Optimal Capacity and Placement of Battery Energy Storage Systems for Integrating Renewable Energy Sources in Distribution System

Srinivas Bhaskar Karanki *Member, IEEE* ,
School of Electrical Sciences,
IIT Bhubaneswar, India

David Xu *Member, IEEE*
Department of Electrical and Computer Engineering
Ryerson Univeristy, Toronto

***Abstract*—Battery energy storage can bring benefits to multiply stakeholders in the distribution system. The integration of the Battery Energy Storage System (BESS) and renewable energy sources with the existing power system networks has many challenges. One of the major challenges is to determine the capacity and connection location of the BESS in the distribution system. The installation of BESS units at suboptimal places may increase the cost, including system losses and installation of larger battery capacity. So, it is essential to have a method capable of analyzing the influence of BESS allocation and sizing on power distribution system performance. In this paper, a loss sensitivity based algorithm is proposed for optimal placement of the BESS in the distribution system to reduce the distribution system losses. This paper also presents an algorithm for determining the optimal size of the BESS using particle swarm optimization technique. An electrical distribution utility system data in Ontario have been used to show the performance of the proposed algorithm.**

***Index Terms*—Distribution system, battery energy sources, particle swarm optimization, optimal battery capacity.**

I. INTRODUCTION

Variable output of the renewable energy sources presents many integration challenges, especially at high level of penetration. Similarly, the uncertainty in the weather-based generating facilities affects the decisions of different activities related to the operation of distribution systems. Electric utility infrastructure costs are driven primarily by the need to serve the load during the peak demand period. Therefore, it is desirable to shave peak demand in order to defer transmission and distribution equipment upgrades, and reduce or avoid the necessity to purchase much higher cost generation assets. An effective way to achieve reduction in peak load is the application of Battery Energy Storage Systems (BESS). The integration of battery energy storage in a distribution grid could mitigate some of the problems of a high penetration of the distributed generation. In addition to the load leveling, BESS can be widely used for frequency control, voltage regulation, improvement of the power quality in the distribution system. All these applications serve to increase the reliability and stability of the grid [1].

The suboptimal BESS location and sizing can cause under or over-voltages in the distribution network. BESS also causes an impact in the losses due to its proximity to the load centers. Therefore, it is necessary to consider an appropriate location in the residential distribution system to install BESS to obtain an optimal effect. The optimal sizing and placement of the BESS in the distribution power system is an important aspect to maximize the benefits of the BESS in the system. The BESS should be located at a bus, where it provides a higher reduction in losses without any violation to the voltage profile. Various algorithms for optimal sizing and placements of the BESS are proposed in the literature [2]. Although most of the investigators state that increasing BESS capacity improves its capabilities and hence power system performance, no suggestion has been made clearly regarding choosing the optimal placement of the BESS in the power distribution network [3].

A method supporting non-radial distribution system for voltage regulation has been proposed to determine the placement of the energy storage units in [4]. In [5], a method based on classification of the distribution substation's main transformer (MTr) is proposed to determine the optimal placement of the BESS in the distribution system. In [6], a method is proposed to reduce the feeder losses by optimally placing the BESS in the network. The major drawback of these methods are, they may not be suitable for complex distribution system.

In this paper, a loss sensitivity index based method is proposed for determining the optimal placement of the BESS in the power distribution system. Even though the BESS is placed at the optimal location, the size of the BESS has a considerable impact on the power distribution system performance. If inappropriate size of the BESS is installed at the optimal location, the loss in the system may decrease but the voltage or power flow violations may increase in the system. Thus in this paper, once the optimal location is determined, the optimal sizing of the BESS to be installed is obtained by optimizing the losses in the system using Particle Swarm Optimization (PSO) technique.

Typically BESS is governed by five important parameters, energy capacity (MWh), power capacity (MW), round trip efficiency (η), discharge rate and state of charge (SOC). State of Charge (SOC) has been considered in sizing and power management of the energy storage in [7]. Similarly various algorithms are proposed in the literature for optimal sizing of the BESS. But most of the proposed algorithms have not considered all the parameters of the BESS while designing

978-1-4799-5141-3/14/$31.00 © 2016 IEEE

the algorithm. In this paper, a heuristic based Particle Swarm Optimization (PSO) technique is proposed to determine the optimal sizing of the BESS by considering all the governing parameters in the objective function of BESS in the distribution system.

