 cm long weighting 1.5 kg and having internal diameter of 5.125 cm is slipped over a vertical shaft 5.0 cm in diameter and allowed to fall under its own weight. Calculate the maximum velocity attained by the falling pipe if a film of oil having viscosity equals 0.5 Ib.s/ft^2 is maintained between the pipe and the shaft.

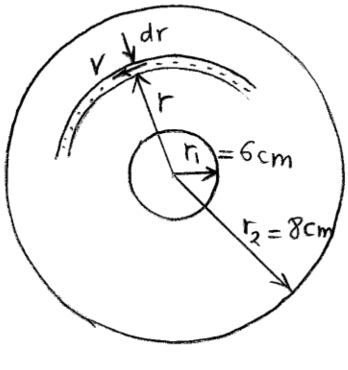


Q13: A cylinder of 0.12 m radius rotates concentrically inside of a fixed cylinder of 0.122 m radius. Both cylinders are 0.30 m long. Determine the viscosity of the liquid which fills the space between the cylinders if a torque of 1 N.m is required to maintain an angular velocity of 2 rad/s.



Q14: The thrust of a shaft is taken by a collar bearing provided with a forced lubrication system. The lubrication system maintains a film of oil of uniform thickness between the surface of the collar and the bearing. The external and internal diameters of collar are 16 and 12 cms. respectively. The thickness of oil film is 0.02 cms. and coefficient of viscosity is 0.91 poise. Find the horse-power lost in overcoming friction when the shaft is rotated at a speed of 350 r.p.m.

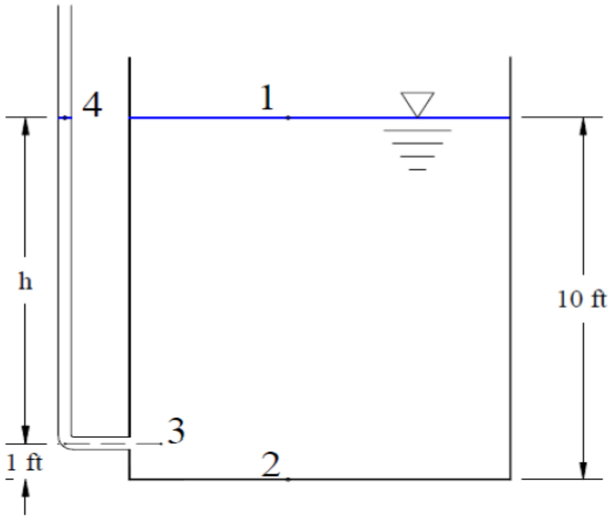




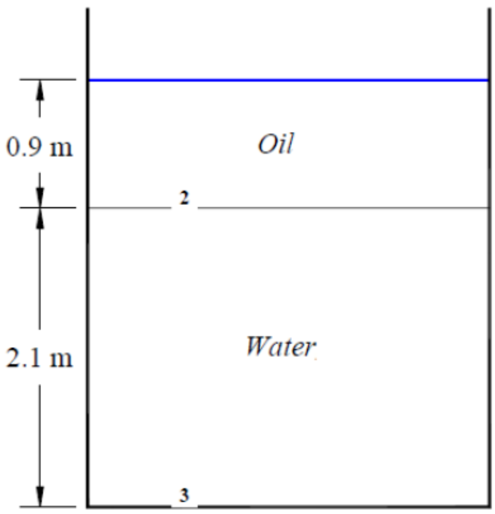
ASSIGNMENT 2

FLUID PRESSURE AND HYDROSTATIC FORCE

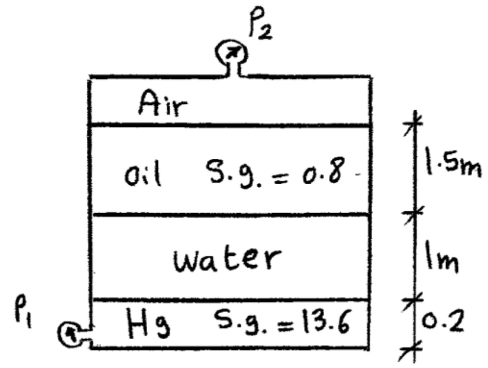
Q1: A tank full of water as shown below. Find the maximum pressure, and h.



Q2: A tank full of water and oil (S.G = 0.80), as shown. Find the pressure at the oil/water interface and the bottom of the tank.

