Albrecht Dold

Lectures on Algebraic Topology

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Albrecht Dold
Mathematisches Institut der Universität Heidelberg
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69120 Heidelberg
Germany

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To Z.

Foreword

This is essentially a book on singular homology and cohomology with special emphasis on products and manifolds. It does not treat homotopy theory except for some basic notions, some examples, and some applications of (co-)homology to homotopy. Nor does it deal with general(-ised) homology, but many formulations and arguments on singular homology are so chosen that they also apply to general homology. Because of these absences I have also omitted spectral sequences, their main applications in topology being to homotopy and general (co-)homology theory. Čech-cohomology is treated in a simple ad hoc fashion for locally compact subsets of manifolds; a short systematic treatment for arbitrary spaces, emphasizing the universal property of the Čech-procedure, is contained in an appendix.

The book grew out of a one-year's course on algebraic topology, and it can serve as a text for such a course. For a shorter basic course, say of half a year, one might use chapters II, III, IV (§§ 1–4), V (§§ 1–5, 7, 8), VI (§§ 3, 7, 9, 11, 12). As prerequisites the student should know the elementary parts of general topology, abelian group theory, and the language of categories – although our chapter I provides a little help with the latter two. For pedagogical reasons, I have treated *integral* homology only up to chapter VI; if a reader or teacher prefers to have general coefficients from the beginning he needs to make only minor adaptions.

As to the outlay of the book, there are eight chapters, I–VIII, and an appendix, A; each of these is subdivided into several sections, § 1, 2, ... Definitions, propositions, remarks, formulas etc. are consecutively numbered in each §, each number preceded by the §-number. A reference like III, 7.6 points to chap. III, § 7, no. 6 (written 7.6) – which may be a definition, a proposition, a formula, or something else. If the chapter number is omitted the reference is to the chapter at hand. References to the bibliography are given by the author's name, e.g. Seifert-Threlfall; or Steenrod 1951, if the bibliography lists more than one publication by the same author.

The exercises are meant to provide practice of the concepts in the main text as well as to point out further results and developments. An exercise or its solution may be needed for later exercises but not for the main text. Unusually demanding exercises are marked by a star, $*$.
I have given several courses on the subject of this book and have profited from many comments by colleagues and students. I am particularly indebted to W. Bos and D.B.A. Epstein for reading most of the manuscript and for their helpful suggestions.

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