Physics of Stellar Interiors

Gravitational Force Pulls on Everything. In Stars, there must be some Pressure to balance Gravity.

Consider a cylinder of mass, dm, located a distance r from center of a spherical star.

Top of the cylinder has an area, A, and it has a height, dr. Net force on the Cylinder is:

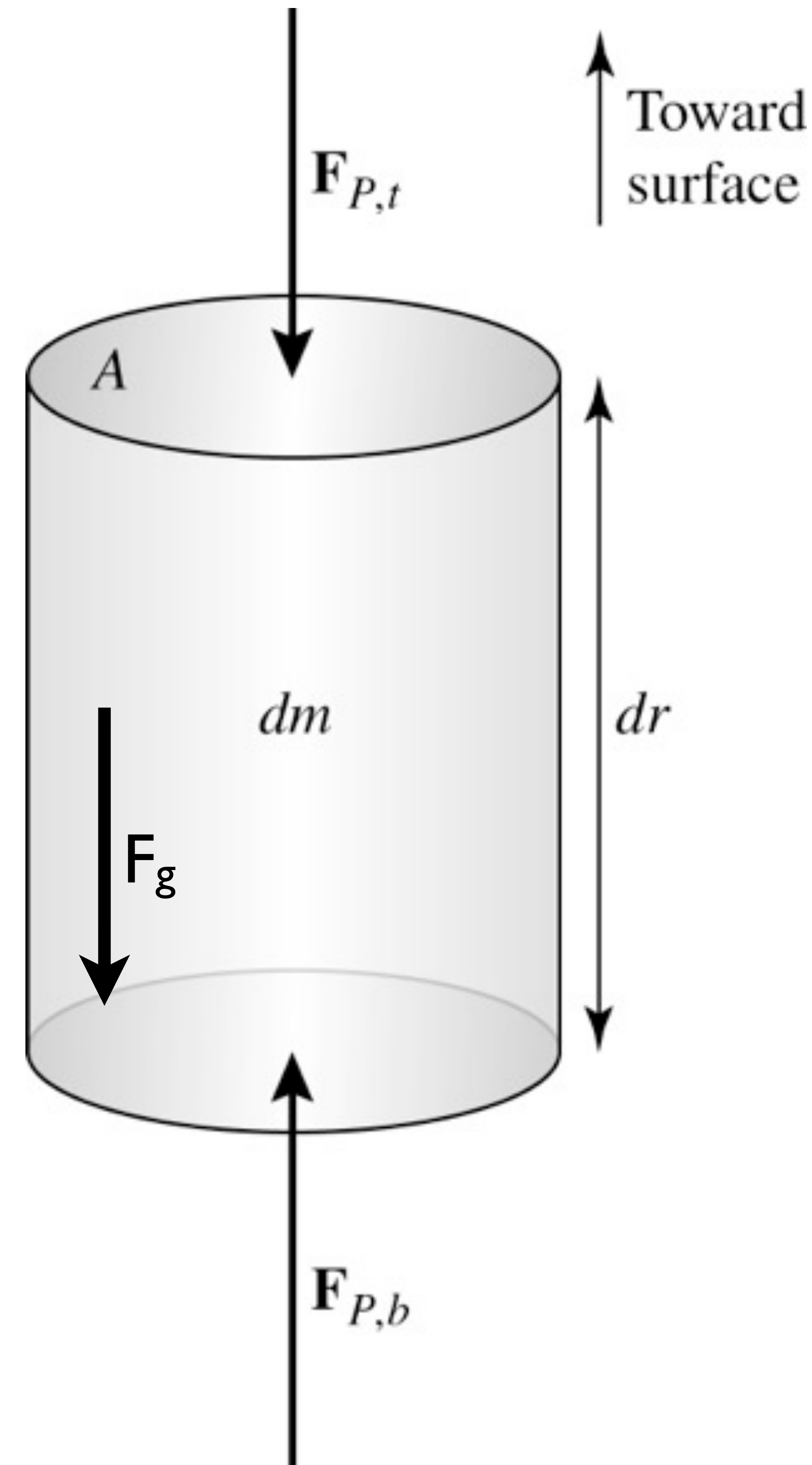
$$dm\frac{d^2r}{dt^2} = F_g + F_{P,t} + F_{P,b}$$

Write $F_{P,t}$ in terms of $F_{P,b}$ and extra force dF_P

$$F_{P,t} = -(F_{P,b} + dF_P)$$

Substitution Gives:

$$dm\frac{d^2r}{dt^2} = F_g - dF_P$$



Physics of Stellar Interiors

$$dm\frac{d^2r}{dt^2} = F_g - dF_P$$

The gravitational force is: $F_g = - GM_r\, dm / r^2$.

The Pressure is $P = F / A$, or $dF_P = A\, dP$. This Yields:

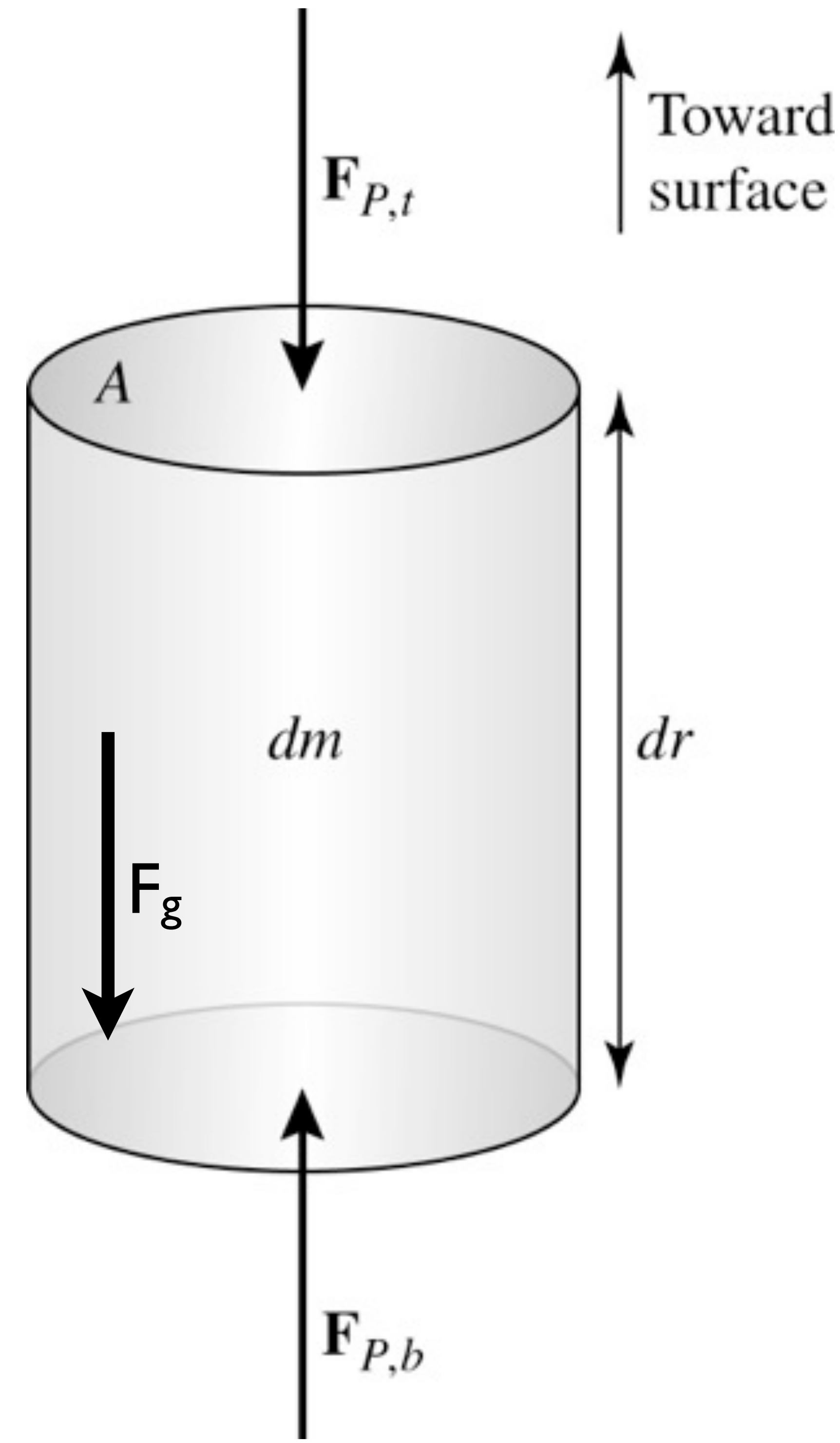
$$dm\frac{d^2r}{dt^2} = -G\frac{M_r dm}{r^2} - A\,dP$$

Rewrite dm in terms of the density, ρ, which gives $dm = \rho A\, dr$.

$$\rho A dr\frac{d^2r}{dt^2} = -G\frac{M_r \rho\, A\, dr}{r^2} - A\,dP$$

Divide by the Area, A, and assume star is static ($d^2r/dt^2 = 0$), and rearrange terms:

$$\boxed{\frac{dP}{dr} = -G\frac{M_r \rho}{r^2} = -\rho\, g}$$



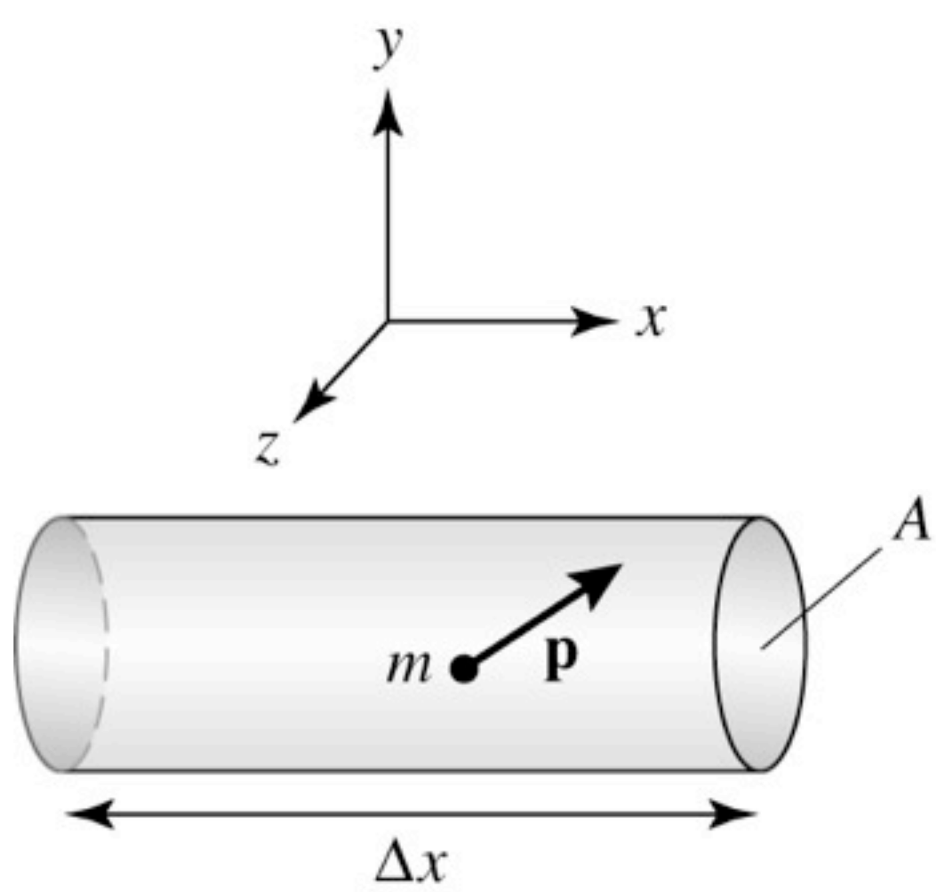
Hydrostatic Equilibrium

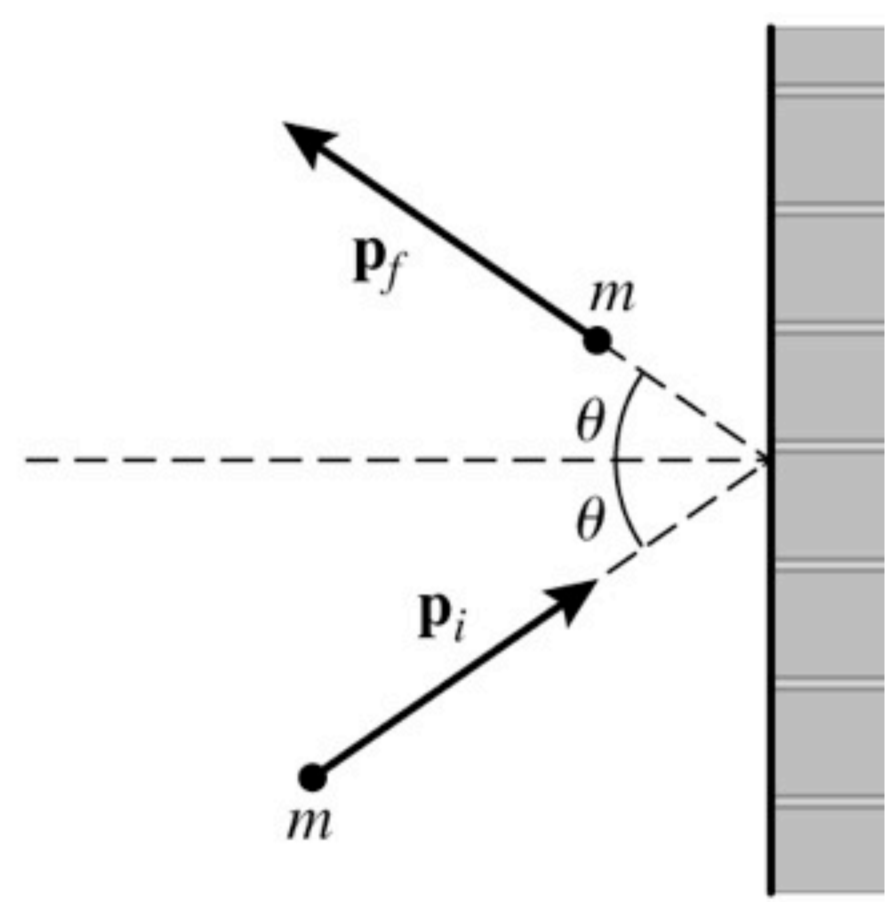
Physics of Stellar Interiors

Where does the Pressure come from ?

Need to derive the equation of state. For example, pressure equation of state for the "ideal" gas law is $PV = N\,k\,T$.

Consider another cylinder of length Δx and area A. Each gas particle has mass m and interacts via collisions only. The impulse ($\mathbf{f}\,\Delta t$) is the negative of the change in momentum, $\mathbf{f}\,\Delta t = -\Delta\mathbf{p} = 2p_x\,\mathbf{i}$.





Physics of Stellar Interiors

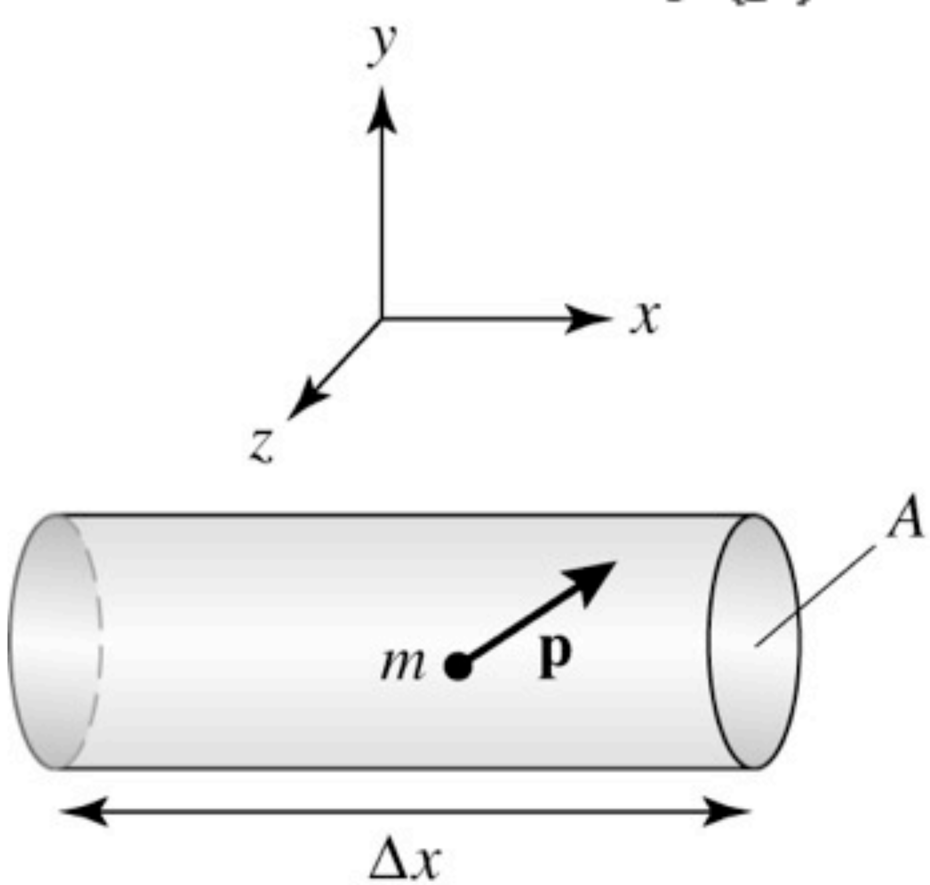
$\mathbf{f}\,\Delta t = -\Delta\mathbf{p} = 2p_x\,\mathbf{i}$

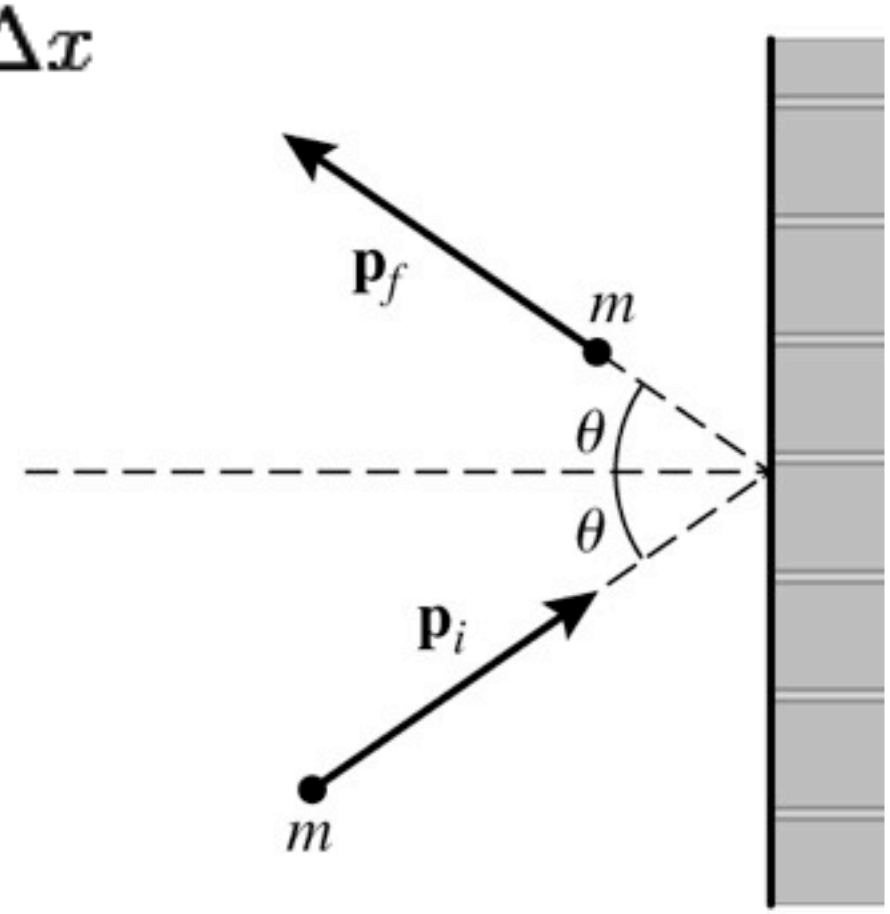
Time interval between "collisions" is $\Delta t = 2\,\Delta x / v_x$. This produces an effective force of $f = 2p_x / \Delta t = p_x v_x / \Delta x. = m\,v_x^2 / \Delta x$.

Note: $v^2 = v_x^2 + v_y^2 + v_z^2$. For large volumes, the average velocity in each direction should be equal, $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = v^2 / 3$.

Substituting, $pv/3$ for $p_x v_x$ gives force per average particle:

$$f(p) = \frac{1}{3}\frac{pv}{\Delta x}$$





Physics of Stellar Interiors

$$f(p) = \frac{1}{3}\frac{pv}{\Delta x}$$

Usually particles have a range of momenta, total number of particles is

$$N = \int_0^\infty N_p \, dp$$

Then total force from all particles is

$$dF(p) = f(p)\, N_p \, dp = \frac{1}{3}\frac{N_p}{\Delta x} p\, v\, dp.$$

Integrating over all momenta gives

$$F = \frac{1}{3}\int_0^\infty \frac{N_p}{\Delta x} pv \, dp$$

Divide both sides by the Area, A. Rewrite number of particles as number per Volume, $n_P = N_P / \Delta V$.

$$\boxed{P = \frac{1}{3}\int_0^\infty n_p \, pv \, dp}$$

Pressure Integral

Physics of Stellar Interiors

$$P = \frac{1}{3}\int_0^\infty n_p\, pv\, dp$$ Rewrite p = mv, which gives $$P = \frac{1}{3}\int_0^\infty m\, n_v v^2\, dp$$

$n_v\, dv$ for an ideal gas is the Maxwell Boltzmann distribution.
Plugging in an evaluating integral gives, $P_g = nkT$, where $n = \int_0^\infty n_v dv$

Express the ideal gas law using the average mass density of particles of different masses, where m is the average mass of a gas particle.

$$n = \frac{\rho}{\bar{m}}$$ Pressure becomes: $$P_g = \frac{\rho k T}{\bar{m}}$$

Define **mean molecular weight**, $$\mu \equiv \frac{\bar{m}}{m_H}$$

Mean molecular weight is the average mass of a free particle in the gas, in units of the mass of hydrogen.

For example, for a gas that is 10% He and 90% H by number, then
$\mu \approx (0.1 \times 4m_p + 0.9 \times m_p) / m_H = 1.3$.

Physics of Stellar Interiors

Therefore, rewrite gas law as $\boxed{P_g = \frac{\rho k T}{\mu m_H}}$

Mean molecular weight depends on the composition (% of different atoms) and ionization state because free electrons must be counted. Complete treatment needs to use Saha Equation to account for everything.

We will consider two cases (1) gas all neutral and (2) gas all ionized.

(1) NEUTRAL GAS: $$\bar{m}_n = \frac{\sum_j N_j m_j}{\sum_j N_j}$$

m_j and N_j are the mass and # of atoms of type j.

divide by m_H, let $A_j = m_j / m_H$: $$\mu_n = \frac{\sum_j N_j A_j}{\sum_j N_j}$$

Physics of Stellar Interiors

(1) NEUTRAL GAS: $$\mu_n = \frac{\sum_j N_j A_j}{\sum_j N_j}$$

(2) IONIZED GAS: $$\mu_i \simeq \frac{\sum_j N_j A_j}{\sum_j N_j (1 + z_j)}$$

Where z_j accounts for the nuclei and # of free electrons.

Invert expressions ($1/\mu$), in terms of mass fractions

$$\frac{1}{\mu_n m_H} = \frac{\sum_j N_j}{\sum_j N_j m_j} = \frac{\text{total \# of particles}}{\text{total mass of gas}}$$

$$= \sum_j \frac{\text{\# of particles from j}}{\text{mass of particles from j}} \times \frac{\text{mass of particles from j}}{\text{total mass of gas}}$$

$$= \sum_j \frac{N_j}{N_j A_j m_H} X_j = \sum_j \frac{X_j}{A_j m_H}$$

X_j is the mass fraction of atoms of type j.

Physics of Stellar Interiors

Multiply both sides by m_H and we get for neutral gas : $$\frac{1}{\mu_n} = \sum_j \frac{X_j}{A_j}$$

Now define **mass fraction**, the fractional abundance (by mass) of an element. Fraction of hydrogen is X. Fraction of Helium is Y, Fraction of everything else is Z.