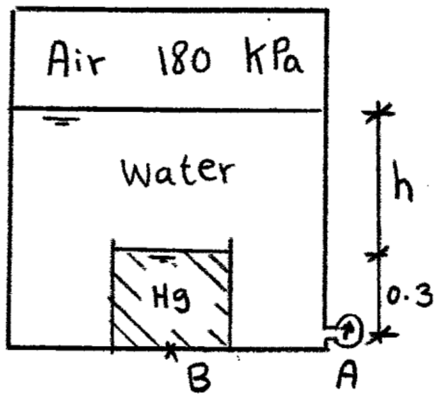


Q3: For the shown figure, find the pressure (P1) if the pressure (P2) = 60 KPa (abs)?

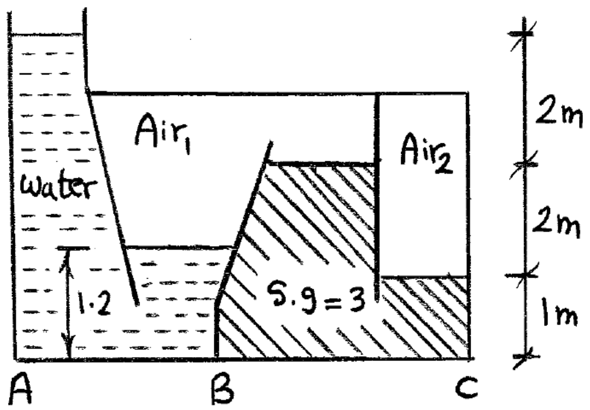


Q4: If the pressure at point (B) = 300 KPa as shown in figure, find the followings:

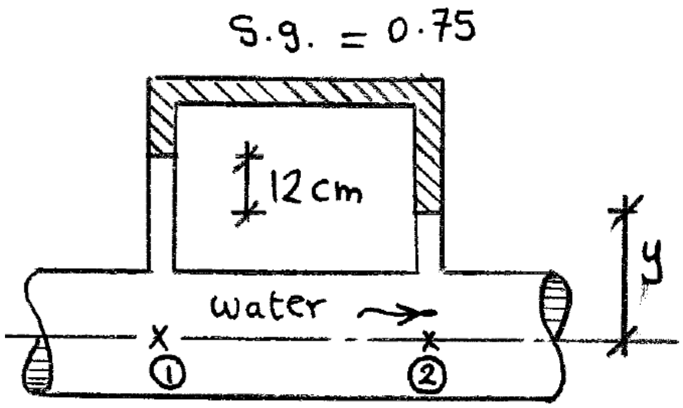
a) The height (h) b) The pressure at point (A)?



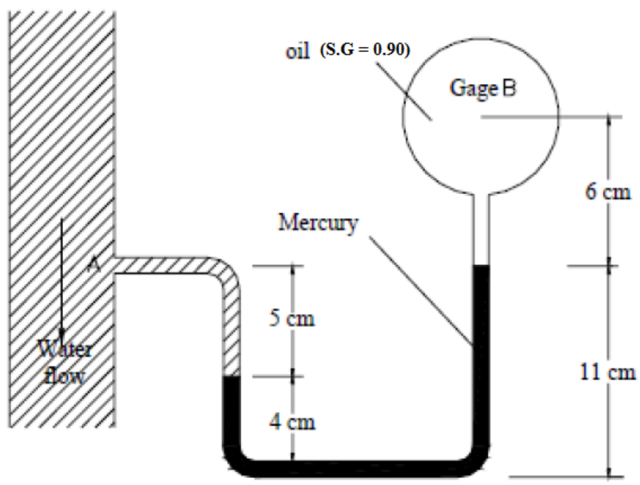
Q5: For the shown figure, where is the maximum pressure (P_{AB} or P_{BC})?



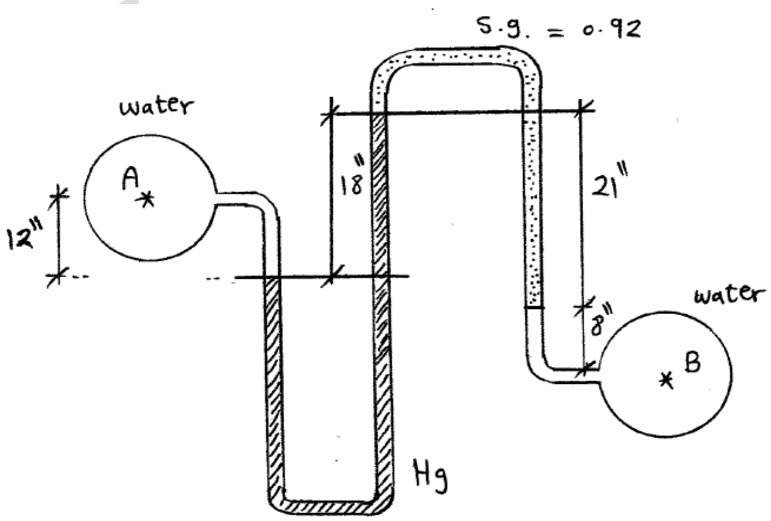
Q6: For the shown figure, what is the difference in pressure between points 1,2?



