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Chapter 8
Basic concepts of reliability analysis by probability methods

8.1 Introduction

This chapter provides the theoretical background for the reliability analysis used in other chapters, Chapter 2 in particular. Some basic concepts of probability theory are discussed as these are essential to the understanding and development of quantitative reliability analysis methods. Definitions of terms commonly used in system reliability analysis are also included. The three methods discussed are the cut-set, the state-space, and the network reduction methods.

8.2 Definitions

The following terms, defined in Chapter 1, are commonly used in system reliability analysis: *component, failure, failure rate, mean time between failures (MTBF), mean time to repair (MTTR)*, and *system*. Additional definitions more specifically related to power distribution systems are given in 1.4. See section 1.4

8.3 Basic probability theory

This subclause discusses some of the basic concepts of probability theory. An appreciation of these ideas is essential to the understanding and development of reliability analysis methods.

8.3.1 Sample space

Sample space is the set of all possible outcomes of a phenomenon. For example, consider a system of three distribution links. Assuming that each link exists either in the operating or "up" state or in the failed or "down" state, the sample space is

$$S = (1U, 2U, 3U), (1D, 2U, 3U), (1U, 2D, 3U), (1U, 2U, 3D), (1D, 2D, 3U), \\ (1D, 2U, 3D), (1U, 2D, 3D), (1D, 2D, 3D)$$

Here iU, iD denote that the component i is up or down, respectively. The possible outcomes of a system are also called "system states," and the set of all possible system states is called "system-state space."

8.3.2 Event

In the example of three distribution links, the descriptions (1D, 2D, 3U), (1D, 2U, 3D), (1U, 2D, 3D), and (1D, 2D, 3D) define an event in which two or three lines are in the failed state. Assuming that a minimum of two lines is needed for successful system operation, this set of states also defines the system failure. The event A is, therefore, a set of system states, and the event A is said to have occurred if the system is in a state that is a member of set A.

8.3.3 Probability

A simple and useful way of looking at the probability of an occurrence of the event is by using a large number of observations.

Consider, for example, that a system is energized at time $t = 0$, and the state of the system is noted at time t. This is said to be one observation. Now, if this process is repeated N times and the system is observed in the failed state N_f times, the probability of the system being in a failed state at time t is

$$P_f(t) = N_f / N \qquad (8\text{-}1)$$

$$N \rightarrow \infty$$

8.3.4 Combinatorial properties of event probabilities

Certain combinatorial properties of event probabilities that are useful in reliability analysis are discussed in this subclause.

8.3.4.1 Addition rule of probabilities

Two events, A_1 and A_2, are mutually exclusive if they cannot occur together. For events A_1 and A_2 that are not mutually exclusive (that is, events which can happen together)

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \qquad (8\text{-}2)$$

where

$P(A_1 \cup A_2)$ is the probability of A_1 or A_2, or both happening; and

$P(A_1 \cap A_2)$ is the probability of A_1 and A_2 happening together.

When A_1 and A_2 are mutually exclusive, they cannot happen together; that is, $P(A_1 \cap A_2) = 0$, therefore Equation (8-2) reduces to

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) \qquad (8\text{-}3)$$

8.3.4.2 Multiplication rule of probabilities

If the probability of occurrence of event A_1 is affected by the occurrence of A_2, then A_1 and A_2 are not independent events.

The conditional probability of event A_1, given that event A_2 has already occurred, is denoted by $P(A_1 \mid A_2)$ and

$$P(A_1 \cap A_2) = P(A_1 \mid A_2)\, P(A_2) \tag{8-4}$$

This formula is also used to calculate the conditional probability

$$P(A_1 \mid A_2) = P(A_1 \cap A_2) \,/\, P(A_2) \tag{8-5}$$

When, however, events A_1 and A_2 are independent, that is the occurrence of A_2 does not affect the occurrence of A_1

$$P(A_1 \cap A_2) = P(A_1)\, P(A_2) \tag{8-6}$$

8.3.4.3 Complementation

$\bar{A}_1$ is used to denote the complement of event A_1. The component $\bar{A}_1$ is the set of states that are not members of A_1. For example, if A_1 denotes states indicating system failure, then the states not representing system failure make $\bar{A}_1$.

$$P(\bar{A}_1) = 1 - P(A_1) \tag{8-7}$$

8.3.5 Random variable

A random variable can be defined as "a quantity that assumes values in accordance with probabilistic laws." A discrete random variable assumes discrete values, whereas a random variable that assumes values from a continuous interval is termed a "continuous random variable." For example, the state of a system is a discrete random variable, and the time between two successive failures is a continuous random variable.

8.3.6 Probability distribution function

Probability distribution function describes the variability of a random variable. For a discrete random variable X, assuming values x_i, the probability density function is defined by

$$P_X(x) = P\,(X = x) \tag{8-8}$$

The probability density function for a discrete random variable is also called the "probability mass function" and has the following properties:

a) $P_X(x) = 0$ unless x is one of the values $x_0, x_1, x_2, \ldots$
b) $0 \le P_X(x_i) \le 1$
c) $\sum_i P_X(x_i) = 1$

Another useful function is the cumulative distribution function. It is defined by

$$F_X(x) = P\,(X \le x) = \sum P_X(x_i),\; x_i \le x \tag{8-9}$$

The probability density function $f_X(x)$ [or simply $f(x)$] for a continuous random variable is defined so that

$$P(a \le X \le b) = \int_a^b f(y)\,\mathrm{d}y \tag{8-10}$$

If, for example, X denotes the time to failure, Equation (8-10) gives the probability that the failure will occur in the interval (a,b). The corresponding probability distribution function for a continuous random variable is

$$F(x) = P(-\infty \le X \le x) = \int_{-\infty}^{x} f(y)\,\mathrm{d}y \tag{8-11}$$

The function $f(x)$ has certain specific properties (see Singh and Billinton [B3][1]) including the following:

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1 \tag{8-12}$$

8.3.7 Expectation

The probabilistic behavior of a random variable is completely defined by the probability density function. It is often, however, desirable to have a single value characterizing the random variable. One such value is the expectation. It is defined by

$$E(X) = \sum_i x_i P_X(x_i) \text{ for a discrete random variable.}$$

$$= \int_{-\infty}^{\infty} x f(x)\mathrm{d}x \text{ for a continuous random variable.}$$

The expectation of X is also called the "mean value of X" and has a special relationship to the average value of X in that, if the random variable X is observed many times and the arithmetic average of X is calculated, it will approach the mean value as the number of observations increases.

