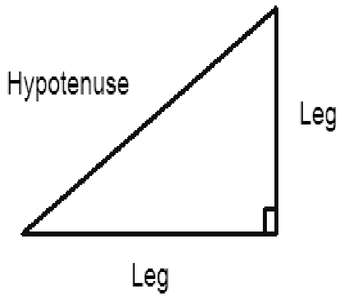
1 Math 116 Supplemental Textbook (Pythagorean Theorem)

1.1 Pythagorean Theorem

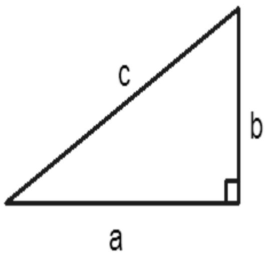
1.1.1 Right Triangles

Before we begin to study the Pythagorean Theorem, let's discuss some facts about right triangles. The longest side of a right triangle which is opposite the right angle is called the hypotenuse. The other two sides that form the right angle are called the legs.



In every right triangle, there is the same relationship between the legs of the right triangle and the hypotenuse of the right triangle. This relationship is know as the the Pythagorean Theorem.

In a right triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse. $c^2 = a^2 + b^2$



1.1.2 Find the missing side of a right triangle

Example 1
Find the length of the missing side where a, b, and c are the sides of a right triangle: a = 9, b = 12, c = ?,

solution
Simple substitute the value of a and b into the Pythagorean Theorem and solve for c.
$c^2 = a^2 + b^2$
$c^2 = 9^2 + 12^2$
$c^2 = 81 + 144$
$c^2 = 225$
$\sqrt{c^2} = \sqrt{225}$
$c = 15$

Example 2
Suppose the two legs of a right triangle are 5 units and 12 units, find the length of the hypotenuse.

Solution To find the solution, substitute the value of the legs into the Pythagorean theorem

and solve for the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c^2 = 169$
$\sqrt{c^2} = \sqrt{169}$
$c = 13$

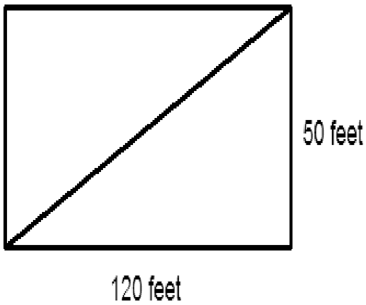
Example 3
Suppose that the hypotenuse of a right triangle is 26 units and one leg is 10 units, find the measure of the other leg.

Solution
To find the answer, substitute the value of the leg and hypotenuse into the Pythagorean theorem and solve for the missing leg.
$c^2 = a^2 + b^2$
$26^2 = 10^2 + b^2$
$676 = 100 + b^2$
$b^2 = 576$
$\sqrt{b^2} = \sqrt{576}$
$c = 24$

1.1.3 Applications of the Pythagorean Theorem

The Pythagorean theorem has several real life applications. This is due to the fact that so many problems can be modeled or represented by a right triangle. If this is the case, then values can be assigned to the sides of the triangle and the unknown value can be found by solving for the missing side of the triangle. Here are some examples of applications of right triangles and the Pythagorean theorem

Example 4
An empty lot is 120 ft by 50 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?



Solution
To find the answer, substitute the value of the legs into the Pythagorean theorem and solve for the hypotenuse. This will give you the distance for walking diagonally across the lot.

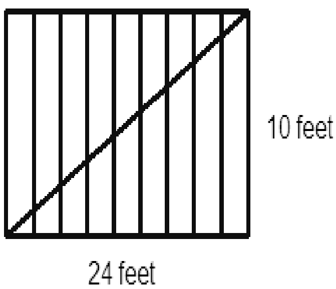
$c^2 = a^2 + b^2$
$c^2 = 50^2 + 120^2$
$c^2 = 2500 + 14400$
$c^2 = 16900$
$\sqrt{c^2} = \sqrt{16900}$
$c = 130$

Now, find the distance walking length and width. Total distance $=120ft + 50ft = 170ft$

Next, you can find the distance you saved by walking diagonally by subtracting the two distances. Distance saved $= 170ft - 130ft = 40ft$

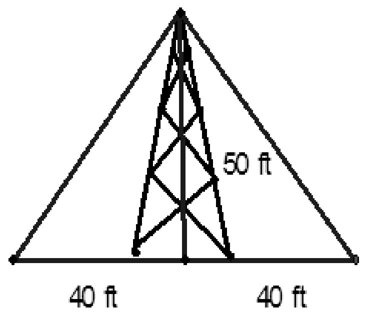
Often construction workers and carpenters use angles and triangle in their profession. For example, carpenters can use the Pythagorean theorem to find measurements when constructing the walls of a house. In this case, we will find the length of a brace that is put inside the wall of the house to reinforce the wall.

