Pathways Algebra II Implementing the Common Core Mathematics Standards

W#7

INTRODUCTION TO FUNCTIONS AND FUNCTION NOTATION

In the previous worksheets in this module you have related values of two quantities by writing formulas to describe how values of one quantity are related to values of another quantity. If the two quantities are related in such a way that each input to the formula generates exactly one output value we can say that the formula defines a function.

As one example, consider the formula $P = 4s$, where P is the perimeter of a square with side length s. Since the formula $P = 4s$ produces exactly one perimeter for each side length s that is input to the formula, we say that this formula defines a function.

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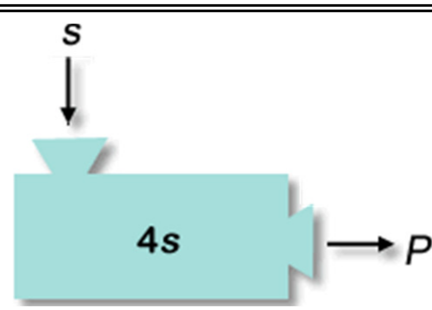
A **function** consists of three parts:

1. A set of input values referred to as the domain of the function.
2. A set of output values referred to as the range of the function.
3. A rule that assigns to each input value *exactly one* output value.



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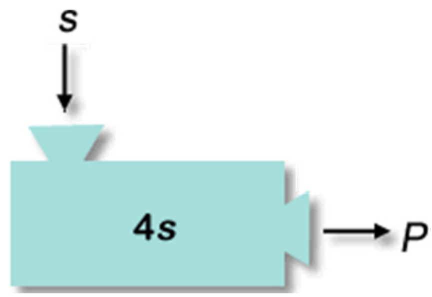

It is also common to say that a function relates values of the independent variable, in this case *s*, to values of the function's dependent or output variable, in this case *P*.

Since the values for the input (or independent) variable determine values of the output (or dependent variable) we say that the output is a function of the input.

1. Given the formula $F = (9/5)C + 32$ that converts Celsius degrees C to Fahrenheit degrees F

a. What are the two quantities that are being related?

The temperature in degrees Fahrenheit and the temperature in degrees Celsius.

b. What does it mean to say that *degrees Fahrenheit F* is a function of *degrees Celsius C*?

The temperature in degrees Fahrenheit is dependent on the temperature in degrees Celsius.

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c. Can we also think about the quantity *degrees Celsius C* as a function of the quantity *degrees Fahrenheit F*? Explain why or why not. If so, what is the input (or independent) variable, and what is the output (or dependent) variable?

Yes. In this case, temperature in degrees Fahrenheit is the independent variable and temperature in degrees Celsius is the dependent variable.

d. What formula (or rule) is conventional for defining *degrees Fahrenheit F* as a function of *degrees Celsius C*?

$$F = \frac{9}{5}C + 32$$

e. How might you rewrite the formula so that it expresses *degrees Celsius C* as a function of *degrees Fahrenheit F*?

$$C = \frac{5}{9}(F - 32)$$

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2. A circle's circumference and radius are related by the formula $C = 2\pi r$. We say that the circumference of a circle is defined in terms of a circle's radius.

a. Determine the circumferences for radius lengths of 2 inches, 3.5 inches, and 4 inches.

For a radius of 2 inches the circumference is about 12.57 inches.
For a radius of 3.5 inches the circumference is about 22.0 inches.
For a radius of 4 inches the circumference is about 25.13 inches.

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2. A circle's circumference and radius are related by the formula $C = 2\pi r$. We say that the circumference of a circle is defined in terms of a circle's radius.

b. Given any value of the radius, will there be a unique value of the circumference? That is, is a circle's circumference a function of the circle's radius? If so, explain your reasoning. If not, give an example of a value for the radius length that is related to more than one circumference value.

Yes, a circle's circumference is a function of the circle's radius. Every radius length produces only one possible measure for the circumference.

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2. A circle's circumference and radius are related by the formula $C = 2\pi r$. We say that the circumference of a circle is defined in terms of a circle's radius.

c. Suppose you are asked to calculate the circumferences for five different radius lengths. You determine the values, $C = 7.54$, $C = 18.85$, $C = 30.79$, $C = 43.98$, and $C = 50.27$. When asked which radius length is associated with each circumference, you find that you can't remember. How might you keep track of which radius length is associated with which circumference?

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In part (c) of Exercise #2, we found that it would be useful to have a method to associate inputs and outputs of functions with one another. In fact, it is often useful to be able to relate input and output values of functions and to represent relationships between quantities more generally.

Because of this, mathematicians devised a notation convention for functions. The notation convention involves assigning a name to the function relationship that exists between quantities.

The letters f or g are frequently used as the name of functions, but almost any letter is acceptable.

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From our previous example, we could define f to be the function that takes an input of a radius length r of a circle and determines an output of the associated circumference C of the circle.

We can then say that $C = f(r)$ when $f(r) = 2\pi r$.

