Sec 7.2 Addition/Subtraction Formulas
Sec. 7.3 Double-Angle, Half-Angle and Product-Sum Formulas

## Previous Trig Identities

$$\csc(\theta)=\frac{1}{\sin(\theta)} \qquad \sec(\theta)=\frac{1}{\cos(\theta)} \qquad \cot(\theta)=\frac{1}{\tan(\theta)}$$

$$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$

$$\sin^2(t)+\cos^2(t)=1 \quad \tan^2(t)+1=\sec^2(t) \quad 1+\cot^2(t)=\csc^2(t)$$

$$\sin\left(\frac{\pi}{2}-u\right)=\cos(u) \quad \tan\left(\frac{\pi}{2}-u\right)=\cot(u) \quad \sec\left(\frac{\pi}{2}-u\right)=\csc(u)$$
$$\cos\left(\frac{\pi}{2}-u\right)=\sin(u) \quad \cot\left(\frac{\pi}{2}-u\right)=\tan(u) \quad \csc\left(\frac{\pi}{2}-u\right)=\sec(u)$$

**http://www.mnwest.edu/fileadmin/static/website/dmatthews/Geogebra/ShiftIdentity.html**

## Addition and Subtraction Formulas

$$\sin(s+t)=\sin(s)\cos(t)+\cos(s)\sin(t)$$
$$\sin(s-t)=\sin(s)\cos(t)-\cos(s)\sin(t)$$
$$\cos(s+t)=\cos(s)\cos(t)-\sin(s)\sin(t)$$
$$\cos(s-t)=\cos(s)\cos(t)+\sin(s)\sin(t)$$

$$\tan(s+t)=\frac{\tan(s)+\tan(t)}{1-\tan(s)\tan(t)}$$
$$\tan(s-t)=\frac{\tan(s)-\tan(t)}{1+\tan(s)\tan(t)}$$

For a visual proof of these see these web pages.
The book has an algebraic proof.

**http://www.ies.co.jp/math/java/trig/kahote/kahote.html**

**http://www.math.ohio-state.edu/~maharry/GeoGebra/SineCosineAdditionFormulas.html**

Consider the Graph of $\frac{1}{2}Sin(x)+\frac{\sqrt{3}}{2}Cos(x)$

**http://math.hws.edu/xFunctions/**

Draw a graph of the function with your calulator.

Can we find a nicer equation for it?

Consider the following:

$$ASin(x)+BCos(x)$$

If we find a number $\phi$ such that $Cos(\phi)=\frac{A}{\sqrt{A^2+B^2}}$ and $Sin(\phi)=\frac{B}{\sqrt{A^2+B^2}}$

So we have the following equation:

$$ASin(x) + BCos(x) = kSin(x + \phi)$$

where $k = \sqrt{A^2 + B^2}$

and $\phi$ such that $Cos(\phi) = \dfrac{A}{\sqrt{A^2 + B^2}}$ and $Sin(\phi) = \dfrac{B}{\sqrt{A^2 + B^2}}$

How could you find a formula for the Sine and Cosine of <u>twice</u> a known angle?

Suppose you know that for a certain angle, Sin(t) = 5/12.
How could you use the sum formulas to find the values of the Trig Functions for twice that angle?

## Double Angle Formulas

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(2x) = 1 - 2\sin^2(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Proof:

## Formulas for Squares of Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\tan^2(x) = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

Proof:

Note: What happens when you add the formulas for $\sin^2(x)$ and $\cos^2(x)$?

# Half Angle Formulas

$$\sin\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1-\cos(u)}{2}}$$

$$\cos\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{\sin(u)}{1+\cos(u)}$$

Proof:

## Applications:

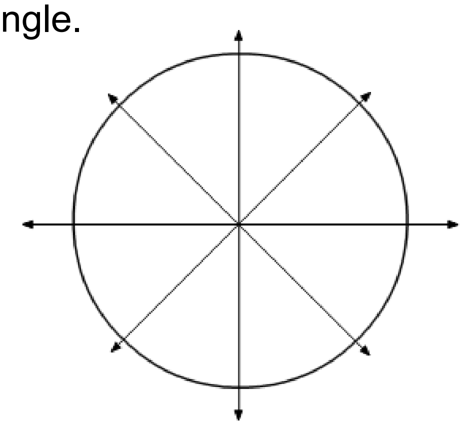
1. Find the exact value of

$$\sin\left(\frac{\pi}{12}\right)=$$

$$\cos\left(\frac{\pi}{12}\right)=$$

## Applications:

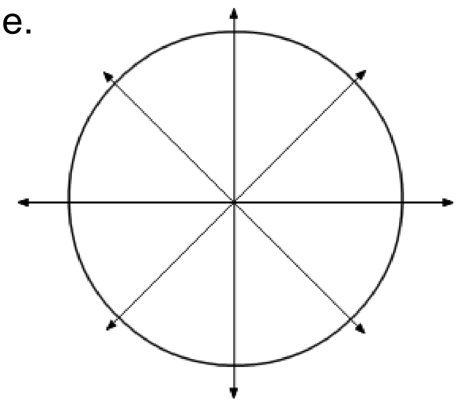
2. Suppose we know that for angle ‘t’, $\sin(t) = \frac{2}{5}$
and ‘t’ is in the $2^{nd}$ quadrant.
Find the values of ‘sin’ for double that angle.



Note we don't need to find the actual measure of the angle. We could though use "inverse sine". Do you get the same answer?

## Applications:

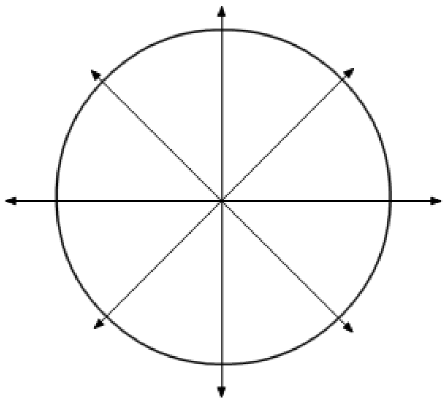
2. Suppose we know that for angle ‘t’, $\sin(t) = \frac{2}{5}$
and ‘t’ is in the $2^{nd}$ quadrant.
Find the value of ‘sin’ for <u>half</u> that angle.



Note we don't need to find the actual measure of the angle. We could though use "inverse sine". Do you get the same answer?

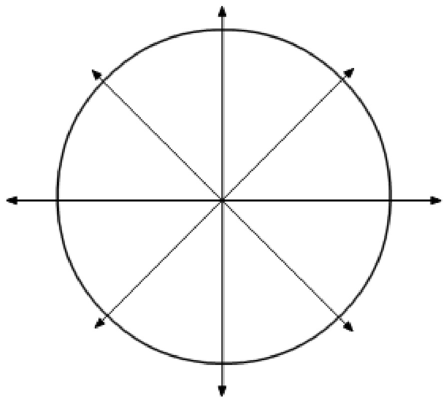
Applications:

4. Suppose we know that for angle ‘t’, $\cos(t) = -\frac{4}{5}$
and ‘t’ is in the 3rd quadrant.
Find the values of Sin(2t) and Cos(2t).



Applications:

4. Suppose we know that for angle ‘t’, $\cos(t) = -\frac{4}{5}$
and ‘t’ is in the 3rd quadrant.
Find the values of Sin(t/2) and Cos(t/2).



Make Up your own Trig Formulas:

What would be a "Triple Angle Formula"? Does it work?

Verify the identity $1 - \cos 2x = \tan x \sin 2x$.