[1]The numbers in brackets preceded by the letter B correspond to those of the bibliography in 8.6.

8.3.8 Exponential distribution

There are several special probability distribution functions (see Singh and Billinton [B3]); but the one of particular interest in reliability analysis is the exponential distribution, having the probability density function of

$$f(x) = \lambda e^{-\lambda x} \tag{8-13}$$

where λ is a positive constant. The mean value of the random variable X, with exponential distribution is

$$d = \int_0^\infty x\lambda e^{-\lambda x} dx = 1/\lambda \tag{8-14}$$

Also the probability distribution is

$$F(x) = \int_0^x \lambda e^{-\lambda y} dy = 1 - e^{-\lambda x} \tag{8-15}$$

If the time between failures obeys the exponential distribution, the mean time between failures is $d = 1 / \lambda$, where λ denotes the failure rate of the component. It should be noted that the failure rate for exponential distribution and only the exponential distribution is constant.

8.4 Reliability measures

The term "reliability" is generally used to indicate the ability of a system to continue to perform its intended function. Several measures of reliability are described in the literature, and some of the meaningful indexes for repairable systems, especially power distribution systems, are described in this subclause.

a) *Unavailability*. Unavailability is the "steady-state probability that a component or system is out of service due to failures or scheduled outages." If only the failed state is considered, this term is called "forced unavailability."

b) *Availability*. Availability is the "steady-state probability that a component or system is in service." Numerically, availability is the complement of unavailability, that is

   Availability = 1 – unavailability

c) *Frequency of system failure*. This index can be defined as the "mean number of system failures per unit time."

d) *Expected failure duration*. This index can be defined as the "expected or long-term average duration of a single failure event."

8.5 Reliability evaluation methods

Numerical values for reliability measures can be obtained either by analytical methods or through digital simulation. Only the analytical techniques are discussed here (a discussion of the simulation approach can be found in (Singh and Billinton [B3]). The three methods described in this chapter are the state-space, network reduction, and cut-set methods. The state-space method is very general but becomes cumbersome for relatively large systems. The network reduction method is applicable when the system consists of series and parallel subsystems. The cut-set method is becoming increasingly popular in the reliability analysis of transmission and distribution networks and has been primarily used in this book. The state-space and network reduction methods are discussed in this chapter for reference and for the potential benefit to the users of this book.

8.5.1 Minimal cut-set method

The cut-set method can be applied to systems with simple as well as complex configurations and is a very suitable technique for the reliability analysis of power distribution systems. A cut-set is a "set of components whose failure alone will cause system failure," and a minimal cut-set has no proper subset of components whose failure alone will cause system failure. The components of a minimal cut-set are in parallel since all of them must fail in order to cause system failure and various minimal cut-sets are in series as any one minimal cut-set can cause system failure.

A simple approach for the identification of minimal cut-sets is described in Chapter 2, but more formal algorithms are also available in the literature (see Singh and Billinton [B3]). Once the minimal cut-sets have been obtained, the reliability measures can be obtained by the application of suitable formulas (see Shooman [B1] and Singh [B2]). Assuming component independence and denoting the probability of failure of components in cut-set C_1 by $P(\overline{C}_i)$, the probability (unavailability) and the frequency of system failure for m minimal cut-sets are given by

$$P_f = P(\overline{C}_1 \cup \overline{C}_2 \cup \overline{C}_3 \cup \ldots \cup \overline{C}_m)$$

$$= P(\overline{C}_1) + P(\overline{C}_2) + \ldots + P(\overline{C}_m)\binom{m}{1}\text{ terms} - [P(\overline{C}_1) \cap (\overline{C}_2)] + \ldots$$

$$+ \ [P(\overline{C}_1 \cap \overline{C}_j)]\text{i} \neq \text{j}\binom{m}{2}\text{ terms}$$

$$\vdots$$

$$(-1)^{m-1}P(\overline{C}_1 \cap \overline{C}_2 \cap \ldots \overline{C}_m)\binom{m}{m}\text{ terms} \qquad (8\text{-}16)$$

where $\overline{C}_1 \cap \overline{C}_2$, for example, denotes the failure of components of both the minimal cut-sets 1 and 2 and, therefore, $P(\overline{C}_1 \cap \overline{C}_2)$ means the probability of failure of all the components contained in $\overline{C}_1$ and $\overline{C}_2$, that is

$$P(\overline{C}_1 \cap \overline{C}_2) = \Pi P_{id} \text{ and } i \in (\overline{C}_1 \cup \overline{C}_2)$$

where

P_{id} is the probability of component i being in the failed state
$= r_i / (d_i + r_i)$.
$= \lambda_i / (\lambda_i + \mu_i)$.
d_i is the MTBF of component i.
λ_i is the failure rate of component i.
$= 1 / d_i$.
r_i is the MTTR of component i.
μ_i is the repair rate of component i
$= 1 / r_i$.
Π is the product.

The frequency of failure is given by

$$f_f = P(\overline{C}_1) W_1 + P(\overline{C}_2) W_2 + \ldots P(\overline{C}_m) W_m - [P(\overline{C}_1 \cap \overline{C}_2) W_{1,2} + P(\overline{C}_1 \cap \overline{C}_3) W_{1,3}$$

$$+ \ldots + P(\overline{C}_i \cap \overline{C}_j) W_{i,j}], i \neq j$$

$$\vdots$$

$$(-1)^{m-1} P(\overline{C}_1 \cap \overline{C}_2 \cap \ldots \overline{C}_m) W_{1,2} \ldots, m \tag{8-17}$$

where

$$W_{i,j} = \sum_{k \in C_i \cup C_j} \mu_k$$

$$k \in C_i \cup C_j$$

The mean failure duration is given by

$$d_f = P_f / f_f$$

When the mean time between the failure of components is much larger than the mean time to repair (or in other words, the component availabilities approach unity). Equation (8-16) and (8-17) can be approximated (see Singh [B2]) by simpler equations:

$$P_f = \sum_{i=1}^{m} P(\bar{C}_i) = \sum_{i=1}^{m} Pcs_i \tag{8-18}$$

and

$$f_f = \sum_{i=1}^{m} P(\bar{C}_i) W_i = \sum_{i=1}^{m} fcs_i \tag{8-19}$$

where Pcs_i and fcs_i are the probability and frequency of cut-set event i, respectively.