X = (total mass of H) / (total Mass)

Y = (total mass of He) / (total Mass)

Z = (total mass of Li through U) / (total Mass)

And X + Y + Z = 1

For neutral gas, we have: $$\frac{1}{\mu_n} \simeq X + \frac{1}{4}Y + \langle 1/A \rangle_n Z$$

Physics of Stellar Interiors

For Ionized gas, include electron in calculation. Ionized hydrogen gives 1 electron, He gives 2, etc. This gives (where z_j is the atomic # of element j).

$$\frac{1}{\mu_i} = \sum_j \frac{1+z_j}{A_j} X_j$$

Including H and He explicitly and averaging over everything else:

$$\frac{1}{\mu_i} \simeq 2X + \frac{3}{4}Y + \langle (1+z)/A \rangle_i Z$$

For elements much heavier than He, $1+z_j \approx z_j$, because $z_j >> 1$. Also holds that $A_j \approx 2z_j$ because sufficiently massive atoms have roughly the same number of protons and neutrons in their nuclei. Therefore,

$$\langle (1+z)/A \rangle_i \sim \frac{1}{2}$$

For X=0.70, Y=0.28, and Z=0.02 (the mass fractions of the Sun), we get μ_n = 1.30 and μ_i = 0.62.

Average Kinetic Energy Per Particle

We can derive the average kinetic energy of a particle from the ideal gas law. We had previously,

$$nkT = \frac{1}{3}\int_0^\infty m\, n_v v^2\, dv$$

Rewriting, we get

$$\frac{1}{n}\int_0^\infty n_v v^2\, dv = \frac{3kT}{m}$$

The left hand side is the average velocity squared given the Maxwell-Boltzmann distribution, so this integral is just <nv^2>. This gives:

$$\overline{v^2} = \frac{3kT}{m}$$

Which implies: $\frac{1}{2}mv^2 = \frac{3}{2}kT.$

Note that the factor of 3 came about because we averaged over 3 coordinate directions (3 degrees of freedom). The average kinetic energy of a particle is kT/2 for each degree of freedom in the system.

Fermi-Dirac and Bose-Einstein Statistics

So far we have ignored the effects of Quantum Mechanics. In some cases we must consider them.

Fermi-Dirac statistics dictate the properties of 1/2 integer spin particles, such as electrons, protons, and neutrons. These are all **fermions**.

Bose-Einstein Statistics dictate properties of integer spin particles (such as photons, **bosons**).

Limits at low densities give classical results that we have so-far derived.

Radiation Pressure

Photons carry momentum, p = hν/c. They are capable of delivering the "impluse" to gas particles (they are absorbed, reflected and scattered, which transfers momentum into the gas atoms/ions).

Rewrite our Pressure Integral in terms of photon momenta. Using $n_p\, dp = n_\nu\, d\nu$ in our pressure integral, we have

$$P_{\rm rad} = \frac{1}{3}\int_0^\infty h\nu\, n_\nu\, d\nu$$

One can solve this using Bose-Einstein statistics for Photons (which are Bosons), or realize that ($h\nu\ n_\nu$) is the energy density distribution of photons. Or,

$$P_{\rm rad} = \frac{1}{3}\int_0^\infty u_\nu\, d\nu$$

For Blackbody radiation, we can solve for P_{rad} using the energy density formula we derived way back in week 3.

$$u_\nu\, d\nu = \frac{8\pi h\nu^3/c^3}{e^{h\nu/kT}-1}\, d\nu$$

which gives

(where a=4π/c)

$$P_{\rm rad} = \frac{1}{3}aT^4$$

Total Pressure

Now we can combine our Pressure terms for stars, combining the thermal pressure for particles of a temperature T and the contribution from the photon pressure.

$$\boxed{P_t = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4}$$

Example: Pressure and Temperature in Sun

We can estimate the pressure and temperature at the center of the Sun.
Use $M_r = 1\ M_\odot$, $r = 1\ R_\odot$, $\rho = \rho_\odot = 1410$ kg m^{-3} (average solar density).
Assume also that the surface pressure is $P_S = 0$.

$$\frac{dP}{dr} \approx (P_S - P_C)/(R_S - 0) = -P_C / R_\odot$$

Inserting this into our hydrostatic equilibrium equation:

$$\frac{dP}{dr} = -G\frac{M_r\rho}{r^2}$$

$$P_C \approx G M_\odot \rho_\odot / R_\odot = 2.7 \times 10^{14} \text{ N m}^{-2}$$

For more accurate value, integrate the hydrostatic equilibrium equation:

$$\int_{P_S}^{P_C} dP = P_C = -\int_{R_S}^{R_C} \frac{GM_r\rho}{r^2}\, dr$$

This gives a central presure of $P_C = 2.34 \times 10^{16}$ N m^{-2}.

1 atm = 10^5 N m^{-2}, so center of the Sun has a pressure of 2.3×10^{11} atm !

Example: Pressure and Temperature in Sun

For Temperature, neglect the radiation pressure and we have:

$$P_g = \frac{\rho k T}{\mu m_H}$$

Rewriting, we have $T_C = \dfrac{P_C \mu m_H}{\rho k}$

Using $\rho_\odot$ = 1410 kg m^{-3} (average solar density) and μ_i = 0.62, and the estimated value for the pressure (last slide) we have $T_C \approx 1.44 \times 10^7$ K.

For fun, you can calculate the radiation pressure, which is

$P_{rad} = (1/3)\, a T^4 = 1.53 \times 10^{13}$ N m^{-2},

or 0.065% that of the gas pressure.

Nature of Stars

Hydrogen makes up 70% of stars ($X \sim 0.7$) and heavy metals a small fraction ($0 < Z < 0.03$)

Assuming that a star forms from a gas cloud that is homogeneous in its heavy metal distribution (a safe assumption), then all Stars should start off using the pp chain or CNO cycle to convert H to He.

During nucleosynthesis, surface of the star is not completely static, *The observational characteristic of the star must change as a consequence of the central nuclear reactions.*

But, these changes are slow ($\sim 10^6$-10^9 yrs), and so are the evolutionary stages of stars.

Recall that: $P_C \approx G\, M\rho / R$ and $T_C = \dfrac{P_C \mu m_H}{\rho k}$

Therefore, as the mass increases, so does the central pressure and temperature.

Nature of Stars

Because nuclear reaction rates depend on temperature, the dominate processes depend on mass. In lower mass stars, the pp-chain dominates. In higher-mass stars (with enough heavy metals), the CNO process will dominate.

For *very* low mass stars, the central temperature will diminish to the point where no nuclear fusion occurs. For a star with Solar compositions this is 0.072 $M_\odot$.

For *very* high mass stars, (>90 $M_\odot$), the radiation pressure can create thermal oscillations that produce variations in their luminosity on 8 hour timescales.

$$\boxed{P_t = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4}$$

Nature of Stars

Very massive stars have high luminosities resulting from very high central temperatures. Radiation pressure can dominate ! Rewrite the pressure gradient:

$$dP/dr \approx -(\kappa\rho/c)\, L/(4\pi r^2)$$

Where κ is a constant (opacity) that measures how readily the material (gas) absorbs the light.

Hydrostatic equilibrium requires:

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$

Combining these equations and solving for L gives the maximum luminosity for a star such that it can remain in hydrostatic equilibrium:

$$L_{edd} = [(4\pi Gc)/\kappa]\, M$$

This is the Eddington Limit (named for Arthur Eddington). It appears in many astrophysical applications.

Arthur Eddington
(1882-1944)

Nature of Stars

This is the Eddington Limit (named for Arthur Eddington).
It appears in many astrophysical applications.

$$L_{edd} = [(4\pi Gc)/\kappa\ ]\ M$$

For high mass stars (highest temperatures, luminosities), $T \sim 50{,}000$ K. Most hydrogen is ionized in these stars' atmospheres.

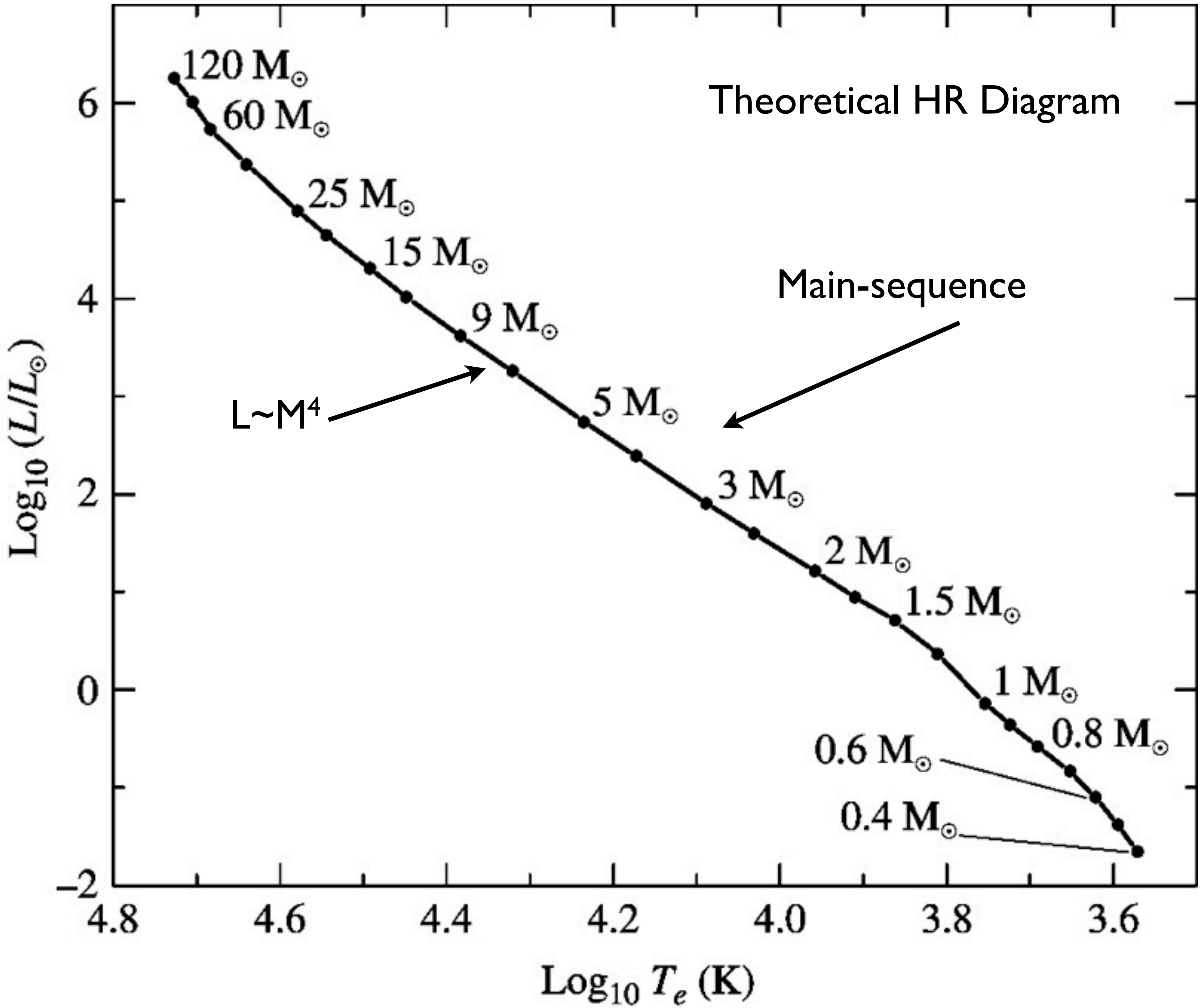
Opacity comes from interactions between photons and electrons.

$\kappa = 0.02(1+X)\ m^2\ kg^{-1}$, where X is the hydrogen mass fraction.