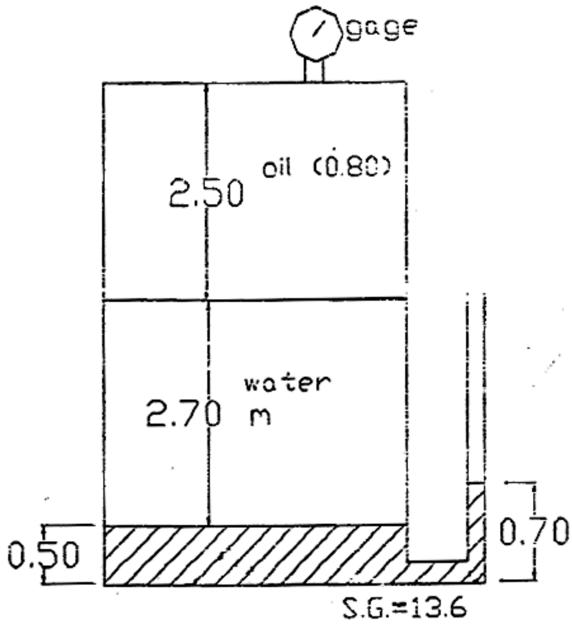
Q7: Pressure gage B is to measure the pressure at point A in a water flow. If the pressure at B is 9 t/m^2, estimate the pressure at A.



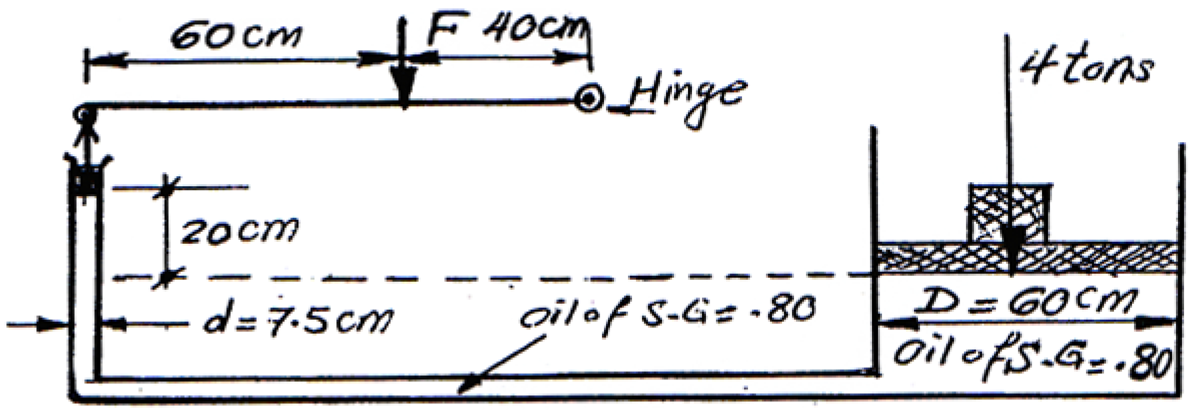
Q8: For the shown figure, what is the difference in pressure between points A, B?



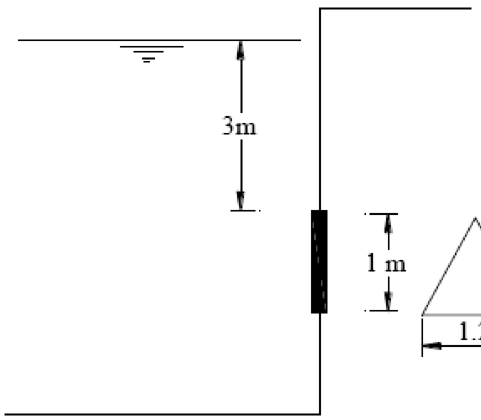
Q9: For the shown figure, what is the pressure at gauge dial P_g?