To determine the circumference when the radius is 3.5 inches, we evaluate $f(3.5)$ and find that $2\pi(3.5) = 7\pi$ inches is the circumference of the circle.

We can express the output of f when the radius is 3.5 inches by referencing $f(3.5)$.

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3. Given that $C = f(r)$ when $f(r) = 2\pi r$,

a. What does $f(5.2)$ represent?

The circumference of a circle whose radius is 5.2.

b. What is the value of $f(5.2)$? What does $f(5.2)$ reference? Why is function notation useful?

Function notation is useful because it provides an easy way to reference the function mapping by using just the letter that we used to name the function, rather than having to write out the formula or say all the words that specify how the independent and dependent variables are related. Function notation also gives us a way to speak about a specific output value of the function, say $f(5.2)$, without actually computing its value or writing the rule for how the value is computed.

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3. Given that $C = f(r)$ when $f(r) = 2\pi r$,

c. What does the statement $f(3) = 18.85$ convey?

A circle whose radius is 3 units has a circumference of 18.85 units.

d. Create a graph of f and label the points $(0, f(0))$, $(1, f(1))$, and $(3, f(3))$ on the graph.

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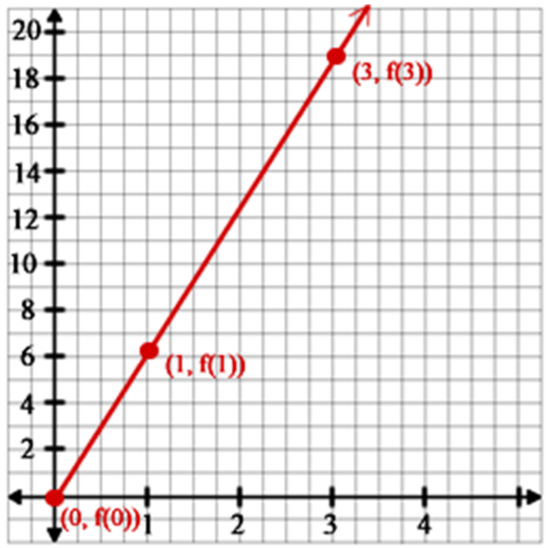

e. What does the graph of *f* represent?

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e. What does the graph of *f* represent?

The graph of *f* represents the relationship between a circle's radius and its circumference. The graph is linear because a circle's circumference is proportional to its radius (with a constant of proportionality of 2π).

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The standard function notation that will be used in this and future mathematics courses involves giving a function a name, say *f*, to define how the values *x* of an input quantity are related to the values *y* of an output quantity. The rule for defining HOW *x* and *y* are related is given by writing

$$y = f(x)$$

where $f(x)$ = <some expression written in terms of *x*>

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So, in summary:

- The values of the input quantity are represented by an input (or independent) variable (*x* is an example).
- The values of the output quantity are represented by an output (or dependent) variable (*y* is an example).
- The function that relates *x* and *y* is given a name (*f* is an example).
- We define HOW *x* and *y* are related by writing a formula, table and/or graph.

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Important:

If we want to refer to particular output values of f that are produced by particular input values of x (take $x = 1$, $x = 1.5$, and $x = 3$ as examples), we use the notation $f(1)$, $f(1.5)$, and $f(3)$.

Coordinate points on the graph of f are of the form $(x, f(x))$.

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4. Recall that the formula for determining the area of a circle with respect to the length of the radius is $A = \pi r^2$. Let g be the name of the function that takes radius length of the circle as an input and outputs the associated area of the circle.

a. Use function notation to represent (***not*** calculate) the area of a circle whose radius is 3.5 inches, 18.2 inches, and 26.92 inches.

$g(3.5)$, $g(18.2)$, $g(26.92)$

b. Explain in your own words what $g(4)$ represents. Determine the value of $g(4)$.

This represents the area of a circle (in square units) whose radius is 4 units. $g(4) \approx 50.265$

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4. Recall that the formula for determining the area of a circle with respect to the length of the radius is $A = \pi r^2$. Let g be the name of the function that takes radius length of the circle as an input and outputs the associated area of the circle.

c. Interpret what $g(4.9) = 75.43$ means in the context of the problem.

This represents a circle whose area is 75.43 square units when the radius of the circle is 4.9 units.

d. What would it mean to ask you to solve $g(x) = 141.026$ for x?

This is asking to determine the radius of the circle whose area is 141.026 square units.

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4. Recall that the formula for determining the area of a circle with respect to the length of the radius is $A = \pi r^2$. Let g be the name of the function that takes radius length of the circle as an input and outputs the associated area of the circle.

e. Determine the value of x such that $g(x) = 141.026$.

$x = 6.7$

f. What is the domain of g? (Recall that the domain is the set of values that can be input to the function. Practical constraints can sometimes limit the domain—as an example, it doesn't made sense for a height or amount of time to be negative.)

The domain of g is all real numbers greater than or equal to 0.

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