Also,

$$d_f = P_f / f_f = \sum_{i=1}^{m} Pcs_i / \sum_{i=1}^{m} fcs_i = \sum_{i=1}^{m} fcs_i rcs_i / \sum_{i=1}^{m} fcs_i \tag{8-20}$$

where:

d_f is the system mean failure duration; and

rcs_i is the mean duration of cut-set event i.

The application of Equations (8-19) and (8-20) to power distribution systems is discussed in Chapter 2. The components in a minimal cut-set behave like a parallel system, and fcs_i (assuming n components in C_i) can be computed as follows:

$$fcs_i = \prod_{j=1}^{n} P_{jd} \sum_{j=1}^{n} \mu_j \tag{8-21}$$

and

$$rcs_i = 1 / \sum_{j=1}^{n} \mu_j \tag{8-22}$$

For example, for a cut-set having three components 1, 2, and 3:

$$fcs_i = \frac{\lambda_1 \lambda_2 \lambda_3 (\mu_1 + \mu_2 + \mu_3)}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)}$$

$$\approx \lambda_1 \lambda_2 \lambda_3 (r_1 r_2 + r_2 r_3 + r_3 r_1), \text{ assuming } \lambda_i << \mu_i$$

and

$$rcs_i = \frac{r_1 r_2 r_3}{(r_1 r_2 + r_2 r_3 + r_3 r_1)}$$

8.5.2 State-space method

The state-space method is a very general approach and can be used when the components are independent as well as for systems involving dependent failure and repair modes. The different steps of this approach are illustrated using a simple example of a component in series with two parallel components, as shown in Figure 8-1.

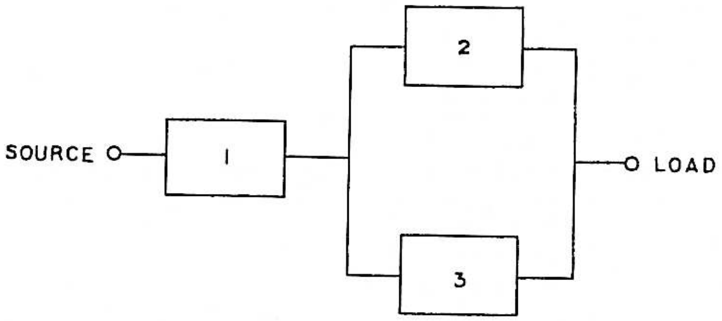

Figure 8-1—One component in series with two components in parallel

a) *Enumerate the possible system states.* Assuming each component can exist either in the up or operating state (U) or in the down or failed state (D) and that the components are independent, there are eight possible system states. These states are numbered 1 through 8 in Figure 8-2, and the description of the component states is indicated in each system state.

b) *Determine interstate transition rates.* The transition rate from s_i (that is, state i) to s_j is the mean rate of the system passing from s_i to s_j. For example, in Figure 8-2 the system can transit from s_1 to s_2 by the failure of component 1 and the repair of component 1, will put the system back into s_1. Therefore, the transition rate from s_1 to s_2 is λ_1, and the transition rate from s_2 to s_1 is μ_1.

c) *Determine state probabilities.* When the components can be assumed to be independent, state probabilities can be found by the product rule as indicated in Equation (8-6).When, however, statistical dependence is involved, a set of simultaneous equations needs to be solved to obtain state probabilities (see Singh and Billinton [B3]). Only the independent case is discussed here and for this, say the probability of being in state 2 can be determined by

$$P_2 = P_{1d} P_{2u} P_{3u} \quad (8\text{-}23)$$

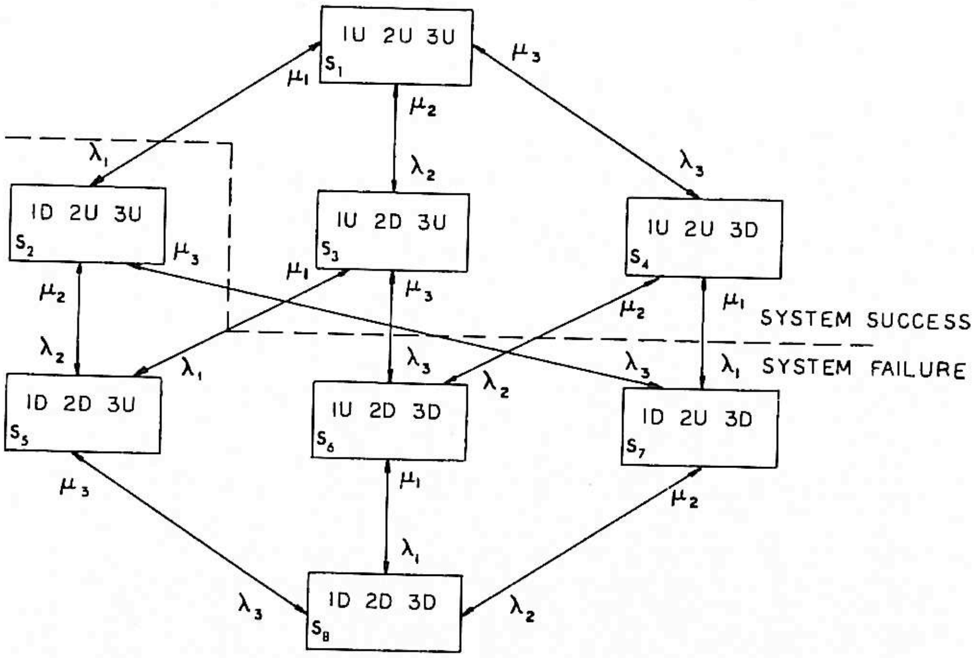

Figure 8-2—State transition diagram for the system shown in Figure 8-1

where

P_{iu} is the probability of component i being in "up" (operating) state
$= d_i / (d_i + r_i)$
$= \mu_i / (\lambda_i + \mu_i)$.

