Calculus I

Lab 1: *Mathematica* Basics and Limits [1]

Objective:

To learn the basic commands of *Mathematica* including algebraic manipulation and plotting. To be able to use this knowledge to explore limits numerically, algebraically, and graphically using *Mathematica*.

Intro to *Mathematica*

Mathematica is a computer algebra system created by Wolfram. It is a very powerful tool that can be used to perform a variety of mathematical tasks. Used correctly, *Mathematica* will hopefully be a useful tool that will help you further understand concepts from this course. First, let's look at some of the things Mathematica can do.

Prerequisites

There a few things to keep in mind when using Mathematica.

1. When using a PC, in order to execute a command you must hit Shift-Enter.
2. All built-in functions must begin with a capital leter. Examples: Plot, Sin, Cos
3. The parameters inside a function are always enclosed w/ square brackets.
4. You can use a semicolon at the end of a line if you want to perform the action, but don't want to see the output.
5. Don't forget about the copy and paste commands. This will be useful if you have to type similar commands and don't want to have to retype the entire command.

Calculations

Try doing the following basic calculations (don't forget about Shift-Enter):
Addition & Subtraction:

```
2+2-7
```

Multiplication & Division:

```
2*2/8
```

Exponents:

```
2^40
```

Trig functions:

```
Sin[Pi/2]
```

[1] Adapted from *Discovering Calculus with MATHEMATICA* by B. Barden, D. Krug, P. McCartney, and S. Wilkinson.

Algebraic Manipulation

Mathematica can perform a variety of algebraic tasks.

```
Simplify[3(x+5)-4(2x-4)^2]
```

```
apples=Expand[ (1-x)^2 (x-3) (x+4) ]
```

```
Factor[apples]
```

```
Solve[x^12-7x^4+7x+9==0,x]
```

```
N[%]
```

This will give the decimal approximation of the solutions to the previous line (you can always use the percent sign to refer to the previous output).

Functions

To define a function, special notation is required. The following defines $f(x) = 3x + 6$.

```
f[x_]:=3x+6
```

```
f[2]
```

```
f[x+h]
```

```
f[bananas]
```

Plotting

If you ever want more info on a command, you can type a ? then the command and get to the help menu.

```
?Plot
```

We begin by doing some basic plotting.

```
Plot[ x^2-9, {x,-5,5}]
```

The next example makes us have equal scales for the x and y axes.

```
Plot[ x^2-9, {x,-5,5}, AspectRatio->Automatic]
```

We can also specify the range of y-values by using the following:

```
Plot[ x^2-9, {x,-5,5}, PlotRange->{-5,5}]
```

Another option that will be useful is adding color to our graphs. The command below makes the first function red and the second function green.

```
 Plot[{Sin[x],Sin[2Pi x]},
{x,0,2Pi},PlotStyle->{ RGBColor[1,0,0], RGBColor[0,1,0]}]
```

Limits

Let's go through an example of the three ways to find a limit using *Mathematica*. Suppose we want to find $\lim_{x \to 0} x \sin(\frac{1}{x})$.
We begin by defining $f(x) = x \sin(\frac{1}{x})$:

```
f[x_]:=x Sin[1/x]
```

Tables

First, we could construct a table using the following commands:

```
Table[{x,f[x]}, {x,-.05,.05,.01}] // TableForm
```

Graphs

We can also examine the limit graphically:

```
Plot[f[x],{x,-1,1}]
```

We can also apply the squeeze theorem if we examine the following graph:

```
Plot[{x,-x,f[x]},{x,-1,1},PlotStyle->{RGBColor[1,0,0],
RGBColor[1,0,0],RGBColor[0,0,0]},AspectRatio->Automatic]
```

Limit command

Finally, *Mathematica* has a built in limit command. We can try it here.

```
Limit[f[x],x->0]
```

However, sometimes the **Limit** command can give false answers. So, be careful when using it, i.e. always use at least one other method to confirm your answer.
Here's some other things the **Limit** command can do:
Left and Right Hand Limits:
Consider: $\lim_{x \to 0} h(x) = \frac{2x + |x|}{|x|} \cos x$.
First, define h:

```
h[x_]:=(2x+ Abs[x])/ Abs[x] Cos[x]
```

We can take the limit:

```
Limit[h[x],x->0]
```

Or the LHL:

```
Limit[h[x],x->0, Direction->1]
```

Or the RHL:

```
Limit[h[x],x->0, Direction->-1]
```

Look at the problem with the first limit:

```
Plot[f[x], {x,-5,5}]
```

Exercises

When doing the exercises for lab, begin by typing your name, class, and date at the top of the lab. You can change the format of typing in a given cell by going to the Cell Menu and clicking on Convert To and then Traditional. This will allow you to type freely. When you're done, hit the down key to get a cell that will allow you to type commands. You can do this technique any time you need to write something for an answer that is not clear from the output.

1. Solve $\dfrac{x-3}{\sqrt{x-8}} - 9x = 0$. Find decimal approximations of the real solutions.

2. Plot the graph of $f(x) = \dfrac{(x-2)^3}{x^2}$. Find a plot which shows the interesting features of the graph of this function.

3. Plot the graphs of $y = x$, $y = -x$, and $y = x \sin x$ together, over the interval $-2\pi \le x \le 2\pi$. Make the first two graphs blue and the third graph red.

4. Use graphical methods to approximate the solution of $|2x - 1| > |x^2 - 7|$. (Note: *Mathematica*'s absolute value function is named **Abs**[ ].) Then use appropriate **Solve** commands to find the endpoints of the solution intervals exactly. (Find equations for the curves, near each crossing point which do not involve the absolute value function).

5. a) Define g to be the function which gives as output the square of the input, plus the input, plus 7.

   b) Find $g[x+h]$ and then use the computer to simplify the expression $(g[x+h] - g[x])/h$.

6. Use the Squeeze Theorem to graphically show that $\lim_{x\to 0} \sqrt{x^3 + x^2} \sin \dfrac{\pi}{x} = 0$, i.e. graph functions f, g, and h (in the notation of the Squeeze Theorem).

   Investigate each of the limits in the following problems a) by graphing, b) numerically, and c) through *Mathematica*'s **Limit** command. After completing parts a)-c), describe in sentences the correct answer for the limit and which method gave that answer.

7. $\lim_{x\to 1} \dfrac{x^3 - 1}{\sqrt{x} - 1}$

8. $\lim_{x\to 0} \dfrac{6^x - 2^x}{x}$