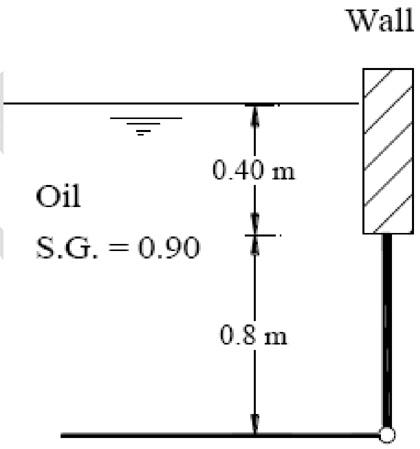
Q10: For the shown hydraulic press, find the force (F) required to keep the system in equilibrium.



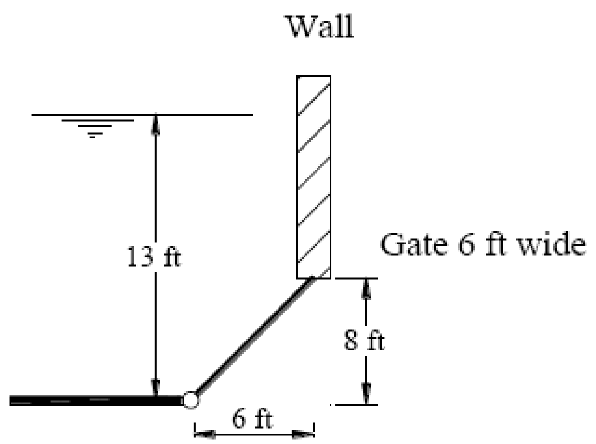
Q11: A vertical triangular gate with water on one side is shown in the figure. Calculate the total resultant force acting on the gate, and locate the center of pressure.



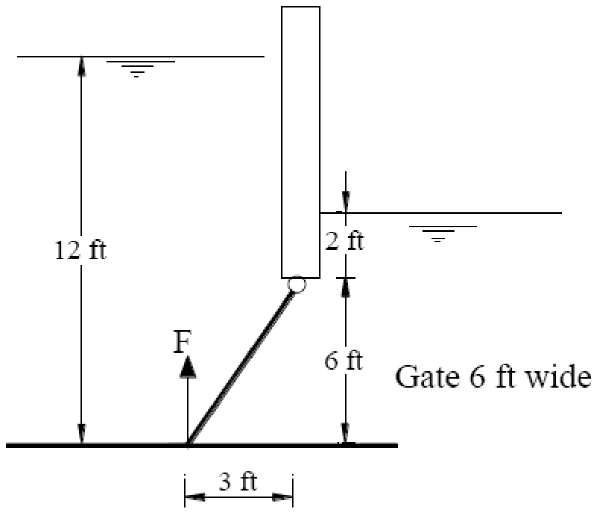
Q12: In the shown figure, the gate holding back the oil is 80 cm high by 120 cm long. If it is held in place only along the bottom edge. What is the necessary resisting moment at that edge.



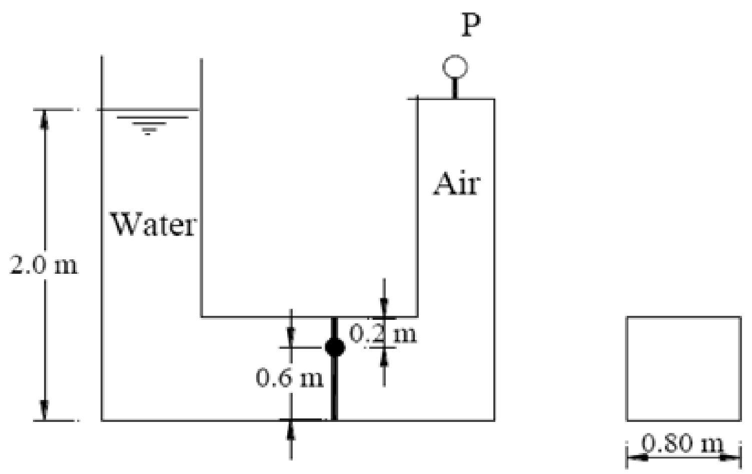
Q13: In the shown figure, the gate holding back the water is 6 ft wide. If it is held in place only along the bottom edge. What is the necessary resisting moment at that edge.



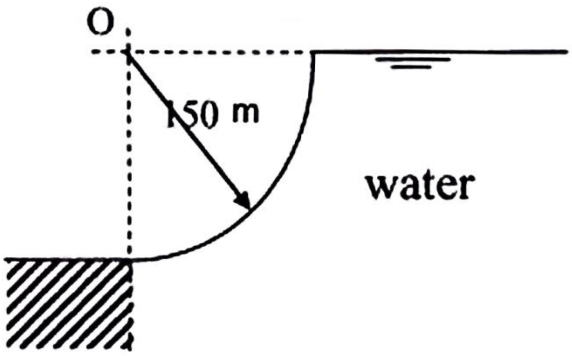
Q14: (A) Find the magnitude and line of action of force on each side of the gate. (B) Find the resultant force due to the liquid on both sides of the gate. (C) Determine F to open the gate if it is uniform and weighs 6000 Ib.