d_i = MTBF of i
r_i = MTTR of i
μ_i = repair rate of i
λ_i = failure rate of i

and

P_{id} is the probability of component i being in "down" (failed) state
$= r_i / (d_i + r_i)$
$= \lambda_i / (\lambda_i + \mu_i)$.

d) *Determine Reliability Measures*. The states contributing the failure, or success, or any other event of interest are identified. For the system shown in Figure 8-1, if the links 2 and 3 are fully redundant, system failure can occur if either component 1 fails, or components 2 and 3 fail, or if all components fail. The state space S is shown in Figure 8-2 is

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

The subset A (representing failure) can be identified as:

$$A = \{2, 5, 6, 7, 8\}$$

and the subset representing the success states is

$$S - A = \{1, 3, 4\}$$

Unavailability or the probability of the system being in the down state is now given by

$$P_f = \sum_{i \in A} P_i \tag{8-24}$$

where $i \in A$ indicates that summation is over all states contained in subset A.

Applied to our example

$$P_f = P_2 + P_5 + P_6 + P_7 + P_8$$

where P_i can be found by the product rule (see Equation (8-23).

The frequency of system failure, that is, the frequency of encountering subset A, can be computed by the following relationship:

$$f_f = \sum_{i \in A} P_i \sum_{j \in A} \lambda_{ij} \tag{8-25}$$

where λ_{ij} equals the transition rate from state i to state j.

$$f_f = P_1\lambda_1 + P_3(\lambda_1 + \lambda_3) + P_4(\lambda_1 + \lambda_2)$$

The mean failure duration can be obtained from P_f and f_f using

$$d_f = P_f / f_f \tag{8-26}$$

In the preceding analysis, it was assumed that the failure of a component does not alter the probability of failure of the remaining components. If, however, it is assumed that after the system failure, no further component failure will take place, the state transition diagram in Figure 8-2 will be modified as shown in Figure 8-3. Once component 1 fails or components 2 and 3 fail, no further failure is possible. The probabilities in this case cannot be calculated by simple multiplication; they can be computed by solving a set of linear equations (see Singh and Billinton [B3]). Once the state probabilities have been calculated, the remaining procedure is the same.

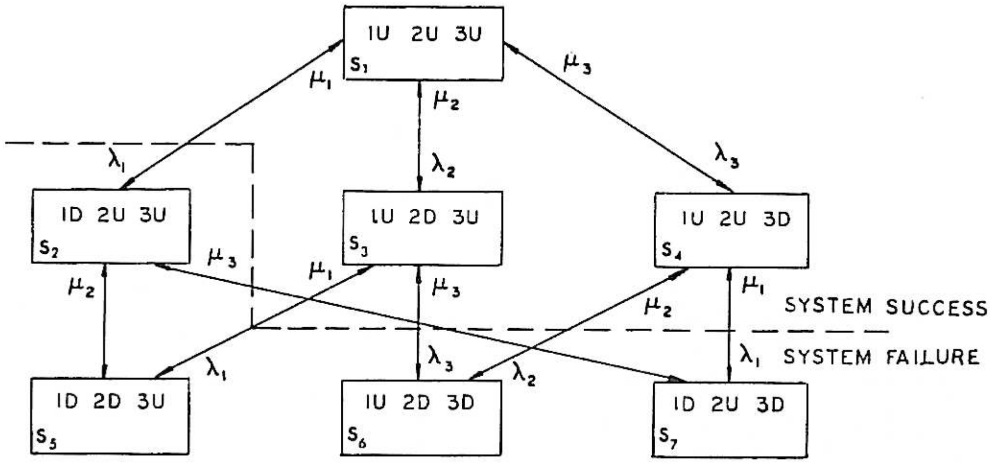

Figure 8-3—State transition diagram for the system shown in Figure 8-1 when components are not independent

8.5.3 Network reduction method

The network reduction method is useful for systems consisting of series and parallel subsystems. This method consists of successively reducing the series and parallel structures by equivalent components. Knowledge of the series and parallel reduction formulas is essential for the application of this technique.

8.5.4 Series system

The components are said to be in series when the failure of any one component causes system failure. It should be noted that the components do not have to be physically connected in series; it is the effect of failure that is important. Two types of series systems are discussed in 8.5.4.1 and 8.5.4.2.

8.5.4.1 Independent components

For the series system of independent components, the failure and repair rate the equivalent component are given by

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \tag{8-27}$$

and

$$\mu_s = \lambda_s / \left(\prod_{i=1}^{n} (1 + \lambda_i / \mu_i) - 1\right) \tag{8-28}$$

where λ_s and μ_s are the equivalent failure and repair rates of the series system and

$\prod_{i=1}^{n}$ denotes the product of values 1 through n (n being the number of components).

Assuming the λ_i is much smaller than μ_i (which, in other words, means that the MTBF is much larger than the MTTR), the quantities involving the products of λ_i can be neglected. Equation (8-27) reduces to

$$r_s = 1 / \mu_s = \sum_{i=1}^{n} r_i \lambda_i / \lambda_s \tag{8-29}$$

8.5.4.2 Components involving dependence

When it is assumed that after the system failure no more components will fail, the equivalent failure and repair parameters are

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \text{ and } r_s = \sum_{i=1}^{n} r_i \lambda_i / \lambda_s \tag{8-30}$$

It can be seen from Equations (8-28) and (8-29) that, for component MTBF to be much larger than MTTR, the r_s for the dependent and independent cases should be practically equal.

8.5.5 Parallel system

Two components are considered in parallel when either can ensure system success. The equivalent failure and repair rates of a parallel system of two components are given by

$$\lambda_p = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{1 + \lambda_1 r_1 + \lambda_2 r_2} \tag{8-31}$$

and

$$\mu_p = \mu_1 + \mu_2 \tag{8-32}$$

If $\lambda_1 r_1$ and $\lambda_2 r_2$ are much smaller than 1, then Equation (8-30) can be written as

$$\lambda_p = \lambda_1 \lambda_2 (r_1 + r_2) \tag{8-33}$$

8.6 Bibliography

[B1] Shooman, L. M., *Probabilistic Reliability: An Engineering Approach*, New York: McGraw-Hill, 1968.

