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Preservice elementary teachers' identification of necessary and sufficient conditions for a rhombus

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Abstract

This study attempts to answer the question: “how well do preservice elementary teachers identify the necessary and sufficient conditions for a rhombus?”. Forty-five elementary preservice teachers were interviewed individually on a rhombus task. They were given four quadrilaterals and asked which of them was not a rhombus. Interview transcripts were analyzed and coded by each researcher independently. Findings have showed that some of the participants noticed many properties of a rhombus, but they did not see the relationships between the properties. Hence they could not reduce the list of properties to a concise definition with necessary and sufficient conditions.

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1. Introduction

The mathematical definition of a quadrilateral is an expression that involves necessary and sufficient conditions for that quadrilateral. Understanding these necessary and sufficient conditions is crucial and before the 2nd Van Hiele Geometric Thinking Level people can't accomplish this. This study is attempting to answer the question: “how well do preservice elementary teachers identify the necessary and sufficient conditions for a rhombus”.

Identifying quadrilaterals and determining necessary and sufficient conditions for them is fundamental content knowledge for the preservice elementary teachers. If they have a sound understanding on them, they will be a good in teaching them to their future students.

2. Related Literature

Literature has shown that preservice teachers possess rote memory rather than a conceptual understanding of geometry (Cunningham & Robert, 2010), they cannot precisely define geometric concepts and cannot determine the minimal characteristics for quadrilaterals (Kuzniak & Rauscher, 2007) and they cannot even identify basic geometry concepts (Çetin & Dane, 2004; Dane, 2008; Pickreign, 2007). Particularly Pickreign (2007) asked 40 preservice teachers to define a rhombus and found that only one of them gave an adequate definition of this quadrilateral.

On the other hand, research studies have revealed that students (Usiskin, 1982), preservice teachers (Duatepe, 2000; Roberts, 1995; Sandt & Nieuwoudt, 2003; Şahin, 2008) and even teachers (Gökbulut, Sidekli & Yangın,

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2010; Sandt & Nieuwoudt, 2003; Şahin, 2008) are not on high van Hiele geometric thinking level. The most commonly-used instrument measuring van Hiele geometric thinking level was developed by Usiskin (1982). The eighth item of this test is on finding the characteristic which is true for every rhombus. In Usiskin's study, which was carried out on 2699 (average age: seventeen years old) students, 69 % of the students chose the correct option for this question. On the other hand, Duatepe (2000) and Roberts (1995) found that only 50.6 and 55.3 % of the preservice teachers gave the correct answer to this question, respectively. The fourth item of this test is on identifying a square between four different quadrilaterals. In answering this question, six per cent of the students in Usiskin's study chose the rhombus option. In other words these students thought that a rhombus holds the properties of a square. On the other hand, nine and fourteen per cent of the preservice teachers held this misconception in Duatepe's and Roberts' study, respectively. For the fifth item of the test responders needed to recognize different representations of parallelogram. In Duatepe's study, 73.2 % of the preservice teachers could recognize different representations of the parallelogram and 23.2 % of them thought that a rhombus is not a parallelogram. On the other hand in Roberts' study 56.3 % of the participants answered this question correctly and 30.1 % of them selected the option which implies that a rhombus is not a parallelogram.

3. Methods

This was a descriptive study examining preservice teachers' reasoning in the process of determining necessary and sufficient conditions for a rhombus. Semi-structured interviews with 45 (25 female and 20 male) elementary preservice teachers were carried out individually on a rhombus task. These teacher candidates had already taken all the mathematics and teaching method courses in their program. They were given four quadrilaterals as seen below and asked which of them was not a rhombus. The task was taken from a 5th grade mathematics textbook so preservice elementary teachers should have been able to answer it easily.

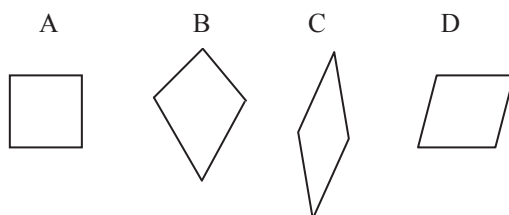


Figure 1. The quadrilaterals used in the interviews

During the interviews preservice teachers were asked to justify their answers. Interview transcripts were coded by each researcher independently and codes were examined for consensus. Consensus rate was 100 %.

4. Findings

Frequencies and percentages of responses in terms of gender and accuracy are shown in Table 1. As can be seen from the table, most of the participants (97.8 %) gave the correct answer to this question. This high correct response rate was expected since the level of the task was so low that preservice elementary teachers would be able to answer it quite easily.