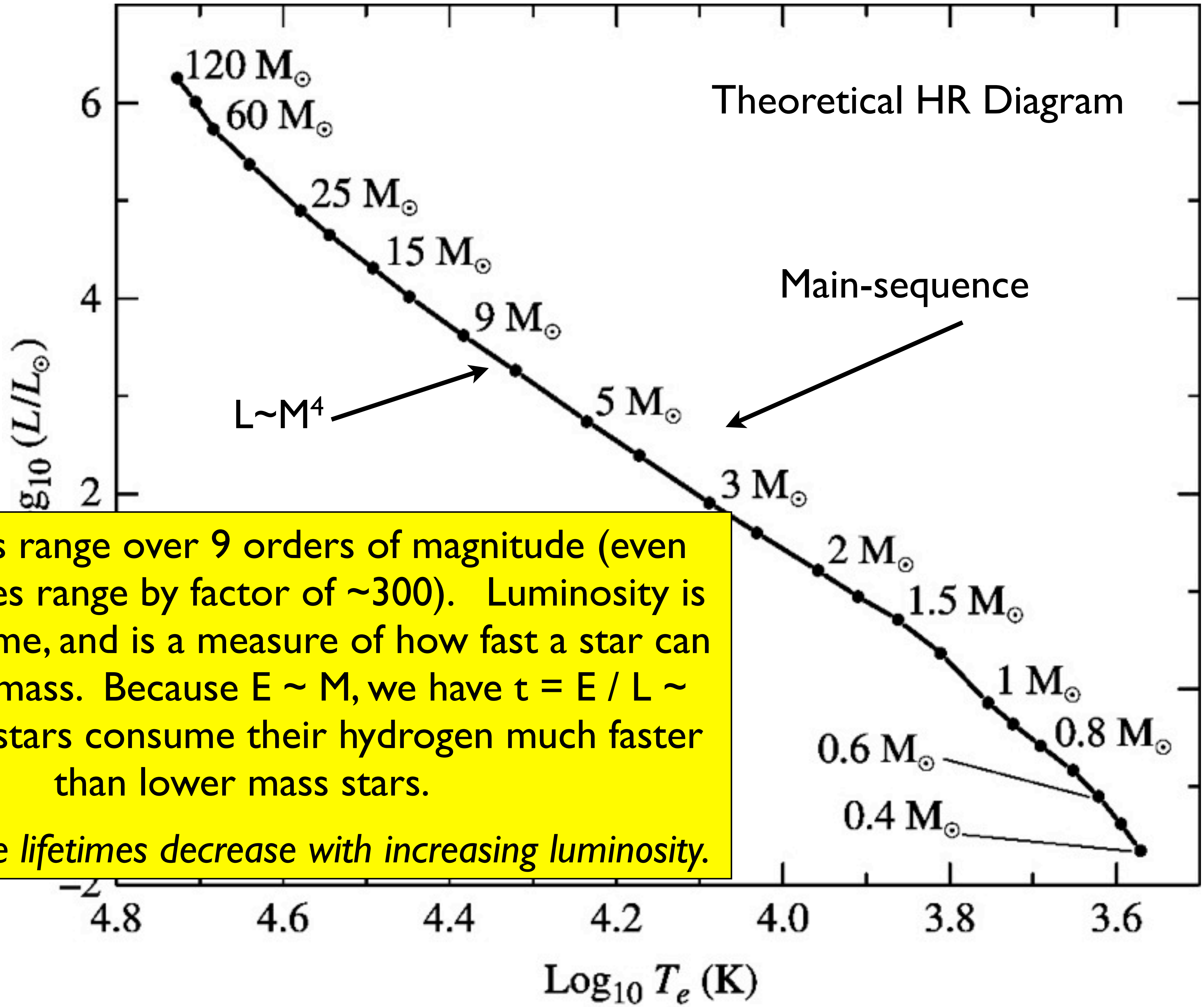
For a 90 $M_\odot$ we find that $L_{edd} = 3.5 \times 10^6\ L_\odot$.

The expected luminosity for such a star's temperature is $L \sim 10^6\ L_\odot$, 3x less than the Eddington limit.

Nature of Stars



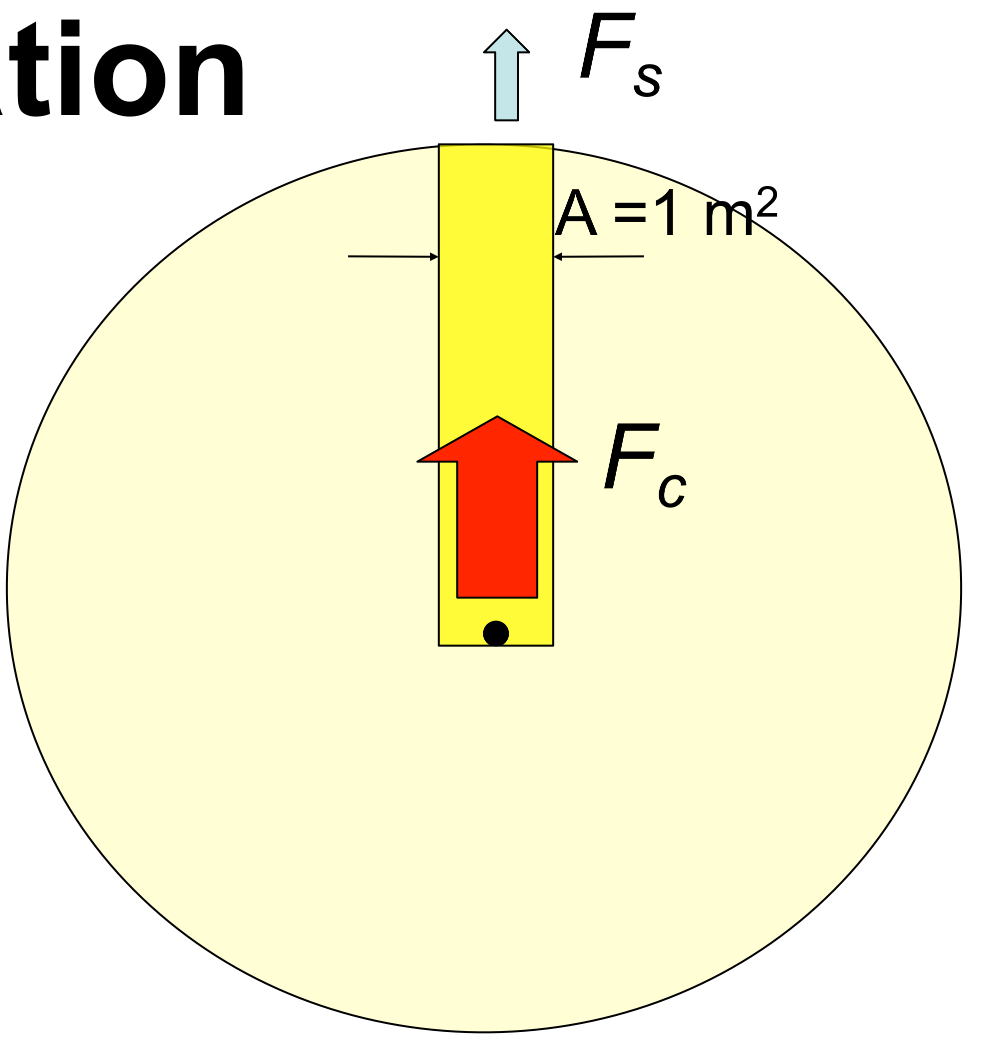
Nature of Stars



Luminosities range over 9 orders of magnitude (even though masses range by factor of ~300). Luminosity is energy per time, and is a measure of how fast a star can burn off its mass. Because $E \sim M$, we have $t = E / L \sim M^{-3}$. Massive stars consume their hydrogen much faster than lower mass stars.

Main-sequence lifetimes decrease with increasing luminosity.

Mass-luminosity relation



Flux from the center:

$$F_c = \sigma T_c^4$$

Flux from the surface:

$$F_s = \sigma T_s^4 = \frac{L}{4\pi R^2}$$

$$F_s = \frac{F_c}{\tau\,(\mathbf{opacity})}$$

$$\boxed{T_c = \frac{\mu G M}{k R} \propto \frac{M}{R}}$$

$$\tau = \alpha\rho R$$

$$L \propto R^2 F_s \propto R^2 \frac{F_c}{\tau} \propto \frac{R^2}{\alpha\rho R}\left(\frac{M}{R}\right)^4 \propto M^3$$

$$\text{Lifetime} = \frac{\text{Amount of hydrogen fuel}}{\text{Rate of energy loss}}$$

Lifetime T ~ M/L ~ 1/M^{p-1} = 1/M$^{2.5}$; p ~ 3.5

M = 4M$_{\odot}$; $\quad \frac{T}{T_{sun}} = \left(\frac{M_{sun}}{M}\right)^{2.5} = \frac{1}{32} \qquad \frac{L}{L_{sun}} = \left(\frac{M}{M_{sun}}\right)^{3.5} = 4^{3.5} = 128$

star mass (solar masses)	time (years)	Spectral type
60	3 million	O3
30	11 million	O7
10	32 million	B4
3	370 million	A5
1.5	3 billion	F5
1	10 billion	G2 (Sun)
0.1	1000's billions	M7

TABLE 9-2

Main-Sequence Stars

Spectral Type	Mass (Sun = 1)	Luminosity (Sun = 1)	Years on Main Sequence
O5	40	405,000	1×10^6
B0	15	13,000	11×10^6
A0	3.5	80	440×10^6
F0	1.7	6.4	3×10^9
G0	1.1	1.4	8×10^9
K0	0.8	0.46	17×10^9
M0	0.5	0.08	56×10^9

What defines an internal structure?

Central temperature $T_c \approx 1.5 \times 10^7$ K

Surface temperature $T_c \approx 5800$ K

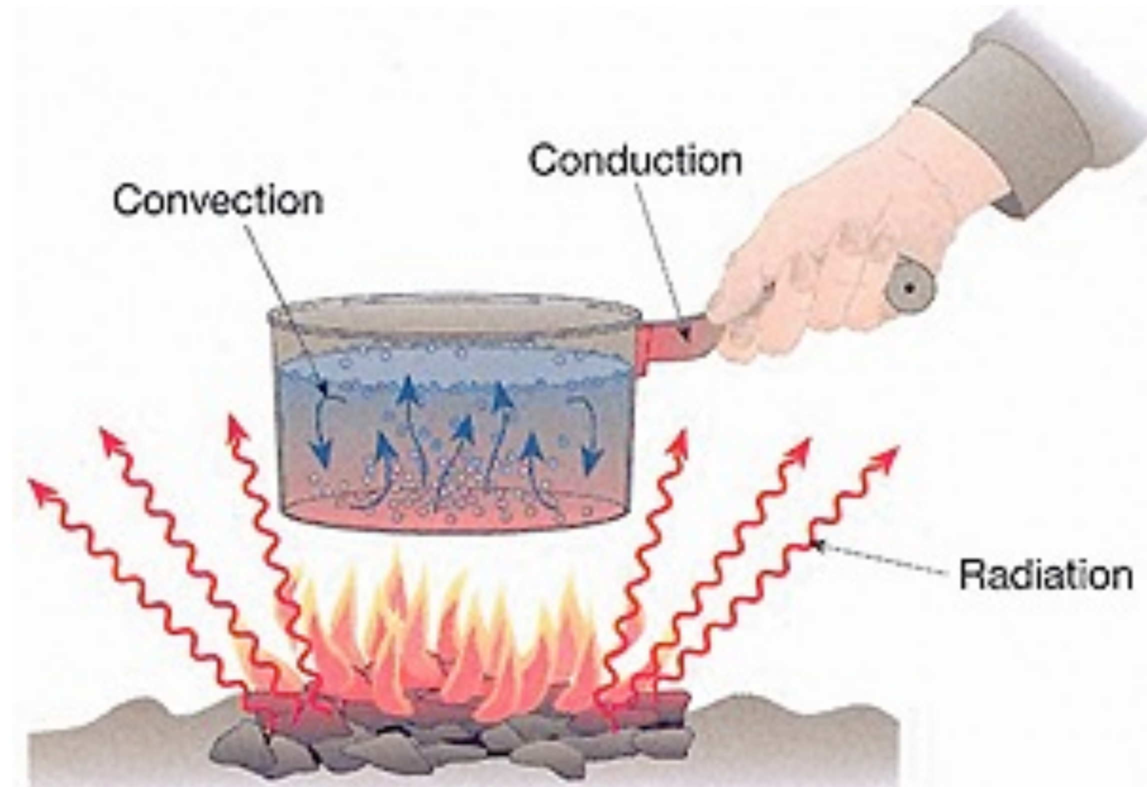
Energy generated in the sun's center must be transported outward.

Heat transfer from the center to the surface!