[B2] Singh, C., "On the behavior of failure frequency bounds," *IEEE Transactions on Reliability*, vol. R-26, Apr. 1977, pp. 63–66.

[B3] Singh, C., and Billinton, R., *System Reliability Modelling and Evaluation*, London, England: Hutchinson Educational, 1977.

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Lec 3.5

Economic Dispatch

- what is the most economic way to produce the needed power??

- How can I operate the power system in the most economic way, while satisfying security constraints?

define:-

1. Generator outputs, P_{g_i}
- generator cost $C_i(P_{gi})$
- total cost $\sum_{i=1}^{m} C_i(P_{g_i})$

- what we want is to minimize to cost subject to a set of constraints.
- so how do we define these $C_i(P_{gi})$s.
- Economic operation can be divided into 2 main problems.

1. Economic dispatch says that :-
   ↳ a certain set of generators in service
   ↳ we want to minimize cost over that set (of generators in service)

2. Unit Commitment :-
   a more challenging problem to solve.
   ↳ which generators do I bring online (to ensure secure operation at minimal cost)
   ↳ start-up costs (associated with getting systems up + running, pre-heating, personell, maintenance etc) once units are online, they do not affect the economic dispatch

- once the unit is committed, economic dispatch is used
- this is the old way of doing things.
- Old way - $\underbrace{\text{vertically integrated utilities}}$

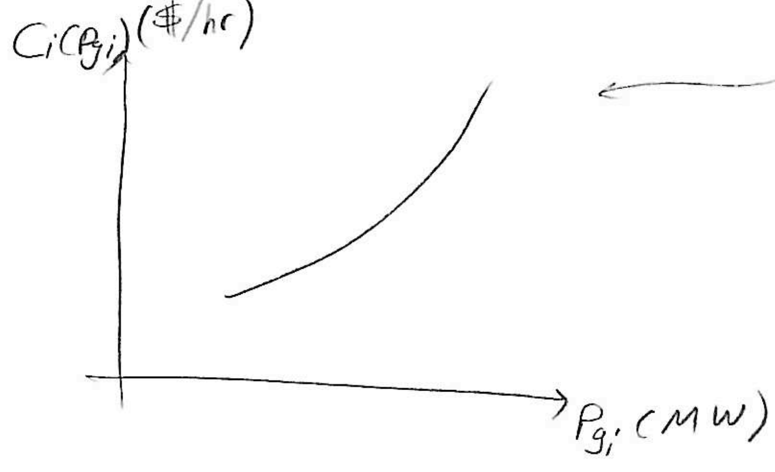
utilities owned generation
transmission
&
distribution

- New Way - electricity markets
  - many more players involved in the generation.
  - acquire offers of energy production from all the possible generators, then to effectively compare their cost and to choose the most appropriate or "cheapest" units to commit.
  - so the general concept of economic dispatch is still valid for new markets
  - cost curves are specified differently in the two ways.

- we will only cover economic dispatch here but we will mention along the way the differences in doing things in the old + new ways.

Formulation

Typical fuel-cost curve



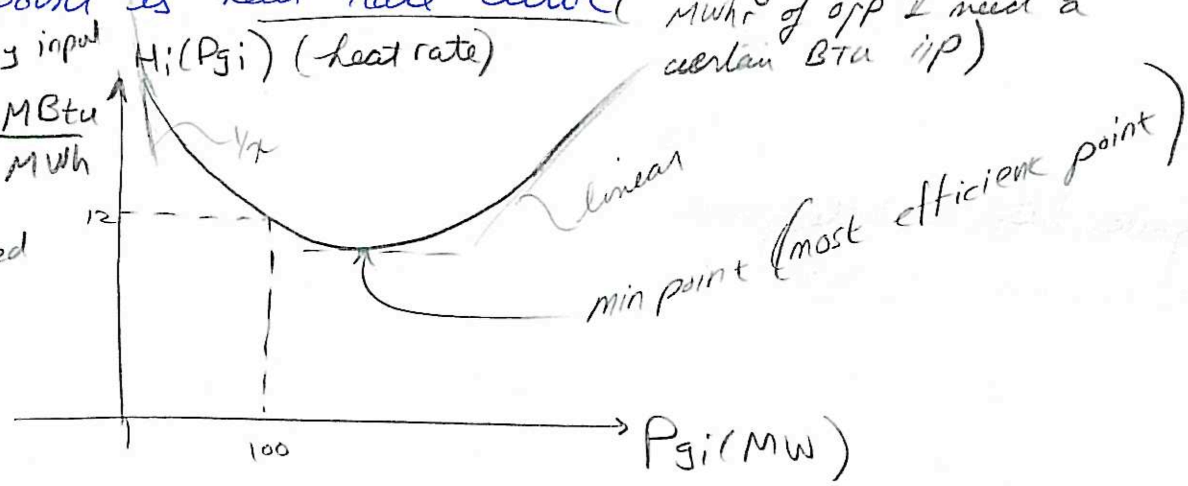
convex function

- where does this curve come from??

- Starting point is heat-rate curve (it says that for every MWhr of o/p I need a certain Btu i/p)

(Btu) amount of energy input → MBtu

amount of energy extracted → MWh



- heat rate is a ratio between $\dfrac{\text{amount of energy i/p}}{\text{amount of energy extracted}}$

- heat energy input rate (this says that if I want to calculate the total heat energy going in/hr I need $F_i(P_{gi})$)

$$F_i(P_{g_i}) = P_{g_i} H_i(P_{gi}) \quad \frac{MBtu}{hr}$$

- if I know the cost of natural gas that produces X MBtu of energy I can get the cost function

- If the fuel cost is $K \$/MBtu$

then

$$C_i(P_{g_i}) = K F_i(P_{g_i})$$

- so it all starts from heat rate curve.
- at low MW the heat rate curve can be approximated by a $1/x$ ie $\Rightarrow \infty$ at $P_{g_i} \Rightarrow 0$
- at high MW the heat rate curve can be approximated by a linear function of generation.
- so we can write the heat rate curve equation as:

$$H_i(P_{g_i}) = \underbrace{\frac{\alpha'}{P_{g_i}}}_{\text{dominant if } P_{g_i} \text{ is small}} + \underbrace{\overset{\text{intersection}}{\beta'} + \overset{\text{slope}}{\gamma'} P_{g_i}}_{\text{dominant part if } P_{g_i} \text{ is large.}}$$

to get cost, we multiply our model by P_{g_i} + then K

The corresponding fuel-cost curve $K\alpha' = \alpha \ldots$

$$C_i(P_{g_i}) = \alpha + \beta P_{g_i} + \gamma P_{g_i}^2 \quad \$/hr$$

where α, β, γ are all +ve and we end up w/ the convex function shape.