II. Optimal Placement of the BESS

An optimal location for BESS has to be identified in the system such that the distribution system losses are minimized. In this work, system loss sensitivity index with respect to the BESS control parameters has been used to optimally place the BESS. In case of optimal BESS placement, the change in the power system performance and the BESS parameter are considered for evaluating the sensitivity index. Total power loss is considered as the parameter for the system performance and the real power injection is the BESS parameter. The power loss in a distribution system is given in (1) [8].

$$P_{Loss} = \sum_{\substack{j=1\\k=1}}^{n} [\alpha_{jk}(P_j P_k + Q_i Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)] \quad (1)$$

where

$$\alpha_{jk} \triangleq \frac{r_{jk}}{|V_j||V_k|} cos(\delta_j - \delta_k) \quad \beta_{jk} \triangleq \frac{r_{jk}}{|V_j||V_k|} sin(\delta_j - \delta_k)$$

The loss sensitivity index with respect to the BESS control parameter is given in (2).

$$\frac{\partial P_{Loss}}{\partial P_B} = 2\sum_{k=1}^{n}(P_k \alpha_{ik} - Q_k \beta_{ik}) \quad (2)$$

where, α_{jk}, β_{jk} are the loss co-efficients. P_j, P_k are the real power injections at the buses j and k. Q_j, Q_k are the reactive power injections at the buses j and k. V_j, V_k are the voltage magnitudes at the buses j and k. r_{jk} is the resistance of the transmission line connected between bus i and j. δ_i, δ_j are the voltage phase angles at the buses i and j respectively.

Since the loss sensitivity on each bus is calculated, this method is suitable for distributed BESS allocation in the power system network, when compared to other methods available in the literature. This method allows the utility to determine various bus locations based on the sensitivity instead of single centralized BESS in the distribution system.

III. Optimal Rating Capacity of the BESS

The BESS is governed by following parameters, energy capacity (MWh), power capacity (MW), round trip efficiency (η), state of charge (SOC) and discharge rate (DR). The round trip efficiency depends on the chemical reaction inside the battery storage and a fixed value is assumed for convenience. The algorithm should determine an optimal energy capacity and the power capacity of the BESS with an inequality on SOC and the discharge rate. Figure 1 shows the block diagram of the proposed methodology. P_L, P_W and P_G are the real time data of the load, distributed power generated and the power available from the grid at the given substation in the distribution system. Typically, the power data sample time is for one hour.

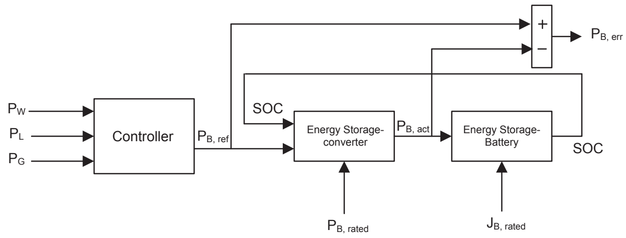


Fig. 1. Block diagram for proposed methodology.

The Controller block determines the desired energy storage power output at the current simulation instant, $P_{B,ref}$, based on the distributed power generation (P_W), Load power (P_L) and grid power (P_G). The energy storage power rating ($P_{B,rated}$) is optimized as following.

$$P_{B,act} = \begin{cases} 0, & SOC = 0 \;\&\; P_{B,ref} > 0 \\ 0, & SOC = 1 \;\&\; P_{B,ref} < 0 \\ P_{B,rated}, & SOC \neq 0 \;\&\; P_{B,ref} > P_{B,rated} \\ -P_{B,rated}, & SOC \neq 1 \;\&\; P_{B,ref} < -P_{B,rated} \\ P_{B,ref} & Else \end{cases} \quad (3)$$

The discharge from the energy source corresponds to the positive $P_{B,ref}$ and the charging of the energy source to negative $P_{B,ref}$. The SOC of the battery pack is updated as a function of the power into or out of the battery. The rated energy capacity J_B, rated is optimized as following.

$$SOC(T) = SOC(T-1) - \frac{\eta.T.P_{Bact}(T-1)}{60.J_{B,rated}} \quad (4)$$

The term T refers to the sample time at which the real time data has been obtained. The round efficiency is defined as following.