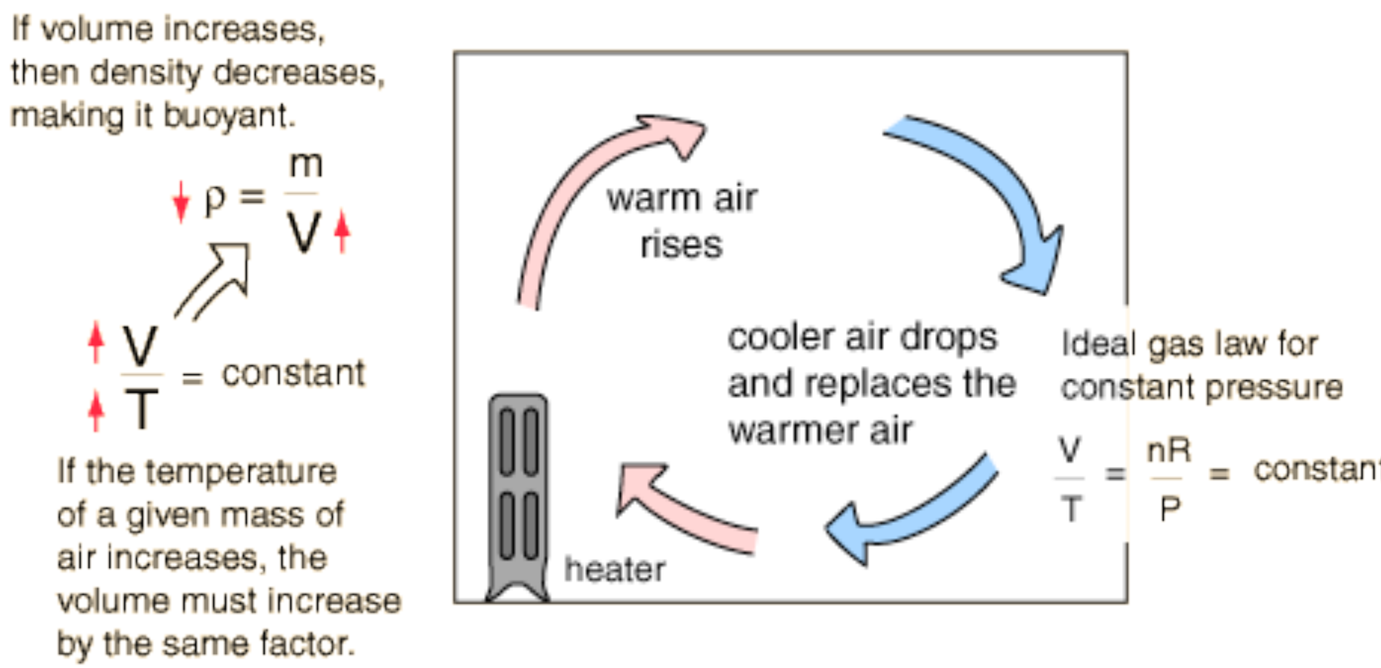
Heat transfer determines the internal structure

Heat transfer mechanisms

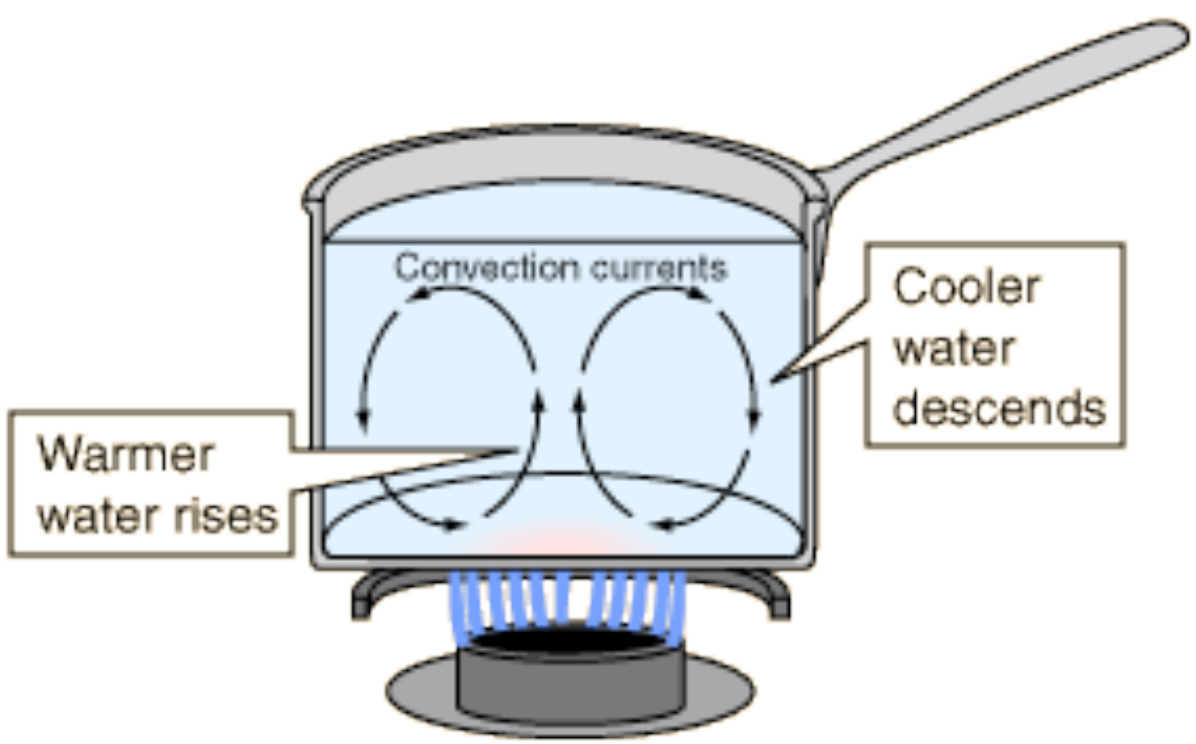
- Conduction
- Convection
- Radiation



Convection



If volume increases, then density decreases, making it buoyant.
$\rho = \frac{m}{V}$
$\frac{V}{T}$ = constant
If the temperature of a given mass of air increases, the volume must increase by the same factor.
warm air rises
cooler air drops and replaces the warmer air
heater
Ideal gas law for constant pressure
$\frac{V}{T} = \frac{nR}{P}$ = constant



Convection currents
Cooler water descends
Warmer water rises

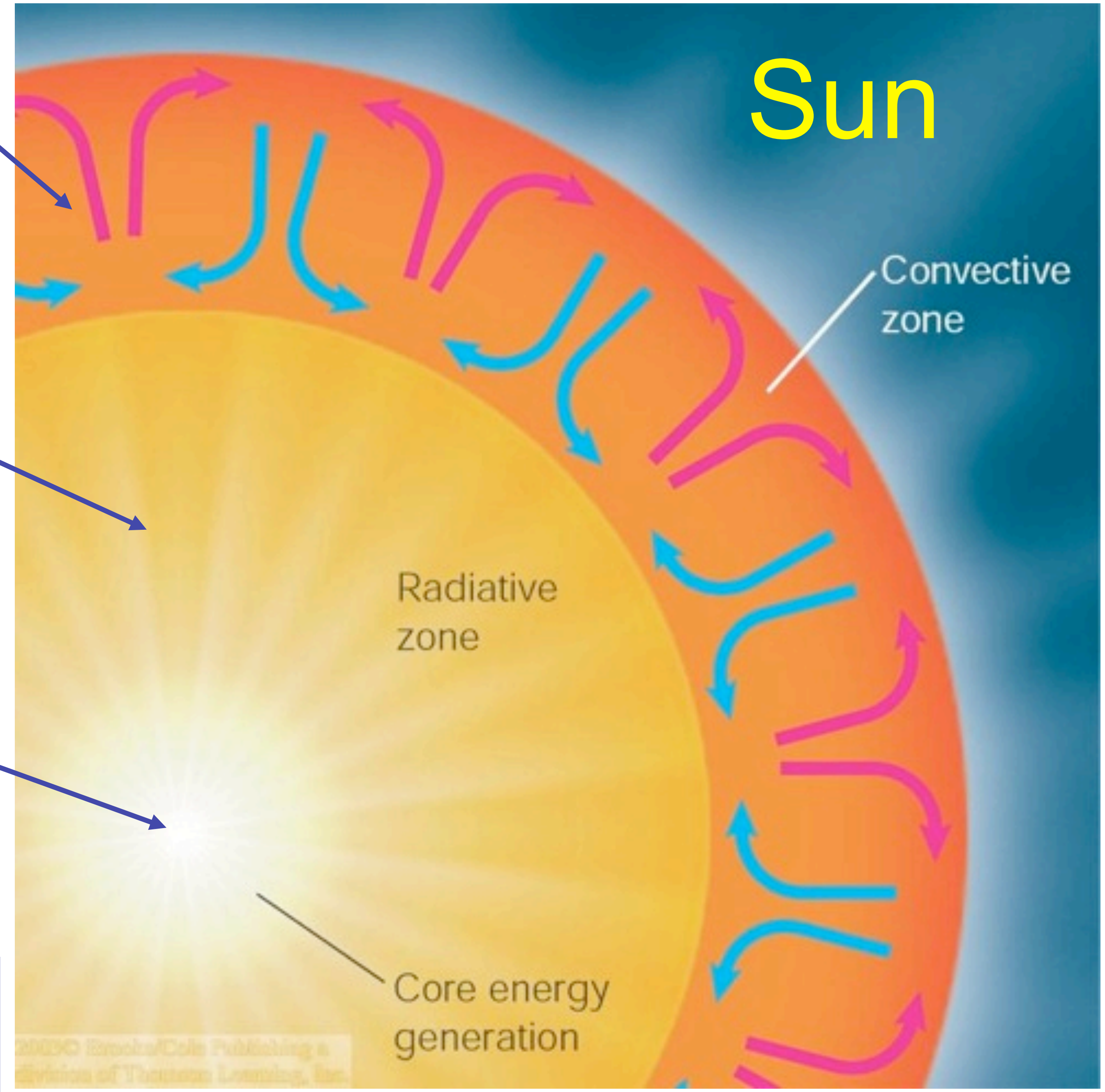
Structure: sun-like stars

Flow of energy

Energy transport via convection

Energy transport via radiation

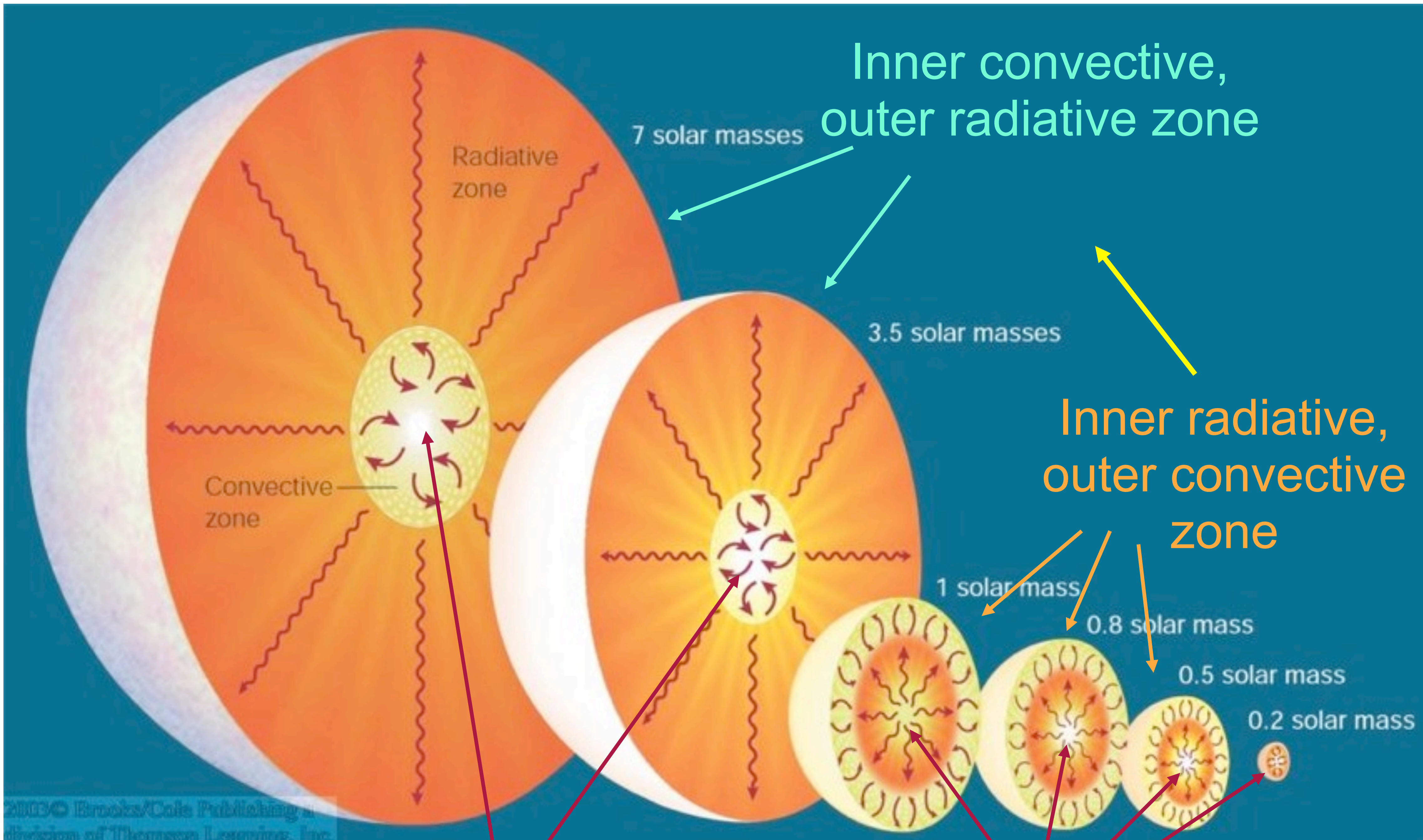
Energy generation via nuclear fusion



Temperature, density and pressure decreasing

Basically the same structure for all stars with approx. 1 solar mass or less.