General Formulation

↳ given the system has m generators that are committed (online)

(note) you can't "choose" to bring units online and offline like small circuits, if a unit is committed, it is "committed" because it may take hours and a good deal of \$ to bring units online + offline.

↳ all the loads S_{d_i} are given
(we will vary the generation, but need to make sure we always satisfy the load)

- Determine the P_{g_i} and $|V_i|$, $i = 1, \ldots, m$ to minimize the total cost.

$$C_T \triangleq \sum_{i=1}^{m} c_i(P_{g_i})$$

subject to all the power flow equations and the "security" inequality constraints.

↓

$$P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max}$$

Transmission flows $|P_{ij}| \leqslant P_{ij}^{max}$ on all lines

- ie can't increase power flow on the line as much as you want, because of line thermal limits, sags → obstruction with trees etc.

$$|V_i|^{min} \leqslant |V_i| \leqslant |V_i|^{max}$$

- regulated voltage limits by authorities

- this problem is also known as :-

Optimal Power Flow:-

↳ solved using a range of methods that fall under the concept of

↳ non-linear programming

- techniques derived from Newtons

↳ ie successive linear programs (LPs)

ie take non-linear → linearize → iterate

- interior point methods (prime choice for large systems)

- where are the max + min limits on generator come from??

- upper limit $P_{g_i}^{max}$ thermal limits on the turbine generator unit. (mainly mech. limits pipes bringing in gas, conveyor of coal, boiler size of steam turbines etc etc ...)

Lower limit : is set by boiler dynamics and other thermodynamic considerations

fuel → boiler → steam

- "flame out" may occur → ie you don't have enough fuel into the furnace to sustain the flame
- this is very dangerous bc. fuel is coming into the furnace but no burning flame, so at some point enough fuel may accumulate and an explosion may occur.
- ex. a 300 MW unit may have a lower limit around 120 MW.

- note : we are optimizing (PV) to reduce cost this is where this comes from in power flow.
- this is the full version of the problem, b/c we will simplify things ALOT.
- we will make some approximations to make the problem a linear problem.

Approximation

- Little coupling between P/Θ and Q/V

ie we don't need to coordinate P variation w/ V variation.

- we can look at the P_{g_i} optimization without considering the V_i.

- we will be able to reduce many of the power flow equations AND ignore some of the inequality constraints.

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this may be a long class

- what we will do in class is a linear simplified version of the real problem.

- in doing so we will neglect:-

  ① transmission line constraints ← called conjesttion

  ② line losses (for now) ~~(conjestion)~~

- Assumptions:- P-θ coupling strong

  Q-V ~ ~

  - neglecting cross couplings

- we are going to assume that P-θ is what really affects economic dispatch, this is not the case in the real world where reactive power does have an effect, ie there is cross coupling.

- Simplified Problem :-
  ↳ optimization

minimize over the generators. $\underset{P_{g_i}}{\text{Minimize}} \quad C_T \quad \sum_{i=1}^{m} C_i(P_{g_i})$

such that generation is equal to demand

$$\sum_{i=1}^{m} P_{g_i} = P_D = \sum_{i=1}^{n} P_{d_i}$$

generated — delivered

is effectively a simplified power flow equation that neglects ALL system topology issues.

- as if all generation and loads are at the same bus.

- this is because we neglected T.L constraints which would require knowledge of detailed power flow in all lines.

where

$$P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max} \qquad i = 1, \dots, m$$

ie there are physical constraints on what we can produce.

this is really a

First approximation to the problem

- what we want is to extract intuition into economic operation.

Solution involves incremental costs (ICs).

$$IC_i = \frac{d\,C_i(P_{g_i})}{dP_{g_i}}$$

- slope of the fuel cost curve

C_i has units \$/hr

so IC_i has units \$/hr/MW ≡ \$/MWhr

- it tells us how the cost changes for the next MW change in generation.

- increase in cost per increase in MW

The fuel cost curve is usually quadratic

$$C_i(Pg_i) = \alpha + \beta Pg_i + \gamma Pg_i^2$$

- generically the coefficients are positive.
(differentiate)

$$IC_i = \beta + 2\gamma Pg_i$$

- linear with positive coefficients.

Economic Dispatch Ignoring Pg Limits

- ie minimize total cost ensuring power balance

Optimal dispatch rule:-
- operate every generator at the same incremental cost.

Intuitively, consider two generators with

$$IC_1 > IC_2$$

- ie next MW from generator ① will cost more than the next MW from generator ②

if we reduce Pg_1 by X MW saves IC_1 \$/MWh

if we increase Pg_2 costs IC_2 \$/MWh

ie transfer of 1MW from generator 1 to gen. 2 we save IC_1 and spend IC_2

net savings $IC_1 - IC_2 > 0$

- so one would keep doing that until $IC_1 < IC_2$
- that is why we want a strategy where $IC_1 \simeq IC_2$

- another way to look at this is mathematically.
- we need to introduce what is called Lagrangian multipliers.

$$\underset{P_{g_i}}{\text{Minimize}} \quad C_T = C_1(P_{g_1}) + \dots + C_m(P_{g_m})$$

subject to $\quad P_{g_1} + P_{g_2} + \dots + P_{g_m} = P_D$

Solve using Lagrangian multipliers.

Replace the cost function C_T by

$$\tilde{C}_T \triangleq C_T(P_{g_i}) - \underset{\text{lagrangian multiplier}}{\lambda} \left(\sum_{i=1}^{m} P_{g_i} - P_D \right)$$

augmented problem

- we know that the minimum of a function can be found by differentiating it and setting that to zero ~~(this is a local~~ (this doesn't gaurantee a minimum in general, BUT we can use other properties about the problem to make sure it is a minimum.