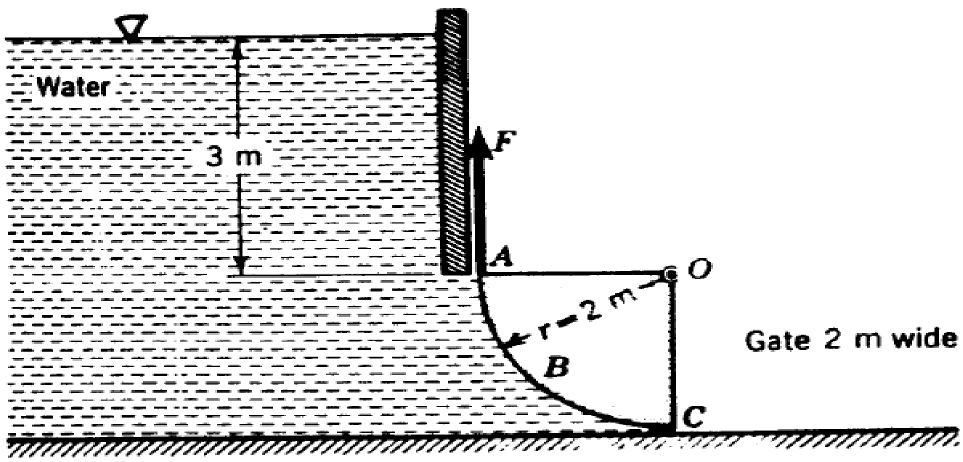
Q15: Find the value of "P" which make the gate in the shown figure just rotate clockwise, the gate is 0.80 m wide.



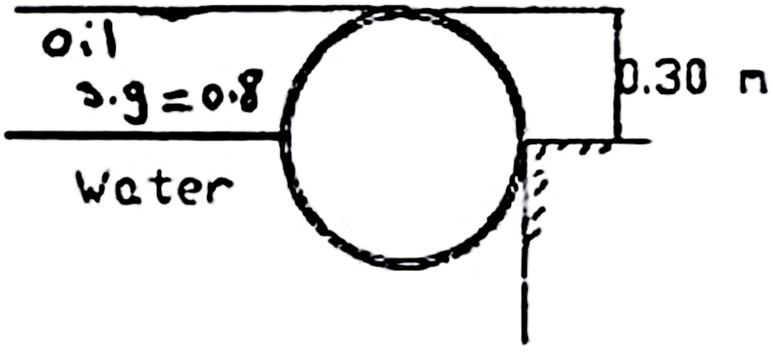
Q16: Determine the value and location of the horizontal and vertical components of the force due to water acting on curved surface per 3 meter length.



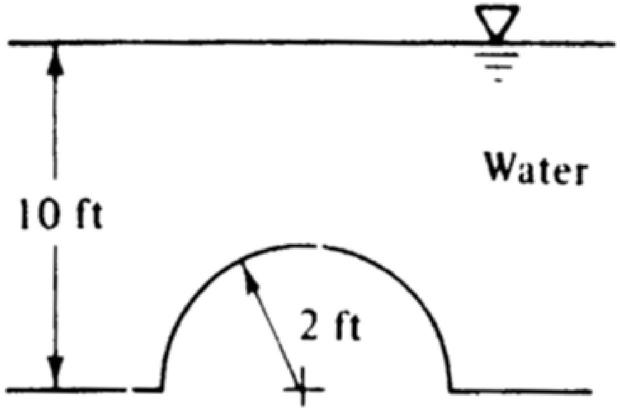
Q17: Determine the horizontal and vertical components of the force acting on radial gate ABC in the shown figure and their lines of action. What F is required to open the gate. Take the weight of the gate W = 2000 kg acting on 1m from O?