Structure: all stars



CNO cycle dominant PP chain dominant

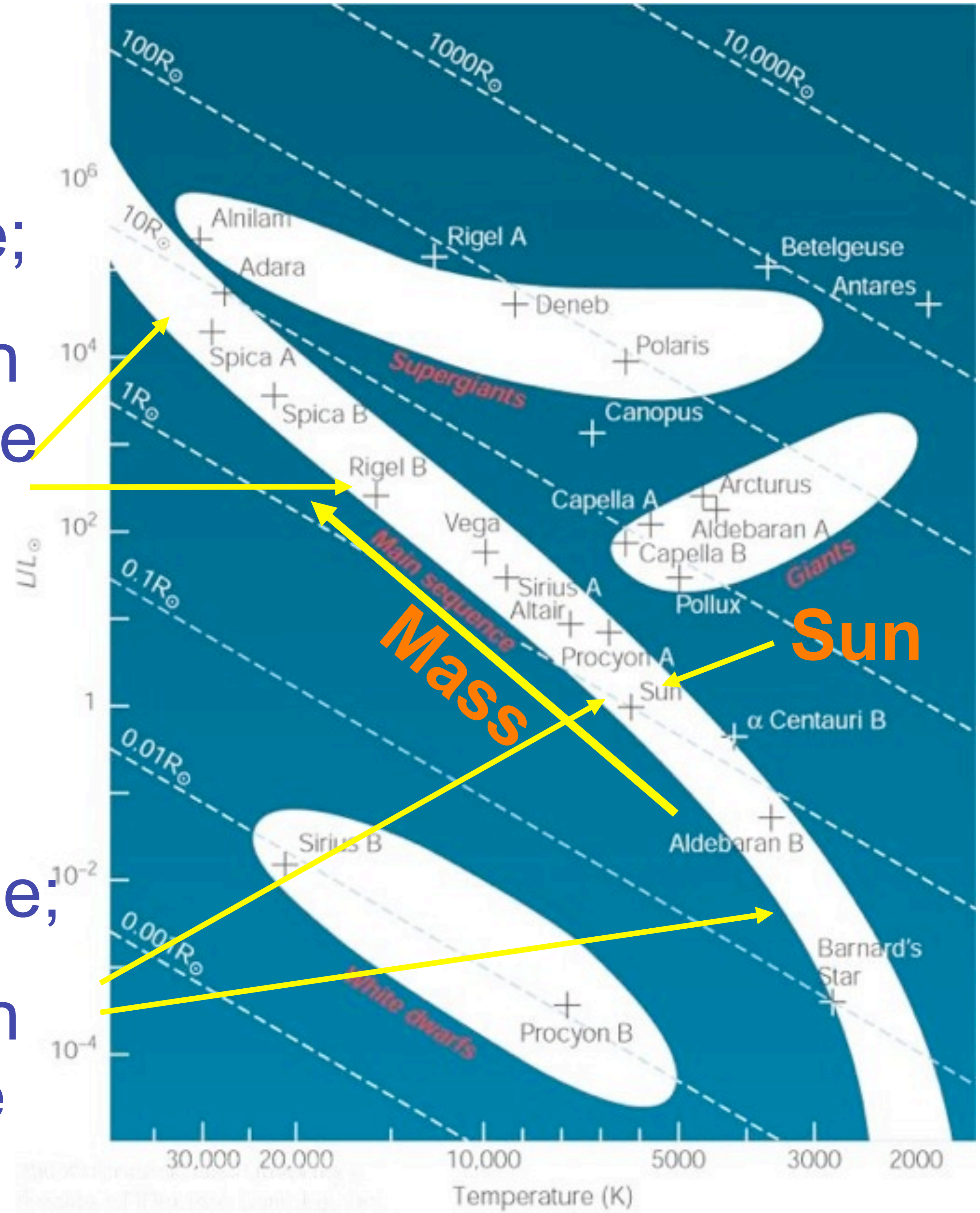
Summary: Stellar Structure

Convective Core, radiative envelope;

Energy generation through CNO Cycle

Radiative Core, convective envelope;

Energy generation through PP Cycle



Energy Transport in the Sun-like stars

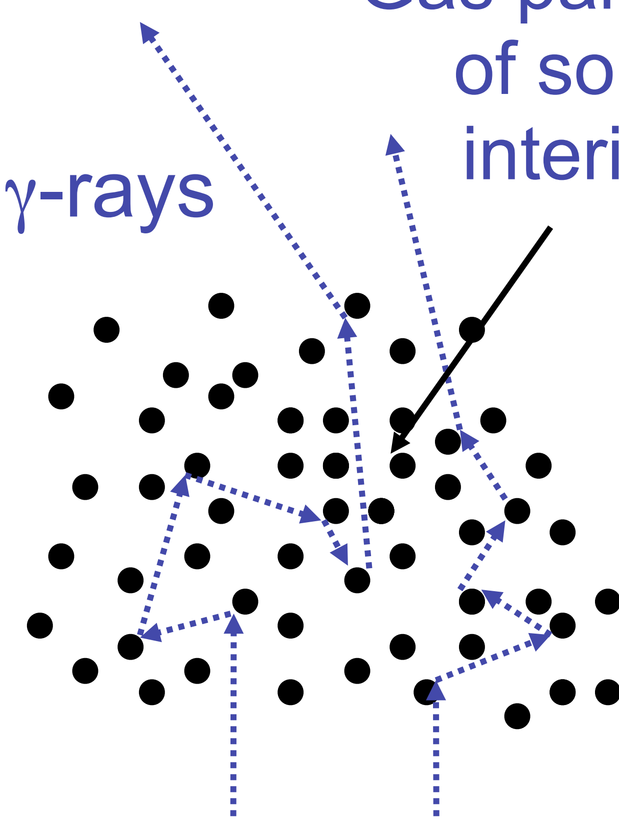
Energy generated in the star's center must be transported to the surface.

Inner layers:

Radiative energy transport

Gas particles of solar interior

γ-rays

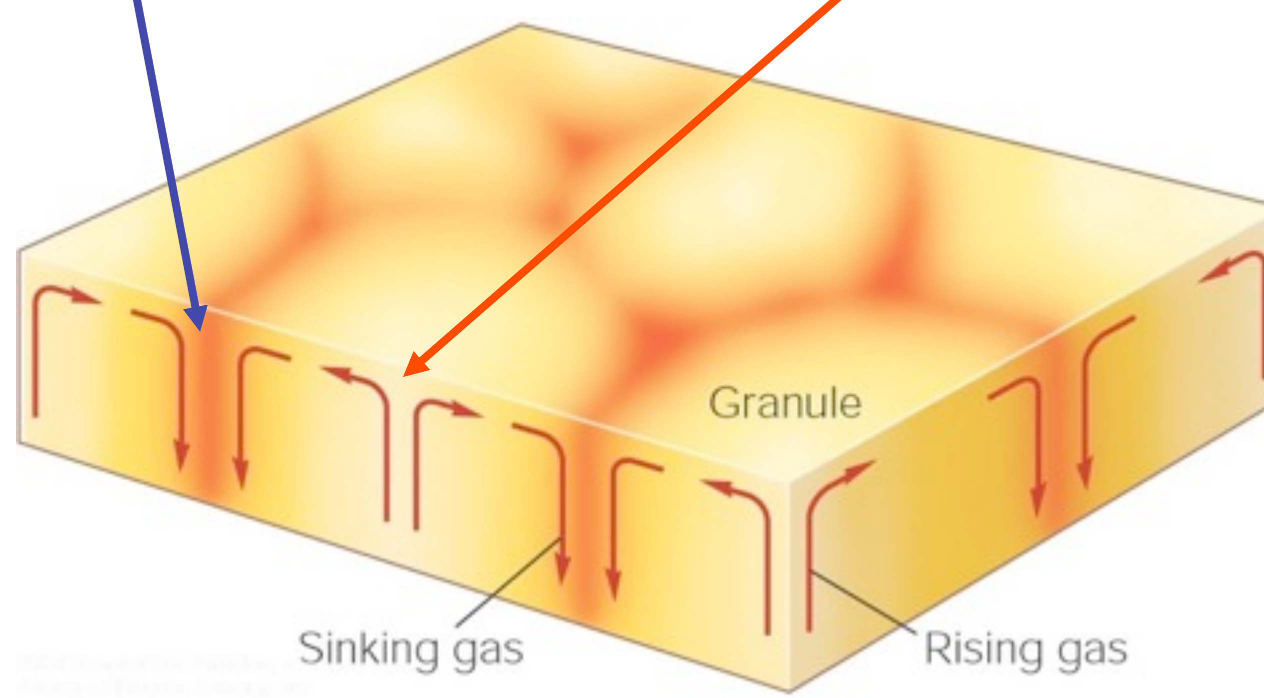


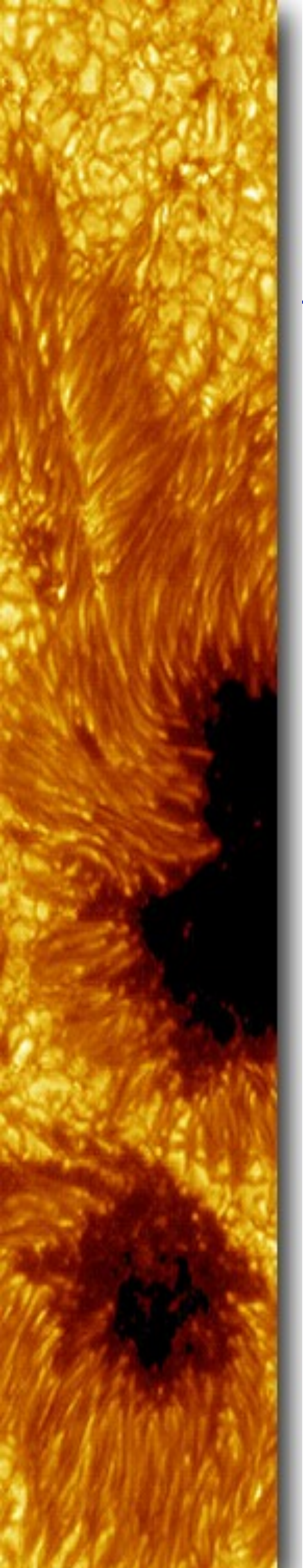
Outer layers (including photosphere):

