Lesson 9: Perimeter and Area of Triangles in the Cartesian plane

Learning Targets
- I can find the perimeter of a triangle in the coordinate plane using the distance formula or Pythagorean theorem
- I can apply the formula to find the area of any triangle on the coordinate grid

Opening Exercises (5 minutes)

Find the perimeter of triangle ∆ABC that goes through A(1,3), B(0,1) and C(3,0)

Find the area of ∆ABC

Find the perimeter of rectangle ABCD that goes through points A(1,3), B(1,1), C(4,1) and D(4,3)

Find the area of rectangle ABCD

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Formulas To Remember (others given on formula sheet)

**Area Formulas (Cover)**

- **Area of** -- multiply length times width  \( A = Lw \)
- **Area of** -- multiply radius squared (\( r^2 \)) times \( \pi \)  \( A = \pi r^2 \)
  
  *if you only need part of the circle then divide*

- **Area of** -- multiply the base times the height then divide by 2  \( A = \frac{bh}{2} \)

**Perimeter/Circumference (around the outside)** For add up the sides on the outside and  \( C = 2\pi r \)
  
  For multiply the radius by 2 and then times by \( \pi \).
Example 1. Consider a triangular region in the plane with vertices \(O(0, 0), A(5, 2),\) and \(B(3, 4).\)

a. What is the perimeter of the triangular region?

b. What is the area of the triangular region?

The steps below show how.....

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
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<tbody>
<tr>
<td>Draw a rectangle enclosing the given triangle (\triangle OAB,) find the area of the rectangle</td>
<td>Find the area of the three triangles inside the rectangle, number the triangles (\text{①}, \text{②},) and (\text{③})</td>
<td>Find the area of triangle (\triangle OAB) by subtracting the areas of triangles (\text{①}, \text{②},) &amp; (\text{③}) from the area of the rectangle.</td>
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We can find the__________ of triangles by finding the area of the ______________ enclosing the triangle and then ______________ the areas of ______________ triangles. This method is called decomposing or box method.
Example 2. Given the triangle below with vertices $A(-2, 3)$, $B(4, 4)$, and $C(-1, -2)$. Calculate its area.

Example 3. Draw a rectangle with vertices $A(-1,1), B(3, -3), C(0, -6)$, and $D(-4, -2)$.

a. Find the perimeter of rectangle $ABCD$

b. Find the area of rectangle $ABCD$
On your own

1. Draw Triangle $\triangle VEX$ that has vertices $V(-5, 2), E(-11, -1)$ and $X(-3, -1)$.
   a. Find the perimeter of $\triangle VEX$
   b. Find the area of $\triangle VEX$

2. Draw the polygon that has vertices at $A(-7, -2), B(-4, -4), C(-6, -6)$ and $D(-9, -6)$.
   a. Find the perimeter of the polygon in the simplest radical form
   b. Find the area of the polygon
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Classwork

1. * Use coordinates to compute the perimeter and area of each polygon.

   a) 
   
   Perimeter: 
   Area: 

   b) 
   
   Perimeter: 
   Area: 

2. * Determine the areas of the following triangles

   a) 
   
   Area: 

   b) 
   
   Area:
3. ** A polygon has vertices at $A(2, 5), B(5, 3), C(5,3)$ and $D(0,4)$. Find the perimeter of the polygon in the simplest radical form.

4. ** Triangle $\triangle ABC$ has the vertices at $A(6,5), B(12,3)$ and $C(6,1)$. Find the area of $\triangle ABC$.

5. *** Rectangle $ABCD$ has an area of 50 units squared. If the coordinates of $A$ are $(1,1)$ and the coordinates of $B$ are $(5, -2)$, what are possible coordinates of $C$ and $D$, respectively?
6. *** A textbook has a picture of a triangle with vertices (3, 6) and (5, 2). Something happened in printing the book and the coordinates of the third vertex are listed as (−1, __ ). The answers in the back of the book give the area of the triangle as 6 square units.

What is the $y$-coordinate of the third vertex?
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Homework

The use of a coordinate grid is optional on all the questions

1. What is the perimeter of triangle \( \triangle ABC \) with vertices \( A(-1, -2), B(2, -6), C(-3, -6) \)? Give your answer in the simplest radical form

   (1) \( \sqrt{5} \)  \hspace{1cm} (2) \( 10 + 5\sqrt{5} \)  \hspace{1cm} (3) \( 10 + 2\sqrt{5} \)  \hspace{1cm} (4) \( 30\sqrt{5} \)

2. The coordinates of triangle \( \triangle ABC \) are \( A(6, 5), B(9, 1), C(3, 1) \). What is the perimeter of triangle \( \triangle ABC \)?

   (1) 8 units  \hspace{1cm} (2) 16 units  \hspace{1cm} (3) 20 units  \hspace{1cm} (4) 32 units

3. The coordinates of triangle \( \triangle ABC \) are \( A(0, 5), B(6, 8), C(3, -1) \). What is the perimeter of triangle \( \triangle ABC \) to the nearest tenth of a unit?

   (1) 13.4  \hspace{1cm} (2) 22.9  \hspace{1cm} (3) 25.6  \hspace{1cm} (4) 27.0

4. What is the perimeter of a parallelogram with vertices at \( A(3, -2), B(7, -4), C(5, -6) \) and \( D(1, -4) \)

   (1) \( \sqrt{56} \) units \hspace{2cm} (3) \( 8\sqrt{2} \) units
   (2) \( 4\sqrt{5} + 4\sqrt{2} \) units \hspace{2cm} (4) \( 8\sqrt{5} \) units

5. Plot the coordinates of the triangles with vertices listed \( O(0, 0), A(1, 6), B(4, 3) \)

   Find the perimeter of \( \triangle OAB \)

   Find the area of \( \triangle OAB \)