$$\eta = \begin{cases} \eta_{out}, & P_{Bact,-1} > 0 \\ \eta_{in}, & P_{Bact,-1} < 0 \end{cases} \quad (5)$$

The terms η_{out} and η_{in} are the efficiency during discharge and charge respectively. The energy storage unit will not source power if the SOC is equal to 0, or sink power if the SOC is equal to 1. The expression for the instantaneous state of charge is given below.

$$SOC(t) = Q_T - \int_{t_o}^{t} i(\tau)d\tau \quad (6)$$

Where Q_T is the state of charge at the initial time t_o. The Discharge rate (DR) of the battery is defined as the current at which a battery is charged or discharged. The rate is expressed in terms as C/h rate, where C is the rated capacity and h is the discharge/ charging time in hours. The instantaneous discharge rate at given time sample can be expressed in terms of SOC as given below.

$$DR(t) = [SOC(t+1) - SOC(t)]/\Delta t \quad (7)$$

According to Peukret's law, as the rate of discharge/ charging increases, the available capacity of the battery decreases. So in order to increase the life span of the battery the discharge/charging rate should be within specified limit. This is considered as the second inequality constraint in determining an optimal energy capacity and the power capacity of the BESS.

At every simulation step, $P_{B,err}$ the error between $P_{B,ref}$ and $P_{B,act}$ is evaluated and verified whether it is within the specified tolerance limit . T_{frac} is the fraction of the samples in the simulation duration in which the error is within the tolerance limit. A reasonable cost model for flow cell battery energy storage device is defined as following [9].

$$Cost = f(T_{frac}).(C_p.P_{B,rated} + C_j.J_{B,rated}) \tag{8}$$

Where C_p and C_J are the cost per Watt and cost per Wh of the energy storage device. The term is added to penalize energy storage systems in the solution time that do not meet the error tolerance band and defined as following

$$f(T_{frac}) = \begin{cases} 1, & T_{frac} \geq 0.85 \\ \inf, & T_{frac} < 0.85 \end{cases} \tag{9}$$

The objective is to minimize the cost of the battery energy storage system and satisfying the inequality constraints on the SOC and DR by finding an optimal value of $P_{B,rated}$ and $J_{B,rated}$. Particle swarm optimization (PSO) is a heuristic based optimization technique which has been employed to power system for its simplicity in mathematical formulation.

Once the optimal size and location of the BESS is determined, the power delivered/observed by the BESS at given operating condition should be determined by optimizing the loss in the system. To determine this, an objective function consisting of loss minimization is formulated. Voltage limit violation at buses, line thermal limit violation are taken as the operational constraints and the BESS injected power at the most sensitive bus is the control variable as given in (10).

$$Fun = min\{\sum_{\substack{j=1 \\ k=1}}^{n} [\alpha_{jk}(P_j P_k + Q_i Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)]\} \tag{10}$$

Such that the following constraints are satisfied.

Equality Constraint

$$\Delta P_i^p = P_i^p - |V_i^p| \sum_{k=1}^{n} \sum_{m=a}^{c} |V_k^m| [G_{ik}^{pm} \cos \delta_{ik}^{pm} + B_{ik}^{pm} \sin \delta_{pm}^{ik}]$$

$$\Delta Q_i^p = Q_i^p - |V_i^p| \sum_{k=1}^{n} \sum_{m=a}^{c} |V_k^m| [G_{ik}^{pm} \sin \delta_{ik}^{pm} - B_{ik}^{pm} \cos \delta_{pm}^{ik}]$$

where i, k = 1,2,3...n buses. p, m = a,b,c phases. G and B represent the conductance and susceptance.

Inequality constraint

$$V_i^{pmin} \leq V_i^p \leq V_i^{pmax} \tag{11}$$

Particle swarm optimization technique is employed in this work for solving the objective function. The PSO is best suitable for optimizing non-smooth and non-linear functions as compared to classical methods like gradient search. It also has the advantage of not being stuck to a local minimum. A detailed step by step procedure of the PSO technique is discussed in [10]

IV. Simulation Results

Electrical Distribution Utility Substation (X)

The proposed algorithm for the optimal placement of the BESS in the distribution system is tested on real time data from an electrical utility in Ontario. Substation (X) is considered as the test system. The CYME model of the substation has been provided by the utility. A total of 99 spot loads are connected to the substation through 161 overhead balanced lines and 92 cables. The substation has 278 nodes and 6 buses. The system also contains 17 shunt capacitors, one regulator, 12 two winding transformers. Two wind farms of 9.9 MW rating of each are connected to the system. Each wind farm has five 1.98 MW rating generators. The wind farms are connected to the substation X approximately through 20 miles transmission line and the load is concentrated around 15 miles away from the wind farms.