Convection

Cool gas sinking down

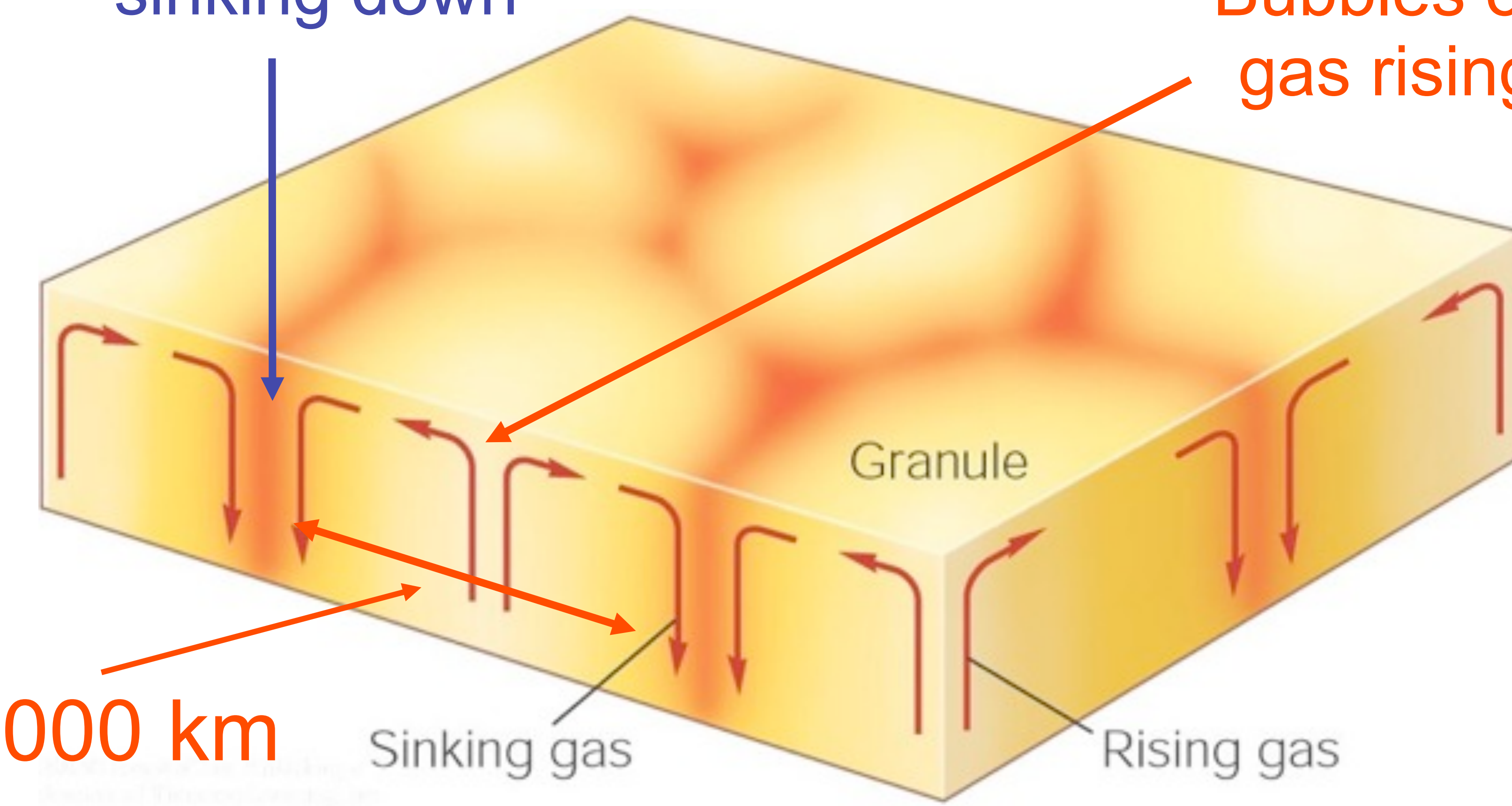
Bubbles of hot gas rising up





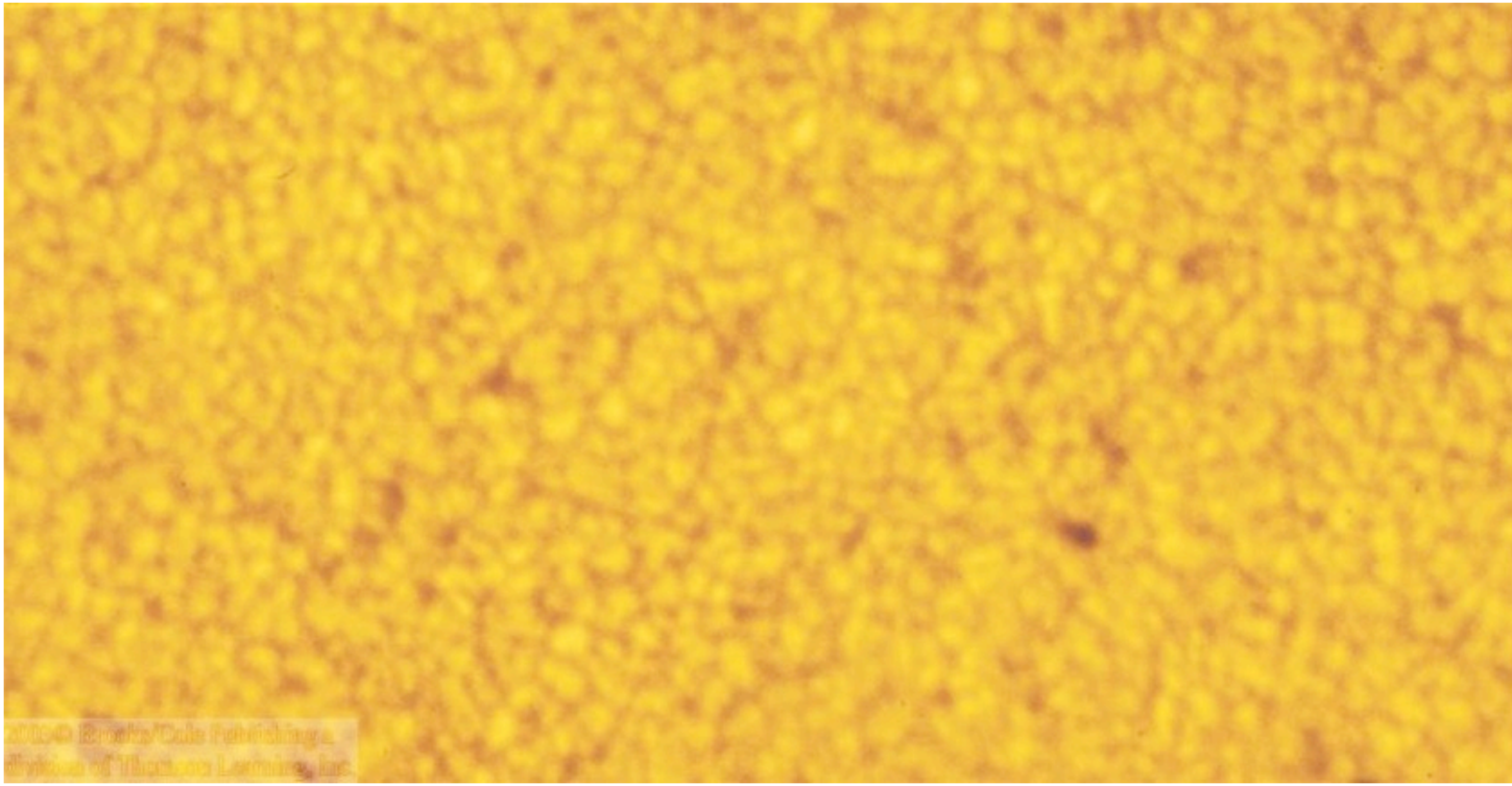
Convection

Convection is the most efficient way to transport heat



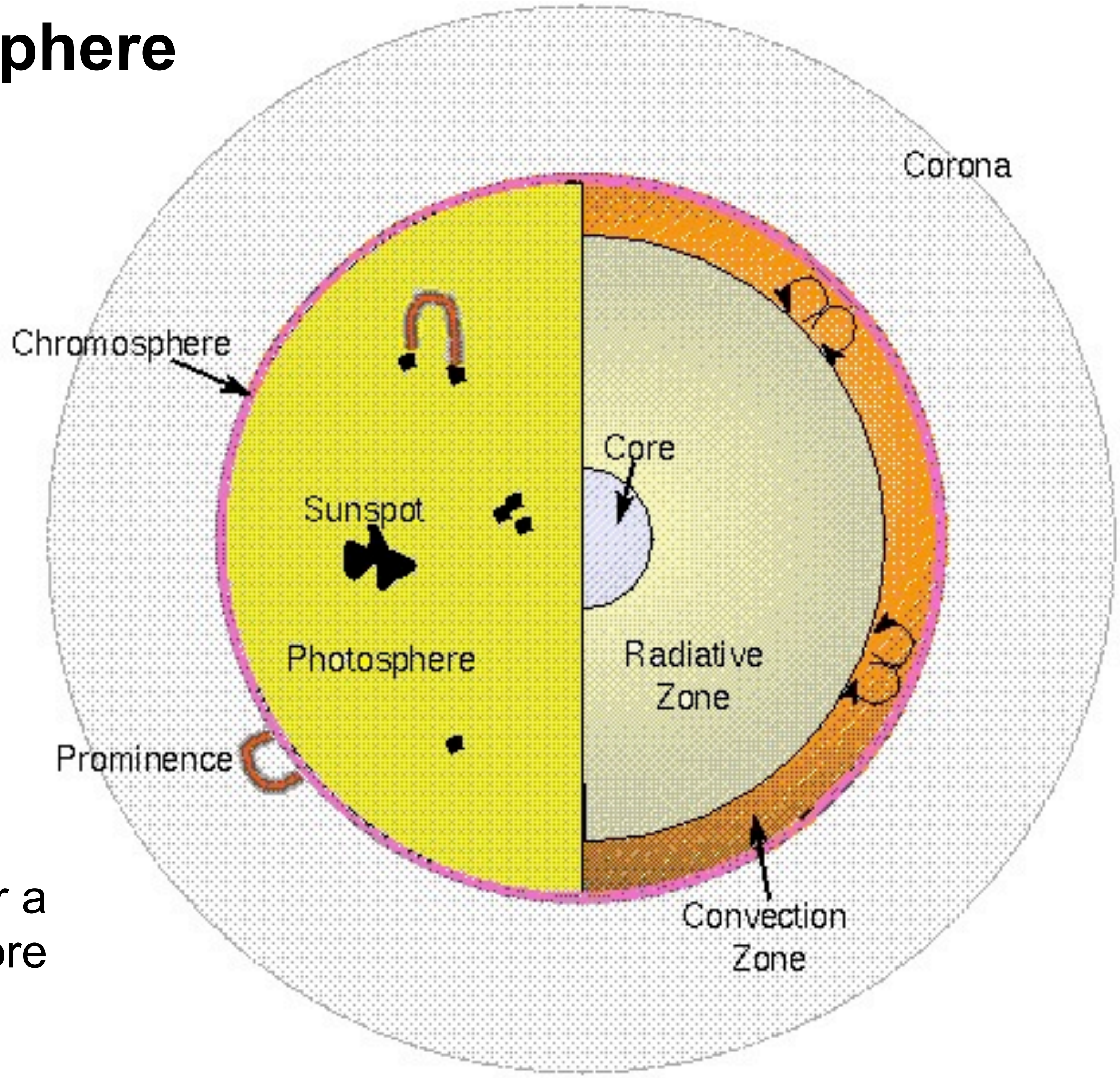
Bubbles last for ≈ 10 – 20 min.

Granulation



… is the visible consequence of convection

The solar atmosphere

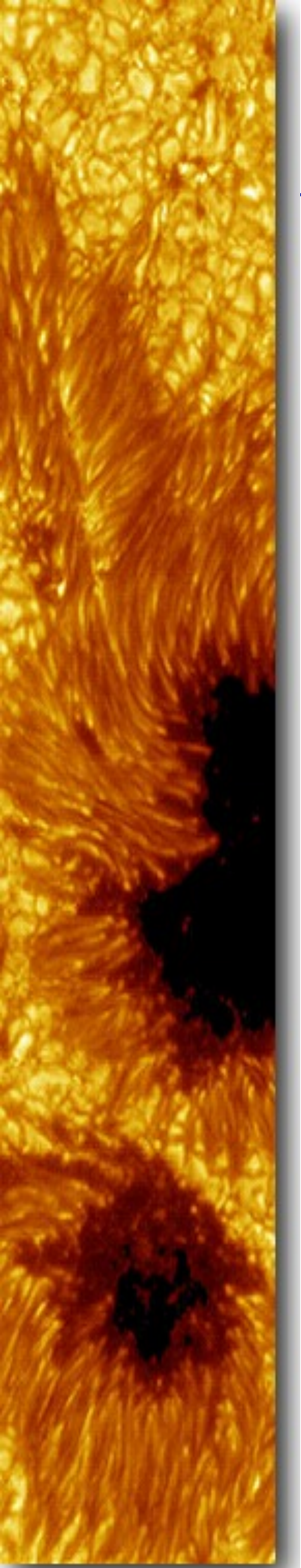


It takes 10,000 years for a photon emitted in the core to reach the surface!