- minima are given by points where all partial derivatives are zero to give the stationary points $\cup \cap$
- what are we going to differentiate w.r.to ??

the P_{g_i}s $\rightarrow P_{g_1}, P_{g_2}, \dots$
- in the augmented problem ~~but~~ we also have λ as a variable.

$$\frac{\partial \tilde{C}_T}{\partial P_{g_i}} = 0 \qquad i = 1, \ldots, m$$

$$\frac{\partial \tilde{C}_T}{\partial \lambda} = 0$$

solving gives

$$\frac{\partial \tilde{C}_T}{\partial P_{g_i}} = \frac{\partial C_T}{\partial P_{g_i}} - \lambda$$

$$= \underbrace{\frac{d C_i(P_{g_i})}{d P_{g_i}}}_{I.C.} - \lambda$$

$$= IC_i - \lambda = 0 \quad \text{for } i = 1, \ldots, m$$

$$\frac{\partial \tilde{C}_T}{\partial \lambda} = -\left(\sum_{i=1}^{m} P_{g_i} - P_D\right) = 0$$

$$\Rightarrow \therefore \sum_{i=1}^{m} P_{g_i} - P_D = 0$$ ← this is nothing but the original constraint we had

- but we also got another relationship. $IC_i = \lambda$ $i=1,\ldots,m$

ie this is our optimal dispatch rule
all gen. at same inc. cost which is equal to λ.

- If the cost curves have +ve coefficients, then they are convex functions, and the stationary point is a minimum.

- If the IC. curves are monotonic (they are here bc they are linear) then the solution is unique.
  ↳ i.e. we can do a global optimization & get a minimum.

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Lagrangian Multiplier, λ

- λ is referred to as the system incremental cost
  its the cost increase that would result from a 1 MW increase in demand at any particular load.
- it relates the increased fuel cost rate (\$/hr) to increased demand (MW)
- lets say we have a given demand P_D°, the optimal $P_{g_i}^\circ$, corresponding to cost C_T° (assuming we have done the economic dispatch) (this is our base case)
- if the load increases (incrementally) → implies we can deal w/ a linearization of the problem.

$$P_D = P_D^\circ + \Delta P_D$$

find the new cost C_T.

Soln

$$C_T = C_T^\circ + \Delta C_T$$

$$C_T = \sum_{i=1}^{m} C_i(P_{g_i})$$

by differentiating

$$\Rightarrow \quad \Delta C_T = \sum_{i=1}^{m} \frac{dC_i(P_{g_i})}{dP_{g_i}} \Delta P_{g_i}$$

this relates small changes in generation to small changes in cost.

but $\dfrac{dC_i(P_{gi})}{dP_{gi}} = \lambda \qquad i = 1, \dots, m$

because this comes from a solution to the economic dispatch problem.

$$\Rightarrow \quad \Delta C_T = \lambda \sum_{i=1}^{m} \Delta P_{g_i} = \lambda \Delta P_D$$

$\because$ total generation = total load!

$$\Rightarrow \quad \Delta C_T = \lambda \Delta P_D$$

- this is why λ is called system incremental cost
ie as the load across my whole system changes, I will see a change in the cost across the whole system that is related through λ.

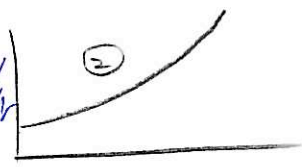
- λ is the additional cost of supplying the next MW

ex Two generators operated on economic dispatch

fixed costs

$C_1(P_{g_1}) = 900 + 45 P_{G_1} + 0.01 P_{g_1}^2$ \$/hr

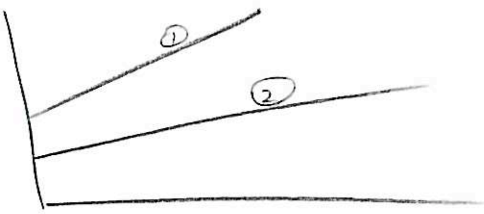
$C_2(P_{g_2}) = 2500 + 43 P_{g_2} + 0.003 P_{g_2}^2$ \$/hr



differentiating we get I C. curves.

$IC_1 = 45 + 0.02 P_{g_1}$ \$/MWh

$IC_2 = 43 + 0.006 P_{g_2}$



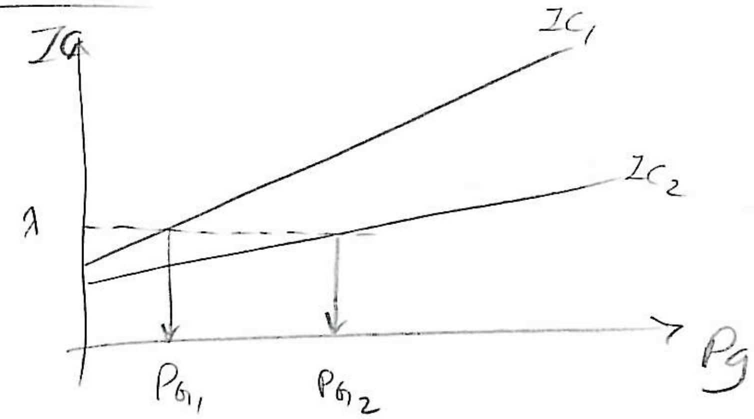
optimal scheduling $\Rightarrow IC_1 = IC_2$

- looking at $C_1 + C_2$ we see that gen 2 is much more expensive for low levels of generation than gen ① is.
- this actually disappears in the optimal dispatch problem, bc it only considers the I.C. curves.
- from IC curves we say that its better to load gen 2 because it has lower IC.

