## MATHEMATICS OF FLIGHT: GLIDE SLOPE-II

Students will have a basic understanding of math applications used in flight. This includes the glide slope.
Students will solve a series of problems. (One in a series)

## LESSON PLAN

## Lesson Objective

The students will:

- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the glide slope using trigonometry.


## Goal

In this lesson, students will gain an understanding of common calculations performed by flight personnel.

## Gliders

A glider is a special kind of aircraft that has no engine. The Wright brothers perfected the design of the first airplane and gained piloting experience through a series of glider flights from 1900 to 1903. During World War II, gliders such as the WACO CG-4 were towed aloft by C-47 and C-46 aircraft then cut free to glide over many miles.

If a glider is in a steady (constant velocity and no acceleration) descent, it loses altitude as it travels. The glider's flight path is a simple straight line, shown in the figure above. The flight path intersects the ground at an angle a called the glide angle. If we know the distance flown and the altitude change, the glide angle can be calculated using trigonometry. The tangent tan of the glide angle a is equal to the change in height $\mathbf{h}$ divided by the distance flown $\mathbf{d}$ :

$$
\tan (a)=h / d
$$

d = distance flown
$\mathrm{h}=$ change in height
$\mathrm{a}=$ glide angle

h

Grade Level: High School
Common Core State Standards for Mathematics:
Algebra: Reasoning with Equations
Trigonometric Functions: Model periodic phenomena with trigonometric functions, prove and apply trigonometric identities

Technology Content Standards (from STL):
Technology and Society

## Materials Required:

- Paper
- Writing utensil
- Trigonometric Tables

There are three forces acting on the glider; lift, weight, and drag. The weight of the glider is given by the symbol " $\mathbf{W}$ " and is directed vertical, toward the center of the earth. The weight is then perpendicular to the horizontal red line drawn parallel to the ground and through the center of gravity.
The drag of the glider is designated by " $\mathbf{D}$ " and acts along the flight path opposing the motion.
Lift, designated "L," acts perpendicular to the flight path. Using some geometry theorems on angles, perpendicular lines, and parallel lines, we see the glide angle "a" also defines the angle between the lift and the vertical, and between the drag and the horizontal.

Assuming that the forces are balanced (no acceleration of the glider), we can write two vector component equations for the forces.

In the vertical direction, the weight $(\mathbf{W})$ is equal to the lift $(\mathbf{L})$ times the cosine $(\cos )$ of the glide angle (a) plus the drag (D) times the sine (sin) of the glide angle.

$$
\mathrm{L} * \cos (\mathrm{a})+\mathrm{D} * \sin (\mathrm{a})=\mathrm{W}
$$

In the horizontal direction, the lift ( $\mathbf{L}$ ) times the sine $(\sin )$ of the glide angle (a) equals the drag (D) times the cosine (cos) of the glide angle.

$$
\mathrm{L} * \sin (\mathrm{a})=\mathrm{D} * \cos (\mathrm{a})
$$

If we use algebra to re-arrange the horizontal force equation we find that the drag divided by the lift is equal to the sine of the glide angle divided by the cosine of the glide angle. This ratio of trigonometric functions is equal to the tangent of the angle.

$$
\mathrm{D} / \mathrm{L}=\sin (\mathrm{a}) / \cos (\mathrm{a})=\tan (\mathrm{a})
$$

We can use the drag equation and the lift equation to relate the glide angle to the drag coefficient (cd) and lift coefficient (cl) that the Wrights measured in their wind tunnel tests.

$$
\mathrm{D} / \mathrm{L}=\mathrm{cd} / \mathrm{cl}=\tan (\mathrm{a})
$$

## See student worksheet and presentation.

## Resources:

National Museum of the United States Air Force
Belcher, Diana. Education in Flight: A Teacher's Guide to the Mathematics of Flight. Department of the Air Force, 2007.

NASA Glenn Research Center
http://www.grc.nasa.gov/WWW/K-12/airplane/bga.html
National Air and Space Museum
http://airandspace.si.edu/exhibitions/wright-brothers/online/fly/1901/index.cfm

## MATHEMATICS OF FLIGHT: GLIDE SLOPE II

STUDENT WORKSHEET
NAME: $\qquad$

Compare the 1901 and 1902 Wright Gliders and determine the height for both.

| Glider | Weight | Distance | Glide Angle | Height |
| :--- | :--- | :--- | :--- | :--- |
| 1901 | 98 lbs | 300 feet | $9^{\circ}$ | $?$ |
| 1902 | 117 lbs | 500 feet | $7^{\circ}$ | $?$ |

$\sin (a)=B C / A B$
$\mathrm{AB}=$ distance flown
$\mathrm{a}=$ glide angle
$\mathrm{BC}=$ height