Table 1. Frequencies and percentages of responses in terms of gender and accuracy

Gender	response [f (%)]		Total
	True	False	
Female	24 (53.4)	1 (2.2)	25 (55.6)
Male	20 (44.4)	0 (0)	20 (44.4)
Total	44 (97.8)	1 (2.2)	2 (100)

When the correct responses were examined, it was found that 5 (11 %) of preservice teachers did not give the necessary properties to claim a rhombus and 10 (22 %) preservice teacher gave more than the necessary properties. The remaining 30 (66 %) of them gave the necessary and sufficient conditions for indentifying a rhombus.

4.1. Insufficient characteristics

Eleven percent of the participants could not state the necessary properties for a rhombus. They thought that opposite sides of a rhombus are equal. This characteristic is true for a rhombus but it is not enough to claim a quadrilateral as a rhombus. By just looking at this we can only say that this quadrilateral is a parallelogram. An example of a response in this category is as follows:

Preservice Teacher 28 (PT28): Opposite sides should be equal in order to claim a quadrilateral as a rhombus.

Another insufficient characteristic stated by two preservice teachers was that “if a quadrilateral has equal opposite sides and parallel opposite sides, it is a rhombus”. Again this characteristic is only enough to claim a quadrilateral as a parallelogram.

PT14: I know that, for a rhombus, opposite sides are equal and also they are parallel to each other.

4.2. More than necessary characteristics

Eight of the preservice elementary teachers thought that “rhombi should have equal sides and that their opposite sides should be parallel”. These two properties are true for a rhombus. Nevertheless these qualities are more than necessary to claim a quadrilateral as a rhombus. In essence, a quadrilateral has equal sides, if and only, if it's opposite sides are parallel. Therefore it is enough to mention only one of these properties.

PT6: In order to claim a quadrilateral as a rhombus, its four sides must be equal and its opposite sides must be parallel.

PT31: The shape in the option B is not a rhombus since the opposite sides are not equal.

Interviewer: What is the necessary characteristic for a rhombus?

PT31: Rhombus is similar to a parallelogram in terms of parallelism. In addition to that the sides of the rhombus must be equal.

One preservice teacher added another characteristic beside the above properties. In addition to parallel opposite sides and equal sides, this participant mentioned that opposite angles should be equal in rhombus.

PT31: Rhombus has to have equal sides, and opposite parallel sides. Besides... there is something more related with the angles. For example this angle and this angle [by pointing out the opposite angles] must be equal.

In addition to having the characteristics of four equal sides, one of the participants thought that a rhombus should be a symmetric. He drew symmetry lines on the shapes and tried to assess which one was symmetrical. When the shapes are analyzed in terms of symmetry, one can say that all options given in the interview were symmetrical in a way. But only option B had only one symmetry line, the others having more than one symmetry line.

PT4: A rhombus has 4 equal sides and it should be symmetric [by drawing symmetry lines for each shape].

Interviewer: Why did you draw these lines?

PT4: In order to see which one is symmetric. Because it must be a symmetrical shape.

4.3. Necessary and sufficient characteristics

For defining a rhombus, 30 preservice elementary teachers used the criteria of it having “4 equal sides” which are the necessary and sufficient characteristics for a rhombus. A typical example of their responses is as follows:

Interviewer: What is the characteristic of a rhombus?

PT23: A rhombus has four equal sides.

5. Discussion

As it was predicted most of the participants (97.8 %) gave the correct answer to this question. This was expected since the level of the task was so low for the participants. However, only 66 % of them stated the necessary and sufficient conditions for indentifying a rhombus. 11 % of preservice teachers did not even cite the necessary properties to claim a rhombus. They thought that having equal opposite sides is enough to claim a quadrilateral as a rhombus. Another insufficient property given by two preservice teachers was that “if a quadrilateral has equal opposite sides and parallel opposite sides, that is a rhombus”. These properties are true for a rhombus but it is not enough to claim a quadrilateral as a rhombus. This result is similar to that of Pickreign (2007) who found that only one of the forty preservice teachers could define a rhombus.

Findings also showed that 22 % of preservice teachers mentioned more than necessary properties. These preservice teachers said that a rhombus should have some properties which are true for a rhombus but the properties they cited were more than the minimum number of properties for this shape. That means these preservice teachers did not reach 2nd van Hiele geometric thinking level (van Hiele, 1986). This finding supports the findings of Kuzniak and Rauscher (2007) who showed that preservice teachers could not identify minimum sets of properties which characterize a square and a rhombus.

This study has revealed that preservice elementary teachers' content knowledge and their geometric thinking is not at the required level. In order to improve their knowledge of geometry, teacher training programmes should be revised so that preservice teachers can receive a better training on geometry. Replication of this study for the other geometry concepts would be beneficial to understand whether the results of this study can be generalized to the other geometry concepts.

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