Example 5
A diagonal brace is to be placed in the wall of a room. The height of the wall is 10 feet and the wall is 24 feet long. (See diagram below) What is the length of the brace?



Solution
To find the answer, substitute the value of the legs into the Pythagorean Theorem and solve for the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 10^2 + 24^2$
$c^2 = 100 + 576$
$c^2 = 676$
$\sqrt{c^2} = \sqrt{676}$
$c = 26 feet$

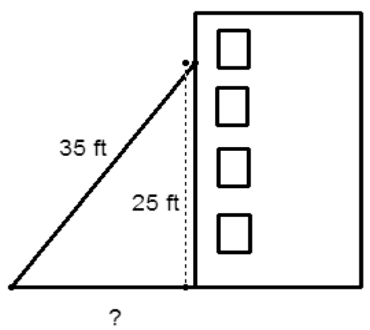
Example 6
A television antenna is to be erected and held by guy wires. If the guy wires are 40 ft from the base of the antenna and the antenna is 50 ft high, what is the length of each guy wire?



Solution

Substitute into the Pythagorean theorem using the values 40 feet and 50 feet in for the legs of the right triangle, and solve for the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 40^2 + 50^2$
$c^2 = 1600 + 2500$
$c^2 = 4100$
$\sqrt{c^2} = \sqrt{4100}$
$c = 64 feet$

Example 7
Given that a 35 foot ladder rest against a window ledge that is 25 feet above the ground, find out how far is the ladder from the edge the building?



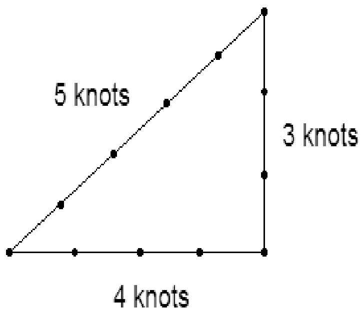
Solution

Use the Pythagorean theorem to find the distance the ladder is from the building by using

the length of the ladder as the hypotenuse and height of the building as the legs of the triangle. After substituting into the Pythagorean theorem, solve for the other leg.
$c^2 = a^2 + b^2$
$35^2 = 25^2 + b^2$
$1225 = 625 + b^2$
$b^2 = 600$
$\sqrt{b^2} = \sqrt{600}$
$b = 24.5 feet$

1.1.4 Historical Excursion: The Pythagorean Theorem

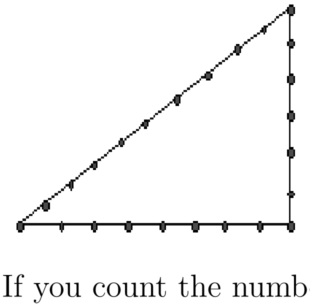
The origins of right triangle geometry can be traced back to 3000 BC in ancient Egypt. The Egyptians used special right triangles to survey land by measuring out 3-4-5 right triangles to make right angles. They mostly understood right triangles in terms of ratios or what would now be referred to as Pythagorean triples. The Egyptians also had not developed a formula for the relationship between the sides of a right triangle. At this time in history, it is important to know that the Egyptians also had not developed the concept of a variable or equation. They studied specific examples of right triangles. For example, they used ropes to measure out distances to form right triangles that were in whole number ratios. In the next illustration, it is demonstrated how a 3-4-5 right triangle can be formed using ropes to create a right angle.



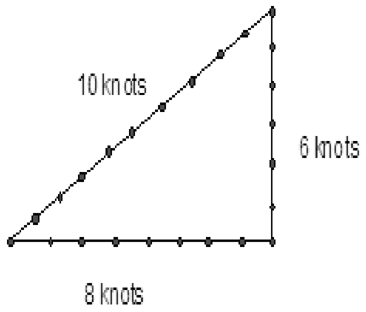
Using ropes that had knots that where equally spaced, the Egyptians could measure out right angles by making a 3-4-5 right angle or other right triangles with the rope.
It wasn't until around 500 BC, when a Greek mathematician name Pythagoras discovered that there was relationship between the sides of a right triangle. This relationship or formula is now known as the Pythagorean Theorem.

