

- 24.20.** Find all solutions on the interval $[0, 2\pi)$ for $\cos t - \sin 2t = 0$.

First use the double-angle formula for sines to obtain an equation involving only functions of t , then solve by factoring:

$$\begin{aligned}\cos t - \sin 2t &= 0 \\ \cos t - 2 \sin t \cos t &= 0 \\ \cos t(1 - 2 \sin t) &= 0 \\ \cos t = 0 \quad \text{or} \quad 1 - 2 \sin t &= 0\end{aligned}$$

The solutions of $\cos t = 0$ on the interval $[0, 2\pi)$ are $\pi/2$ and $3\pi/2$. The solutions of $1 - 2 \sin t = 0$, that is, $\sin t = 1/2$, on this interval, are $\pi/6$ and $5\pi/6$.

Solutions: $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

- 24.21.** Find all solutions on the interval $[0, 2\pi)$ for $\cos 5x - \cos 3x = 0$.

First use a sum-to-product formula to put the equation into the form $ab = 0$.

$$\begin{aligned}\cos 5x - \cos 3x &= 0 \\ -2 \sin\left(\frac{5x + 3x}{2}\right) \sin\left(\frac{5x - 3x}{2}\right) &= 0 \\ -2 \sin 4x \sin x &= 0 \\ \sin 4x \sin x &= 0 \\ \sin 4x = 0 \quad \text{or} \quad \sin x &= 0\end{aligned}$$

The solutions of $\sin 4x = 0$ on the interval $[0, 2\pi)$ are $0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2$, and $7\pi/4$. The solutions of $\sin x = 0$ on this interval are 0 and π , which have already been listed.

SUPPLEMENTARY PROBLEMS

- 24.22.** Derive the sum formulas for sine and tangent.

- 24.23.** Derive the cofunction formulas (a) for tangents and cotangents; (b) for secants and cosecants.

- 24.24.** Use sum or difference formulas to find exact values for (a) $\sin \frac{5\pi}{12}$; (b) $\cos 105^\circ$; (c) $\tan(-\frac{\pi}{12})$.

Ans. (a) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2} + \sqrt{6}}{4}$; (b) $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2} - \sqrt{6}}{4}$; (c) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ or $\sqrt{3} - 2$

- 24.25.** Given $\sin u = -\frac{2}{5}$, u in quadrant III, and $\cos v = \frac{3}{4}$, v in quadrant IV, find (a) $\sin(u + v)$; (b) $\cos(u - v)$; (c) $\tan(v - u)$.

Ans. (a) $\frac{-6 + 7\sqrt{3}}{20}$; (b) $\frac{-3\sqrt{21} + 2\sqrt{7}}{20}$; (c) $\frac{-6 - 7\sqrt{3}}{3\sqrt{21} - 2\sqrt{7}}$ or $\frac{-32\sqrt{21} + 75\sqrt{7}}{161}$

- 24.26.** Derive (a) the double-angle formula for tangents; (b) the third form of the half-angle formula for tangents.

- 24.27.** Given $\sec t = -3$, $\frac{\pi}{2} < t < \pi$, find (a) $\sin 2t$; (b) $\tan 2t$; (c) $\cos \frac{t}{2}$; (d) $\tan \frac{t}{2}$.

Ans. (a) $-\frac{4\sqrt{2}}{9}$; (b) $\frac{4\sqrt{2}}{7}$; (c) $\frac{1}{\sqrt{3}}$; (d) $\sqrt{2}$.

- 24.28.** Use a double-angle identity to derive an expression for (a) $\sin 4x$ in terms of $\sin x$ and $\cos x$; (b) $\cos 6u$ in terms of $\cos u$.

Ans. (a) $\sin 4x = 4 \sin x \cos x (1 - 2 \sin^2 x)$; (b) $\cos 6u = 32 \cos^6 u - 48 \cos^4 u + 18 \cos^2 u + 1$

- 24.29.** Use a half-angle identity to derive an expression in terms of cosines with exponent 1 for (a) $\sin^2 2t \cos^2 2t$; (b) $\sin^4 \frac{x}{2}$.

Ans. (a) $\frac{1 - \cos 8t}{8}$; (b) $\frac{3 - 4 \cos x + \cos 2x}{8}$

- 24.30.** Complete the derivations of the product-to-sum and sum-to-product formulas (see Problems 24.13 and 24.14).

- 24.31.** (a) Write $\sin 120\pi t + \sin 110\pi t$ as a product. (b) Write $\sin \frac{\pi n}{L} x \cos \frac{k\pi n}{L} t$ as a sum.

Ans. (a) $2 \sin 115\pi t \cos 5\pi t$; (b) $\frac{1}{2} \sin \frac{\pi n}{L} (x + kt) + \frac{1}{2} \sin \frac{\pi n}{L} (x - kt)$

- 24.32.** Verify that the following are identities: (a) $\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x} = \tan x$; (b) $\tan \frac{u}{2} = \csc u - \cot u$;

(c) $1 + \tan \alpha \tan \frac{\alpha}{2} = \sec \alpha$; (d) $\frac{\cos a - \cos b}{\sin a - \sin b} = -\tan \left(\frac{a + b}{2} \right)$

- 24.33.** Verify the reduction formulas:

(a) $\sin(n\pi + \theta) = (-1)^n \sin \theta$, for n any integer; (b) $\cos(n\pi + \theta) = (-1)^n \cos \theta$, for n any integer.

- 24.34.** If $f(x) = \cos x$, show that the difference quotient for $f(x)$ can be written as

$$\cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \frac{\sin h}{h}.$$

- 24.35.** Find all solutions on the interval $[0, 2\pi)$ for the following equations:

(a) $\sin 2\theta - \sin \theta = 0$; (b) $\cos x + \cos 3x = \cos 2x$.

Ans. (a) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$; (b) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{3}, \frac{7\pi}{4}$

- 24.36.** Find approximate values for all solutions on the interval $[0^\circ, 360^\circ)$ for $\cos x = 2 \cos 2x$.

Ans. $32.53^\circ, 126.38^\circ, 233.62^\circ, 327.47^\circ$