Using the basic network equations, the CYME model has been reduced to fewer nodes and buses. The reduced CYME model contains 63 nodes and 45 spot loads connected to the substation through 62 overhead transmission lines and cables. The reduced model parameters are used for Matlab simulation study to find the loss sensitivity of each bus.

Newton Raphson Iterative technique has been used to solve the power flow in the three phase unbalanced distribution system in Matlab. The loss sensitivity index for each bus in the system is determined using equations (1) and (2). The bus voltages, voltage phase angles and the power flows are obtained by solving the load flow analysis. The overall system losses under various operating conditions are given in Table I. From the table it is clear that the maximum losses in the system occur when the load is minimum (3.01 MW) and the wind generation is maximum (19.8 MW). Similarly the number of phases which are violating the voltage limits are maximum when the load and wind generation are minimum.

TABLE I
Load Flow Analysis Under Various Operating Conditions

Load in MW	Wind Generation in MW	Losses in kW	Voltage Variation	
			< 0.94	> 1.06
8.84	0	341.96	0	106
3.01	0	56.09	0	162
8.84	19.80	1624.06	0	66
3.01	19.80	1896.54	0	105

The operating condition in which the losses are highest is considered for calculating the loss sensitivity index in this work. Figure 2 shows the bus number versus total real power loss sensitivity index value. It can be observed that 41st Bus (Bus ID 753 in CYME model) is the most negative sensitive bus and it is the best location to place BESS to reduce the losses in the system. In this particular test system, the most negative sensitive bus coincides with the most negative sensitive bus when individual phase sensitivity is considered.

Whereas, for the next best position the most negative sensitive bus does not coincides with the most negative sensitive bus when individual phase sensitivity is considered. In this case bus 29 in a-phase and b phase, bus 54 in c phase are the best location after the 41st bus for the placement of the BESS. Bus ID 753 in CYME model is the most negative sensitive bus, for all the above mentioned conditions, but the next best option varies with the operating condition.

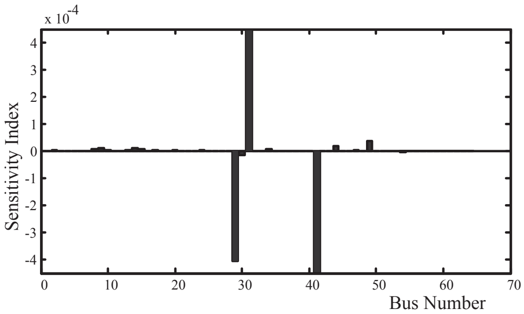


Fig. 2. Bus number versus sensitivity index plot

The optimal rating of the battery can be obtained using the load and generation profile for specific period of time. The performance of the proposed algorithm in the Section III is evaluated by running the Matlab simulation of the combined system shown in Fig. 1 over a week of wind power data. The energy storage power rating ($P_{B,rated}$) and energy storage energy capacity ($J_{B,rated}$) are optimized using the algorithm given in section III. The load profile and wind power generation profile on substation (X) from March 1st to March 7th 2013 are shown in Fig. 3. The peak load on the substation is 8.91 MW and the peak wind power generation is 19.8 MW. Similarly the substation has lowest load of 3.01 MW and lowest generation of 0.20 MW. From the load and generation profile it is clear that high amount of power has been exchanged from the grid, which will eventually increase the losses in the system.

Simulation parameters for determining the optimal sizing of the BESS are given in Table II. The SOC of the BESS at the start of the simulation is considered as 50 % in this study. The two optimizing variables are $P_{B,rated}$ and $J_{B,rated}$, when the substation is run on stand alone mode i.e., without any exchange of power from the grid. Keeping the error $P_{B,ref}$ within the given tolerance band of $\pm 4\%$, the optimization algorithm determined the $P_{B,rated}$ and $J_{B,rated}$ as 11.52 MW and 198 MWh respectively. Out of 168 sample duration 145 times (86%) the error is within the specified limit. Since the energy rating is higher than the power rating, the discharge rate is always less than 1C. SOC is maintained within the specified limits and 23 times out 168 samples it is reaching the depth of the discharge (DOD).