Example 8

Determine if the triangle measured out by ropes has a right angle.



If you count the number of knots on each side of the triangle you get a ratio of 6-8-10.



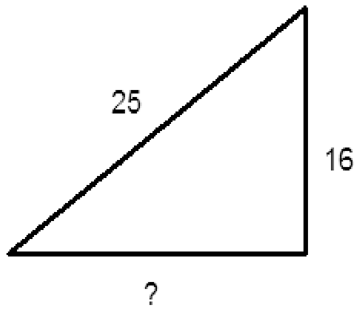
Substituting these values into the Pythagorean Theorem using 10 as the hypotenuse and the other two sides as the legs, you can determine if the triangle is a right triangle.

$c^2 = a^2 + b^2$
$10^2 = 6^2 + 8^2$
$100 = 36 + 64$
$100 = 100$

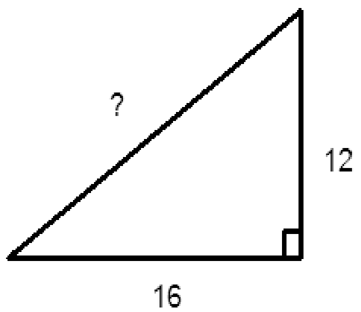
After substituting these values into the Pythagorean Theorem, the final values obtained on both sides of the equation are equal. This verifies that the triangle is a right triangle and therefore, has a right angle.

1.1.5 Exercises

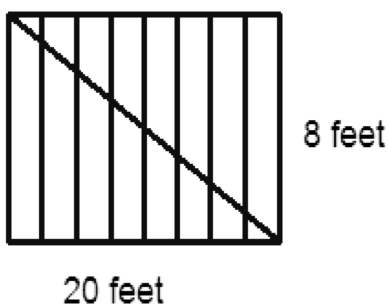
1. Find the length of the missing side: a = 6, b = 8, and c = ?
2. Find the length of the missing side. a = 12, c = 20, b = ?
3. Find the length of the missing leg in the diagram below: (Round answer to the nearest tenth)



4. Find the length of the missing side.

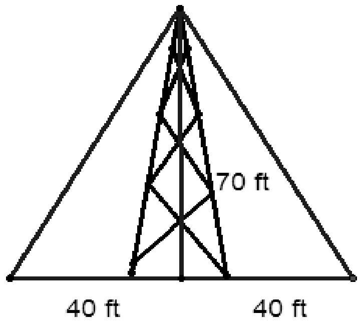


5. A rectangular shaped lot is 80 ft by 60 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?
6. A diagonal brace is to be placed in the wall of a room. The height of the wall is 8 feet and the wall is 20 feet long. (See diagram below) What is the length of the brace?

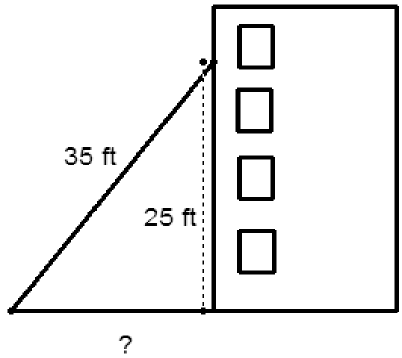


7. Find the length of the diagonal of a rectangle that is 30 ft by 40 ft.
8. Find the length of the diagonal of a rectangle that is 30 ft by 30 ft.

9. A television antenna is to be erected and held by guy wires. If the guy wires are 40 ft from the base of the antenna and the antenna is 70 ft high, what is the length of the guy wire?



10. Given that 35 foot ladder rest against a window ledge that is 25 feet above the ground, find out how far is the ladder form the edge the building?



Solutions

1. c = 10
2. b = 16
3. 19.2
4. 20
5. Distance walking diagonally = 100 ft: Distance saved walking diagonally = 40 ft
6. 21.5 ft
7. 50 ft
8. 42.4 ft
9. 80.6 ft
10. 24.5 ft