Chapter 14A: Apportionment.

In this chapter, we will talk about the many ways and procedures of dividing up or apportioning a house size. We start with an example, then with definitions. This chapter will be using lots of decimals, so we will go by this convention: three digits after the decimal point. Also, when descriptive words larger and smaller are used, they are to describe state populations, not land areas. Don't look at me weird when I say Georgia is larger than Alaska.

Example 1: The Congressional Apportiontment.
In 1792, George Washington vetoed the Congressional Apportiontment Bill. Using the 1790 Census, the US population was at 3,615,920. The House of Representatives was to have 105 members, distributed amongst the 15 states. The US Constitution required that the house size *"shall be apportioned among the several states within this union according to their respective Numbers"*. Table 14.2 on p. 488 of your book has the chart and numbers for the bill. Each state's population is listed, as well as the state's **quota**.

Before we describe quota, we start by describing the **standard divisor**. This value is computed by taking the population count and dividing it by the house size. This is a constant value and will be independent of the state populations. In our example, the standard divisor is 3,615,920 ÷ 105 = 34,437.333. *This means that for every 34,437 people, there should be one house representative*.

The **state quota** is then computed by taking that state's population and dividing it by the standard divisor. Georgia's population at the time was 70,835, so its state quota was 70,835 ÷ 34,437 = 2.057.

Formula Recap:

$$\text{Standard Divisor} = \frac{\text{Total Population}}{\text{House Seats}} \qquad \text{State Quota} = \frac{\text{State Population}}{\text{Standard Divisor}}$$

The crux of the problem is now this: should we round up or down? Should Georgia receive just two seats or three? Rounding up seems quite unreasonable because its quota is barely above 2. However, rounding down would imply underrepresentation. Throughout this chapter, we will use the terminologies of "populations", "states", and "house sizes" for other applications not involving US politics. Here's another example:

Example #2: Creating Classes.
The High School Math Teacher A high school has one math teacher who teaches all geometry, precalculus, and calculus classes. She has time to teach a total of five sections. 100 students are enrolled as follows: 52 for geometry, 33 for pre-calculus, and 15 for calculus. The population here is the 100 students. The "states" are the three math classes, and the "house size" is the five sections. The standard divisor is 100 ÷ 5 = 20.

The quotas can then be computed. They are as follows: Geometry: 2.600, Pre-Calc: 1.650, Calculus: 0.750. A simple rounding scheme would then apportion 3 classes of Geometry, 2 of Precalculus, and 1 of Calculus. This of course is impossible because we only have five classes that we can assign. The problem now is this: which class should be rounded down?

Example #3: Field Hockey Coach.
A field hockey team finished their season with a record of 18-4-1 (wins-losses-ties). Their coach wants to create a poster that reflects this, but he wants to simplify it and uses whole number percentages to reflect their season. The population here is the 23 games, the states are wins, losses, and ties, and the house size is 100, or the percentages that are to be broken down amongst the "states". The standard divisor is 23 ÷ 100 = 0.230. The quotas are as follows: Wins: 78.261, Losses: 17.391, and Ties: 4.348.

Apportionment Methods
We will discuss four apportionment methods this chapter: The Hamilton Method, the Jefferson Method, the Webster Method, and the Hill-Huntington Method. All four of these have been used at some point in US history to allocate representation in the House of Representatives.

Method #1: The Hamilton Method.
Alexander Hamilton, a founding father and the first US Secretary of the Treasury, was the creator of the Congressional Apportionment bill that George Washington vetoed. With the Hamilton Method, each state receives either its **lower quota**, $\lfloor q \rfloor$, which is the state quota rounded down, or its **upper quota**, $\lceil q \rceil$, which is the state quota rounded up. States that receive their upper quotas are those whose quotas have the largest decimal parts.

The Hamilton Method essentially uses three steps:

- Compute the standard divisor and each state's quota
- Round each state's quota down and add the result.
- Take the remaining seats and apportion them to the states with the largest decimal part.

Example #4: Math Teacher
Apply Hamilton's Method to the math teacher in Example #2. The pertinent information are as follows:

Math Teacher: House size = 5.

Class	Population	State Quota
Geometry	52	2.600
Pre-Calculus	33	1.650
Calculus	15	0.750

Example #5: Field Hockey Coach.
Apply Hamilton's Method to the Field Hockey coach in Example #3. The pertinent information are as follows:

Creating a Poster: House size = 100.

Category	Population	State Quota
Wins	18	78.261
Losses	4	17.391
Ties	1	4.348

Properties of Hamilton's Method of Apportionment:
When using Hamilton's Method of Apportionment, you have the following properties:

- Every state is guaranteed to get at least their lower quota. A state with a state quota of 7.453 will have at least seven seats.
- If Hamilton's Method underfills the house, a state can only receive one additional seat.
- From the two statements above, a state will receive either its lower quota or upper quota.

The US Congress eventually adopted Hamilton's method in 1850 and it remained in use until 1900. During its 50 years of usage, it revealed the *Alabama Paradox*.

As the nation's population grew, so too did the house size. By 1873 the house size was at 292. In 1881, to prepare for the 1883 reapportionment, the US Census Bureau supplied congress with a table of congressional apportionments for a range of different house sizes from 275 to 350. The tables revealed a strange phenomenon:

House Size = 299

State	Quota	App.
Alabama	7.646	8
Illinois	18.640	18
Texas	9.640	10

House Size = 300

State	Quota	App.
Alabama	7.671	7
Illinois	18.702	19
Texas	9.672	10

In the table for 299 house seats, Alabama was the last state to receive its upper quota. Illinois and Texas both received their lower quota. However, with 300 seats, both Illinois and Texas surpassed Alabama's quota. This stems from a mathematical fact that as the house size increases, the larger state populations will experience a higher quota increase.

This is called the **Alabama Paradox**: *A situation where a state loses a seat as a result of an increase in the house size.* This is what George Washington warned about 90 years earlier: this fractional part apportionment does not take into account state population sizes and generally under-represents the larger states.