In case of the stand alone system operation, the energy rating of the battery is very high and it is not economical. If you consider power exchange from the grid, the energy rating can be decreased. In this work, two methods are discussed to find the optimized rating of the battery when power exchange is allowed from the grid. The first method is to fix the power

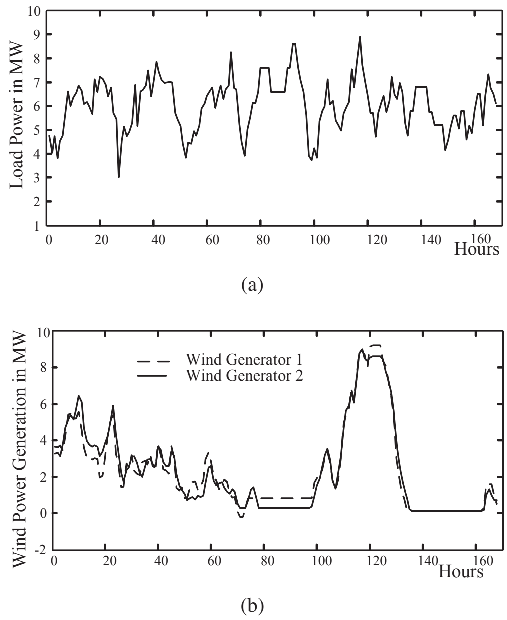


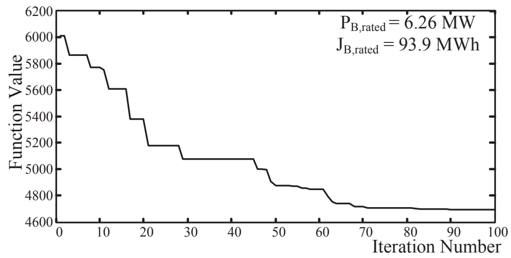
Fig. 3. a) Load Demand on Malden M7 Substation for 7 days (168 hours) b) Wind Power Generation.

TABLE II
SIMULATION PARAMETERS FOR OPTIMAL SIZING

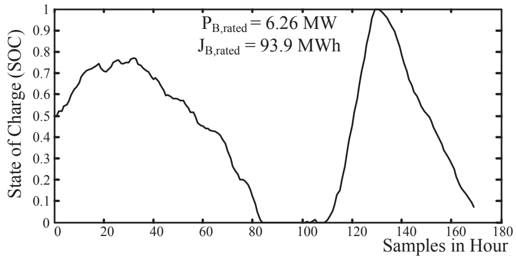
Variable	Value
Discharge efficiency (η_{out})	95 %
Charging efficiency (η_{in})	90 %
Error Limit	± 4
C_p	0.20 $\$/W$
C_j	0.48 $\$/Wh$
Number of Particles	50
Number of Iterations	100
Sampling time	1 Hour
Data duration	7 Days

to be exchanged between the grid and wind generation and second method is to take this power to be exchanged as a optimization variable. When the exchanged power is fixed to 4 MW, the optimized rating of the battery is obtained as 7.41 MW and 127.9 MWh power and energy rating respectively. The energy exchange to the grid is 348 MWh for one week duration. When the same is taken as optimization variable, the objective function has been modified. The modified objective function will minimize the energy exchange with the grid along with the cost minimization of the BESS. The optimized power to be exchanged to the grid is 2.74 MW, with a power and energy rating of 6.26 MW and 93.89 MWh respectively. The energy exchange to the grid is 306.85 MWh for one week duration. The convergence curve and the variation of State of Charge is shown in Fig. 4.

Table III gives the values of the $P_{B,rated}$ and $J_{B,rated}$ for various fixed power exchanged limits and the optimized solution. It can be observed from the table that the PSO based optimized solution simultaneously reduced the energy exchange with the grid and the rating of the BESS . The



(a)



(b)

Fig. 4. Results for Malden substation a) PSO Convergence curve b) Variation of SOC .

Variation of the SOC for various operating conditions is shown in Fig. 5. The exchange of power is reduced when the power exchange is considered as an optimization variable. This also reduce the losses that occur during the transmission and the problem of overloading also reduces.