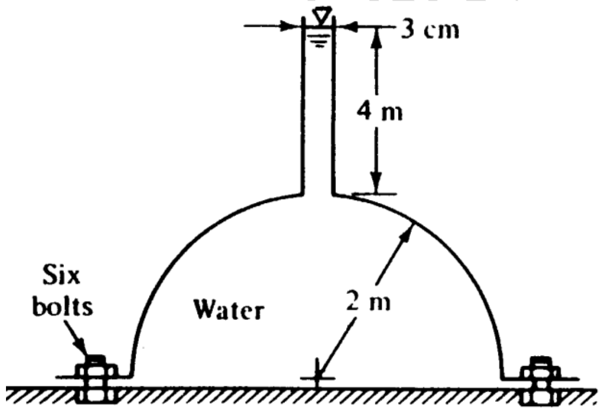
Q18: A cylinder barrier (0.30 m) long and (0.60 m) diameter as shown in figure. Determine the magnitude of horizontal and vertical components of the force due to water pressure exerted against the wall.



Q19: Compute the horizontal and vertical components of the hydrostatic force on the hemispherical dome at the bottom of the shown tank.



Q20: The hemispherical dome in the figure weighs 30 kN, is filled with water, and is attached to the floor by six equally spaced bolts. What is the force on each bolt required to hold the dome down.



ASSIGNMENT 3

FLUID MASSES SUBJECTED TO ACCELERATION

Q1: Calculate the total forces on the sides and bottom of the container shown in Figure 1 while at rest and when being accelerated vertically upward at 3 m/s^2. The container is 2.0 m wide. Repeat your calculations for a downward acceleration of 6 m/s^2.

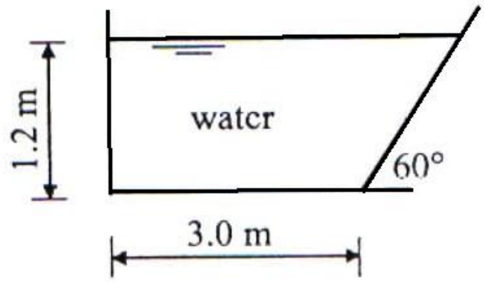


Figure 1

Q2: For the shown container in Figure 2, determine the pressure at points A, B, and C if:

- The container moves **vertically** with a constant linear acceleration of 9.81 m/s^2.
- The container moves **horizontally** with a constant linear acceleration of 9.81 m/s^2.

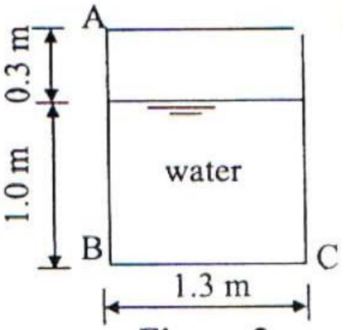


Figure 2

Q3: A tank containing water moves horizontally with a constant linear acceleration of 3.5 m/s^2. The tank is 2.5 m long, 2.5 m high and the depth of water when the tank is at rest is 2.0 m. Calculate:

a) The angle of the water surface to the horizontal.
b) The volume of spilled water when the acceleration is increased by 25%.
c) The force acting on each side if (a_x =12 m/s^2).

Q4: An open cylindrical tank 2.0 m high and 1.0 m diameter contains 1.5 m of water. If the cylinder rotates about its geometric axis, find the constant angular velocity that can be applied when:

a) The water just starts spilling over.
b) The point at the center of the base is just uncovered and the percentage of water left in the tank in this case.

Q5: An open cylindrical tank 1.9 m high and 0.9 m diameter contains 1.45 m of oil (S.G = 0.9). If the cylinder rotates about its geometric axis,

a) What constant angular velocity can be attained without spilling the oil?
b) What are the pressure at the center and corner points of the tank bottom when (ω = 5.0 rad/s).

Q6: An open cylindrical tank 2.0 m high and 1.0 m diameter is full of water. If the cylinder is rotated with an angular velocity of 2.5 rev/s, how much of the bottom of the tank is uncovered?

Q7: A closed cylindrical container, 0.4 m diameter and 0.8 m high, two third of its height is filled with oil (S.G = 0.85). The container is rotated about its vertical axis. Determine the speed of rotation when:

a) The oil just starts touching the lid.
b) The point at the center of the base is just clear of oil.

Q8: A closed cylindrical tank with the air space subjected to a pressure of 14.8 psi. The tank is 1.9 m high and 0.9 m diameter, contains 1.45 m of oil (S.G = 0.9). If the cylinder rotates about its geometric axis,

a) When the angular velocity is 10 rad/s, what are the pressure in bar at the center and corner points of the tank bottom.
b) At what speed must the tank be rotated in order that the center of the bottom has zero depth?

Q9: A closed cylindrical tank 2 ft diameter is completely filled with water. If the tank is rotated at 1200 rpm, what increase in pressure would occur at the top of the tank at that case?