ECONOMIC DISPatch $\boxed{VS}$ UNIT COMMITMENT — Dilema — QEWC + Kahramaa

- economic dispatch assumes that we $\boxed{have}$ to use these generators + they are committed whether we like it or not.
- so we have to live with the fixed costs + the best we can do is alter the generation to reduce the remaining part of the cost function. (the fixed cost is a unit commitment problem)

(Good idea)



λ where $P_{g_1} + P_{g_2} = P_D$

we want to satisfy total demand = 700 MW

$$IC_1 = IC_2$$

$$\Rightarrow 45 + 0.02\, P_{g_1} = 43 + 0.006\, P_{g_2}$$

$$\Rightarrow P_{g_1} + P_{g_2} = 700$$

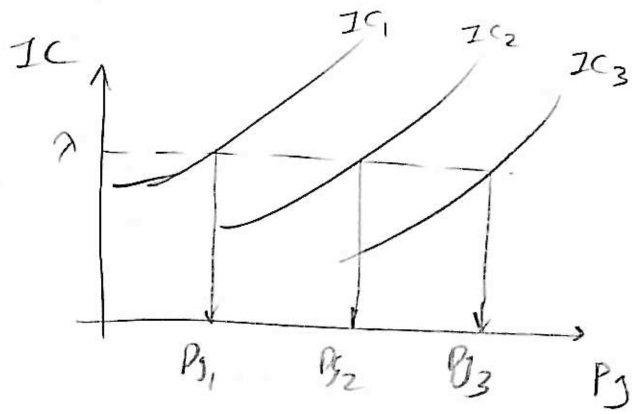
2 eqns + 2 unknowns

$$P_{g_1} = 84.6\ \text{MW}$$

$$P_{g_2} = 615.4\ \text{MW}$$

$$IC_1 = IC_2 = 46.69\ \$/\text{MWh}$$

what if we have a more general, nonlinear kind of setting



$$P_{g_1} + P_{g_2} + P_{g_3} = P_D$$

- if sum is less than P_D $\Rightarrow$ increase λ and vice versa.
- note an algorithm that relies on the monotonic nature of these curves can be designed.

Iterative Process :-

1) pick a value for λ

2) Find correspondly $P_{g_1}(\lambda), P_{g_2}(\lambda) \ldots$

3) If $\Sigma P_{g_i}(\lambda) - P_D < 0$, increase λ + go to 2

If $\Sigma P_{g_i}(\lambda) - P_D > 0$, decrease λ + go to 2

" " " $= 0$, stop.

- the electricity markets task the participants to bid in curves of IC.

- note we have simplified things alot

① no losses

② no congestion.

- what if we have limits on the generators ??

If $P_{g_2} \leq 600$ MW, the dispatch 64 cannot be implemented.

$\therefore$ now $P_{g_2} = 600$ MW

can no longer achieve equality in I.C.

we know that $P_{g_1} + P_{g_2} = 700$

$\therefore P_{g_1} + 600 = 700$

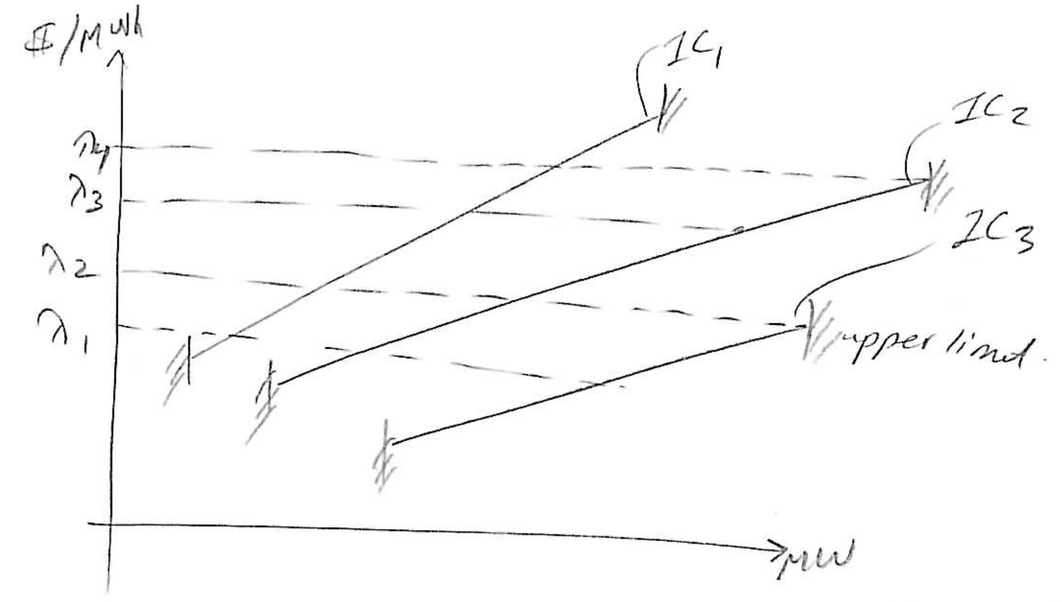
$\therefore P_{g_1} = 100$ MW

then $IC_1 = 45 + 0.02(P_{g_1})$ (100)

$= 47$ \$/MWhr

ie the next MW of demand would cost 47\$/hr vs 46.69\$/hr

- ie by constraining P_{g2} we operate in a suboptimal system.
- this will occur any time a generator in the system hits a ~~lim~~ generation limit.



- at λ_3 generator ③ cannot be economically dispatched and sits ~~at~~ at its generation limit while generators ① + ② can be economically dispatched.
- at λ_4 only generator ① is responding to load changes + beyond λ_4 there is no longer equality in any of the incremental costs
- gen① is called "marginal unit"

- if P_D increases, λ increases to provide more generation
  - ↳ eventually reach λ_2 where P_{g_3} reaches a limit.
  - ↳ further load increase must be met by $P_{g_1} + P_{g_2}$ with equal I.C. ($IC_1 = IC_2 \neq IC_3$)

- process continues as P_D continues to increase, until P_{g_2} also hits its upper limit

- The sensitivity of the cost rate C_T to increases in P_D (total demand) is still given by λ.

$$\Delta C_T = \lambda \sum \Delta P_{g_i} = \lambda \Delta P_D$$

- where the summation is over the units that are not at their limits, and λ is the common IC of those units.

- λ is still called the system incremental cost.

- extend the example by including a minimum of 50MW + 200 MW max for P_{g_1} for every generator, how does the generation look like?? 350

	P_{g_1}	P_{g_2}
λ_1	50	50
λ_2	50	100
λ_3	50	500
λ_4	100	600