TABLE III
COMPARISON OF OPTIMIZATION RESULTS

Power Exchange with Grid (MW)	$P_{B,rated}$ (MW)	$J_{B,rated}$ (MWh)	Energy Exchange with Grid (MWh)	T
① 2.0	6.69	105.52	248	143
② 3.0	7.41	104.90	309	143
③ 4.0	6.41	127.70	348	144
④ 5.0	6.59	132.44	290	149
⑤ 2.74	6.26	93.88	306.85	143

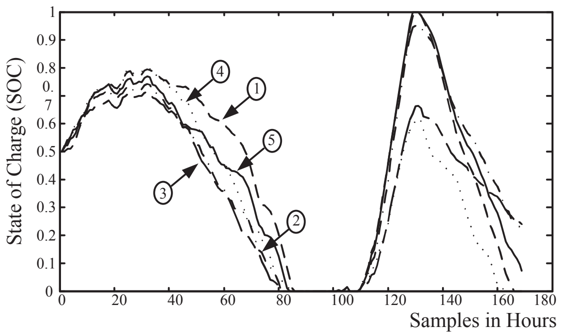


Fig. 5. State of charge under varying operating conditions.

Once the optimal location and sizing of the BESS is determined, the optimal power to be injected/observed by the BESS at a particular operating condition is obtained by minimization of the losses in the system. PSO algorithm is used for solving the optimization problem. For this, two optimization criteria are considered. The first one is loss optimization and equation (10) is minimized without considering any constraints. The second one is loss optimization along with minimizing the number of phases violating the voltage profile (i.e. considering the constraints). Table IV gives the optimization results with 4 different cases. When the wind generation is at its peak, the BESS will be in the charging mode (i.e., it acts as load). It can be observed that, when BESS is not installed, case 4 gives the maximum power losses. But when the BESS is installed at an optimal location, the losses have been reduced by 10.33 % when compared to losses without BESS (under light load conditions), even though the BESS is acting as an additional load.

The variation of the total system losses with respect to the loading on the system is shown in Fig. 6. It can be observed from the figure that, at 0.7 per unit (pu) loading the losses with and without BESS are same. The BESS injects power, when the loading on the system is above 0.7 pu and it absorbs power when the loading is below 0.7 pu. When optimal power is injected into or absorbed from the system, the losses in the system would be reduced with the installation of the BESS at all the operating points.

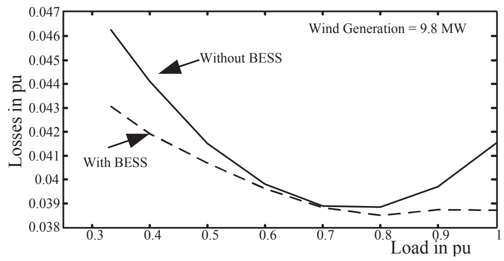


Fig. 6. The variation of total losses w.r.t to load changes.

Table V gives the optimization results using CYME model. Bus number 753 is chosen as the optimal location for the BESS installation based on the loss sensitivity index. The optimal power injected/absorbed at various operating conditions obtained from Matlab simulations are used in CYME model. The losses in the system have reduced with the BESS installation. Even in Case 3 and Case 4, where BESS is in charging mode (acts as extra load on the system) the losses have decreased and the loading on the system is also reduced.

V. CONCLUSIONS

An algorithm based on loss sensitivity index is proposed for optimal placement of the BESS in the unbalanced distributed system. The proposed algorithm has been verified using the data from the Ontario electrical distribution utility system. PSO based controller is proposed to determine the optimal rating of the BESS to be installed in the given substation. The optimal power to be injected/absorbed by the BESS at various operating conditions is determined by loss minimization criteria and voltage profile variations. It is possible to conclude that optimally-allocated and sized BESS will potentially reduce the losses in the system even when the BESS is in charging