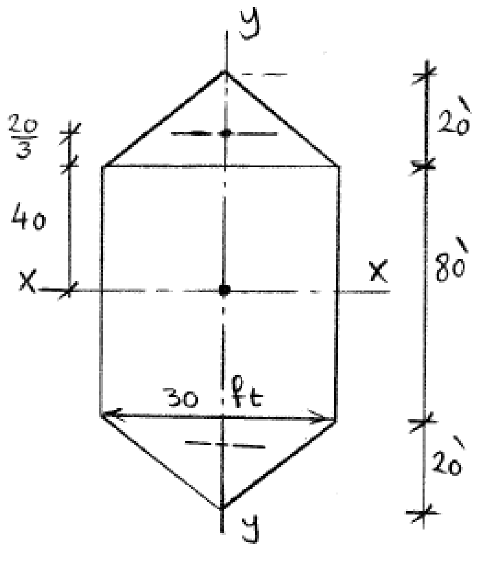
ASSIGNMENT 4

EQUILIBRIUM OF FLOATING BODIES

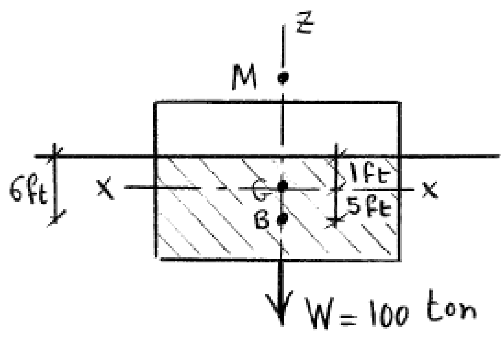
Q1: Will a beam of S.G. = 0.65 and length 1500 mm long with a cross section 136 mm wide and 96 mm height float in stable equilibrium in water with two sides horizontal?

Q2: A floating body 100 m wide and 150 m long has a gross weight of 60,000 ton. Its center of gravity is 0.5 m above the water surface. **Find** the metacentric height and the restoring couple when the body is given a tilt as shown 0.5m.

Q3: A ship displacing 1000 ton has a horizontal cross-section at water-line as shown in the figure, its center of bouyancy is 6 ft below water surface and its center of gravity is 1 ft below the water surface. **Determine** its metacentric height for rolling (about y-axis) and for pitching (about x-axis).



Plan

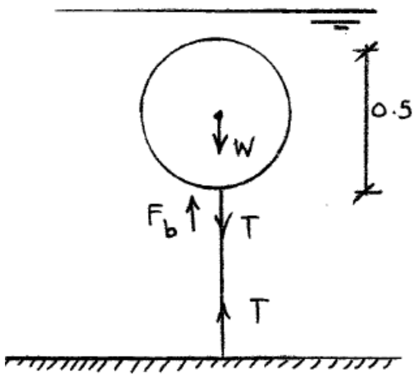


Elevation

Q4: An empty tank rectangular in plan (with all sides closed) is 12.5m long, and its cross section 0.70 m width x 0.60 m height. If the sheet metal weights 363 N/m^2 of the surface, and the tank is allowed to float in fresh water (Specific weight 9.81 KN/m^3) with the 0.60m wedge vertical. **Show**, whether the tank is stable or not?

Q5: A cylindrical buoy 1.8 m diam., 1.2 m high and weighing 10 KN is in sea water of density 1025 kg/m^3. Its center of gravity is 0.45 m from the bottom. If a load of 2 KN is placed on the top; find the maximum height of the C.G. of this load above the bottom if the buoy is to remain in stable equilibrium.

Q6: A spherical Buoy شمندورة (floating ball) has a 0.50 m in diameter, weights 500 N, and is anchored to the seafloor with a cable. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed. **What is the tension on the cable**?



Q7: A wooden cylinder 60 cm in diameter, S.G. = 0.50 has a concrete cylinder 60 cm long of the same diameter, S.G. = 2.50, attached to one end. Determine the length of wooden cylinder for the system to float in stable equilibrium with its axis vertical.

Q8: A right solid cone with apex angle equal to 60° is of density **k** relative to that of the liquid in which it floats with apex downwards. Determine what range of **k** is compatible with stable equilibrium.

Q9: A cylindrical buoy is 5 feet diameter and 6 feet high. It weighs 1500 Ib and its C.G. is 2.5 feet above the base and is on the axis. **Show** that the buoy will not float with its axis vertical in sea water. If one end of a vertical chain is fastened to the centre of the base, **find** the tension in the chain in order that the buoy may just float with its axis vertical.