TABLE IV
OPTIMAL BATTERY POWER DELIVERED/OBSERVED UNDER VARIOUS OPERATING CONDITION

Operating Condition	Optimization criteria	Optimal Battery Power observed/delivered in kW			Losses (kW)	Voltage variations	
		a	b	c		< 0.94	> 1.06
Case 1 Wind Generation = 0 MW Load Power = 8.84 MW	Basic	No BESS			341.96	0	106
	Loss Minimization	1552.7	1603.5	141.74	108.02	0	150
	Loss +Voltage Profile	226.7	1802	76.90	216.60	0	106
Case 2 Wind Generation = 0 MW Load Power =3.01 MW	Basic	No BESS			56.02	0	162
	Loss Minimization	418	548	507	30.49	0	162
	Loss +Voltage Profile	480	1000	34.7	39.69	0	140
Case 3 Wind Generation =19.8 MW Load Power =8.84 MW	Basic	No BESS			1624.06	0	66
	Loss Minimization	-468	-216	-131	1612.54	0	55
	Loss +Voltage Profile	-1930	-1733	-243	1771.58	0	0
Case 4 Wind Generation =19.8 MW Load Power =3.01 MW	Basic	No BESS			1896.54	0	105
	Loss Minimization	-1414.8	-1271.8	-1085.3	1700.37	0	104
	Loss +Voltage Profile	-1953.7	-766.2	-1828.7	1736.67	0	79

TABLE V
OPTIMIZATION RESULTS USING CYME MODEL

Operating Condition	BESS Status	Grid Power (MW)	Optimal Battery Power observed/delivered (kW)			Losses (kW)	Over loading		
			a	b	c		a	b	c
Case 1 Wind Generation = 0 MW Load Power =8.84 MW	Without BESS	9.19	0	0	0	316	1	1	1
	With BESS	4.37	1552.7	1603.5	141.74	130	0	0	0
Case 2 Wind Generation = 0 MW Load Power =3.09 MW	Without BESS	3.13	0	0	0	42.57	0	0	0
	With BESS	1.64	418	548	507	22.16	0	0	0
Case 3 Wind Generation =19.8 MW Load Power =8.84 MW	Without BESS	-9.59	0	0	0	1336.6	11	11	11
	With BESS	-8.8	-468	-216	-131	1280.6	11	11	11
Case 4 Wind Generation = 19.8 MW Load Power =3.09 MW	Without BESS	-15.23	0	0	0	1479.58	14	14	14
	With BESS	-11.45	-1448.8	-1271.8	-1085.3	1319.48	11	11	11

mode. The losses in system has been reduced by 10.35% when compared to losses without BESS. Similarly the number of buses violating the voltage profile limits also reduced by 24.76 % in the system.

REFERENCES

[1] R. Dell and D. Rand, "Energy storage a key technology for global energy sustainability," *Journal of Power Sources*, vol. 100, no. 1, pp. 2–17, 2001.

[2] S. Dutta and R. Sharma, "Optimal storage sizing for integrating wind and load forecast uncertainties," in *Innovative Smart Grid Technologies (ISGT),*. IEEE PES, 2012, pp. 1–7.

[3] K. Divya and J. Østergaard, "Battery energy storage technology for power systems an overview," *Electric Power Systems Research*, vol. 79, no. 4, pp. 511–520, 2009.

[4] A. Barnes, J. Balda, A. Escobar-Mejia, and S. Geurin, "Placement of energy storage coordinated with smart pv inverters," in *Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES*, jan. 2012, pp. 1 –7.

[5] K. Jung, H. Kim, and D. Rho, "Determination of the installation site and optimal capacity of the battery energy storage system for load leveling," *IEEE Transactions on Energy Conversion,*, vol. 11, no. 1, pp. 162–167, 1996.

[6] F. Alvarado, "Locational aspects of distributed generation," in *Power Engineering Society Winter Meeting,*, vol. 1. IEEE, 2001, pp. 140.

[7] C. Abbey and G. Joos, "Sizing and power management strategies for battery storage integration into wind-diesel systems," in *34th Annual Conference of IEEE Industrial Electronics, IECON,*. IEEE, 2008, pp. 3376–3381.

[8] O. Elgerd, *Electric energy systems theory: an introduction*. Tata McGraw-Hill Education, 2007.

[9] T. Brekken, A. Yokochi, A. von Jouanne, Z. Yen, H. Hapke, and D. Halamay, "Optimal energy storage sizing and control for wind power applications," *IEEE Transactions on Sustainable Energy,*, vol. 2, no. 1, pp. 69–77, 2011.

[10] S. B. Karanki, D. Xu, B. Venkatesh, and B. N. Singh, "Optimal location of battery energy storage systems in power distribution network for integrating renewable energy sources," in *2013 IEEE Energy Conversion Congress and Exposition*, Sept 2013, pp. 4553–4558.