ASSIGNMENT 4

KINEMATICS OF FLUID FLOW

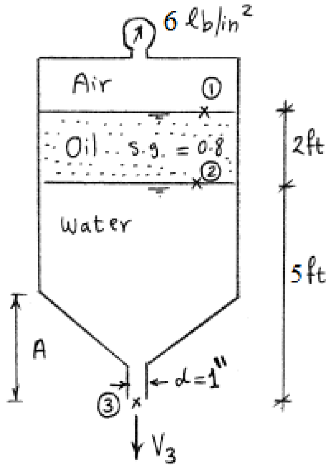
Q1: Determine the type of flow in a 400 mm diameter pipe when

a) Water flows at average velocity of 1.2 m/sec and $\nu = 1.13 \times 10^{-6}$ m^2/sec.

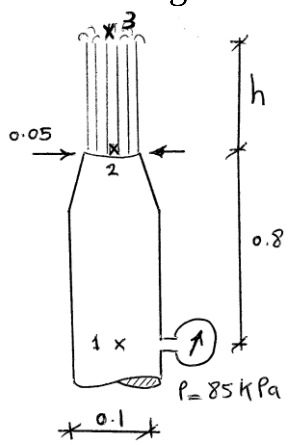
b) Glycerin flows at a rate of 2.0 lit/min having ρ = 1260 kg/m^3 and μ = 0.9 N.s/m^2.

Q2: An inclined pipe carrying water gradually changes from 10 cm at A to 40 cm at B which is 5.00 m vertically above A. If the pressure at A and B are respectively 0.70 kg/cm^2 and 0.5 kg/cm^2 and the discharge is 150 liters/sec. Determine a) the direction of flow b) the head loss between the sections.

Q3: A cylindrical tank contains air, oil, and water as shown. A pressure of 6 lb/in^2 is maintained on the oil surface. What is the velocity of the water leaving the 1.0-inch diameter pipe (neglect the kinetic energy of the fluids in the tank above elevation A).



Q4: For the shown figure calculate the discharge and the value "h".

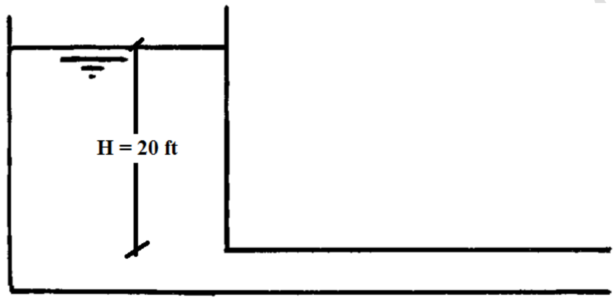


Q5: A tank where the water level is 30.0 m above an arbitrary datum feeds a pipeline AB ending at B with a nozzle 5.0 cm diameter. Pipe AB is 20.0 cm diameter with point A being 23.0 m above datum and point B at datum. Find:

i- The discharge through the pipeline, the pressures and water velocities at A & B.

ii- If friction losses in the nozzle are 0.5 m, and between A & B are 5.0 m, resolve (i) and plot the hydraulic gradient and total energy lines.

Q6: The losses in the shown figure equals $3(V^2/2g)$ft, when H is 20 ft. What is the discharge passing in the pipe? Draw the TEL and the HGL.



Q7: A siphon filled with oil of specific gravity 0.9 discharges 300 lit/s to the atmosphere at an elevation of 4.0 m below oil level. The siphon is 0.2 m in diameter and its invert is 5.0 m above oil level. Find the losses in the siphon in terms of the velocity head. Find the pressure at the invert if two thirds of the losses occur in the first leg.

Q8: A horizontal venturi-meter carries a liquid (S.G.= 0.9) and has an inlet to throat diameters of 18 / 9 cm. If the actual discharge is 45 l/s, and Cd = 0.94, find:

i)the pressure difference between the inlet and the throat, ii) the deflection in the mercury U-tube manometer connecting these points, and iii) the energy lost between the inlet and the throat. Also sketch the H.G.L. and the T.E.L. for the system.

Q9: An orifice-meter having a 0.15/0.075 m pipe to orifice diameters and C_d = 0.65 is used in measuring the flow of water in a pipe. If a U-tube mercury differential manometer gives 0.35 m deflection when connected to the meter, find the rate of flow. What is the maximum possible rate of flow of water if the pressure at inlet to the meter is maintained at 4 KN/m^2 and its vapor pressure is 2 KN/m^2 abs.

Q10: To what height will water rise in tubes A and B? (P = 25 Kpa, Q = 60)