Energy is generated in the **core** where the temperature reaches 16 million K and the density is 160 g/cm^3 and then transported outward by radiation. In the **convection zone** rising and falling gas is used to tranfer the energy to the **photosphere** (''surface'' of the Sun). **Sunspots** are cooler, dimmer regions with strong magnetic fields. Some sunspots have **prominences** forming over them. The **chromosphere** is a thin pink layer above the photosphere that is hotter than the photosphere. The temperature increases outward into the **corona**, the very hot (1–2 million K) but tenuous atmosphere of the sun. Fast moving ions in the corona escape the Sun to form the **solar wind.**

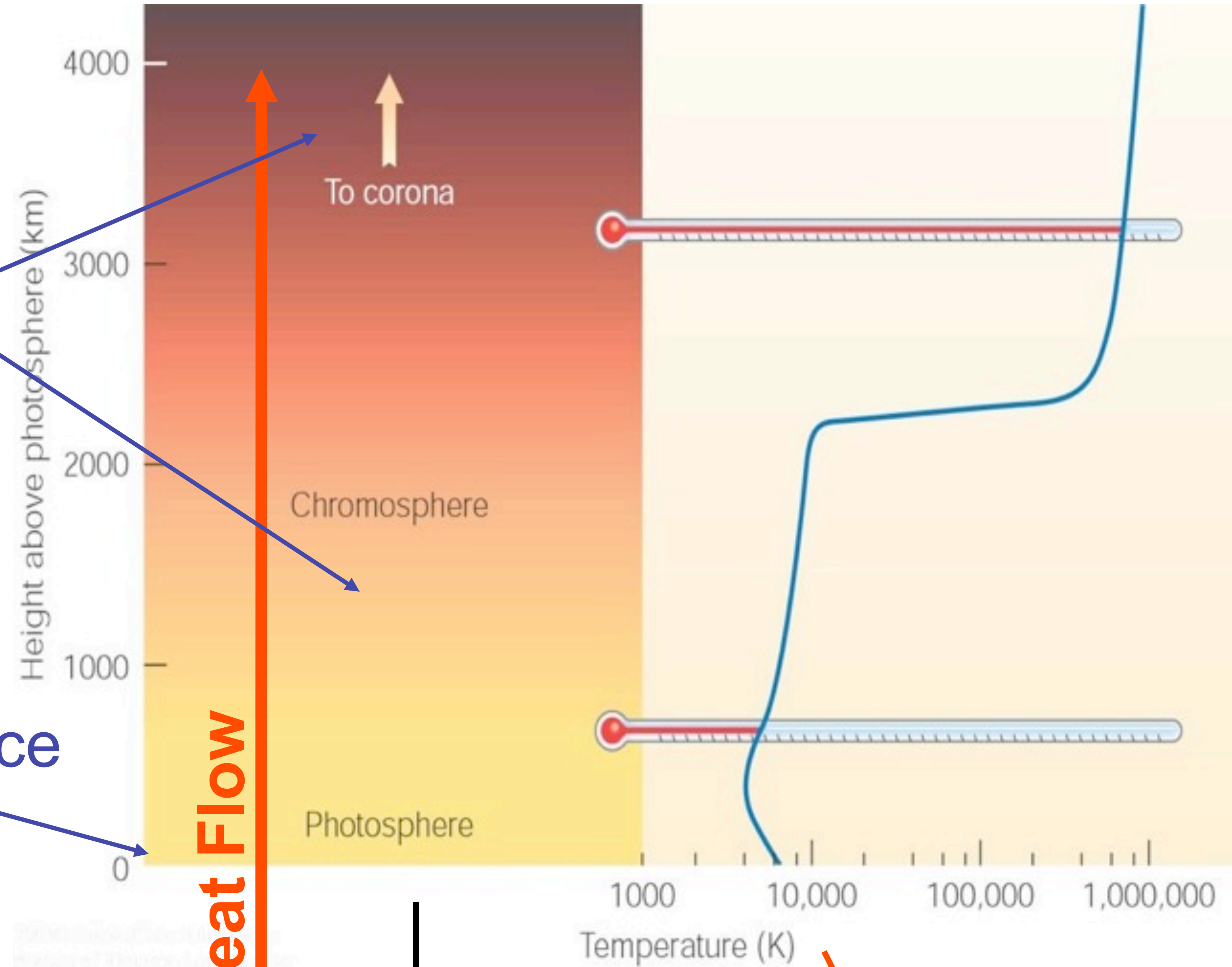
Slide

The Solar Atmosphere



Only visible during solar eclipses

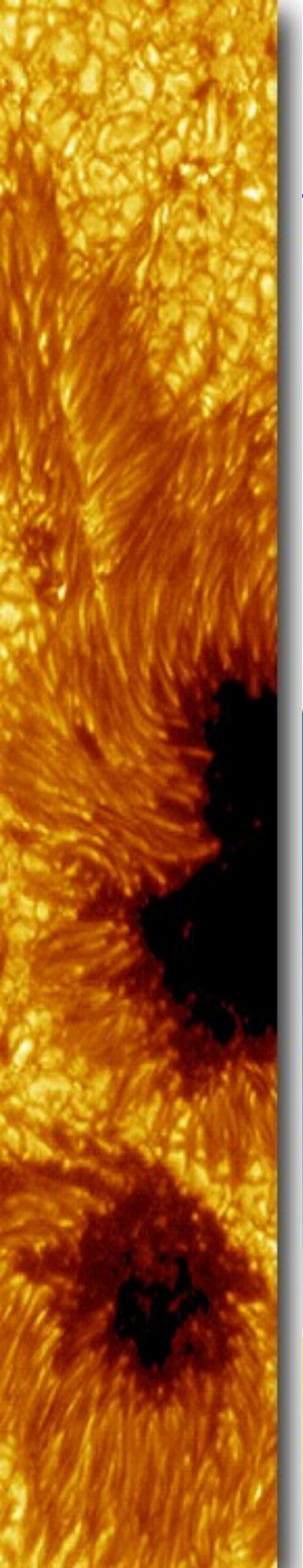
Apparent surface of the sun



Heat Flow

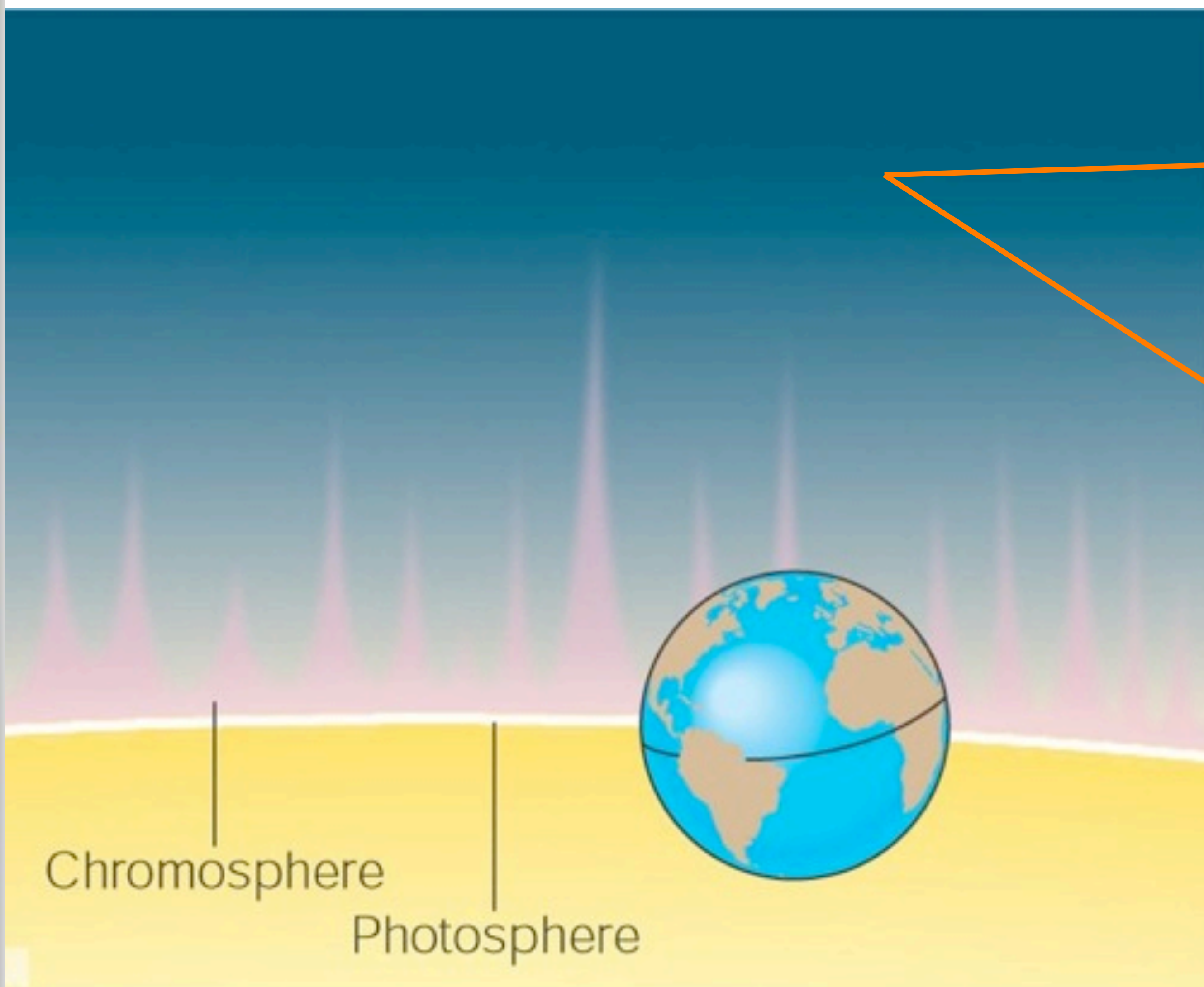
Solar interior

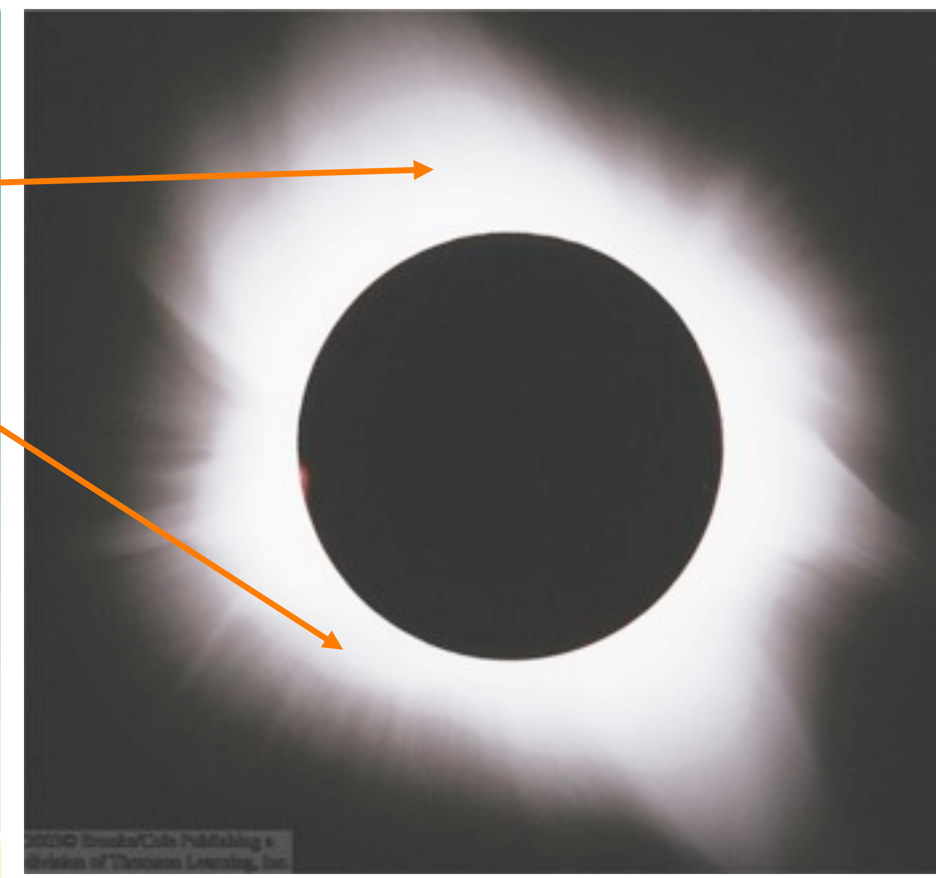
Temp. incr. inward



The Photosphere

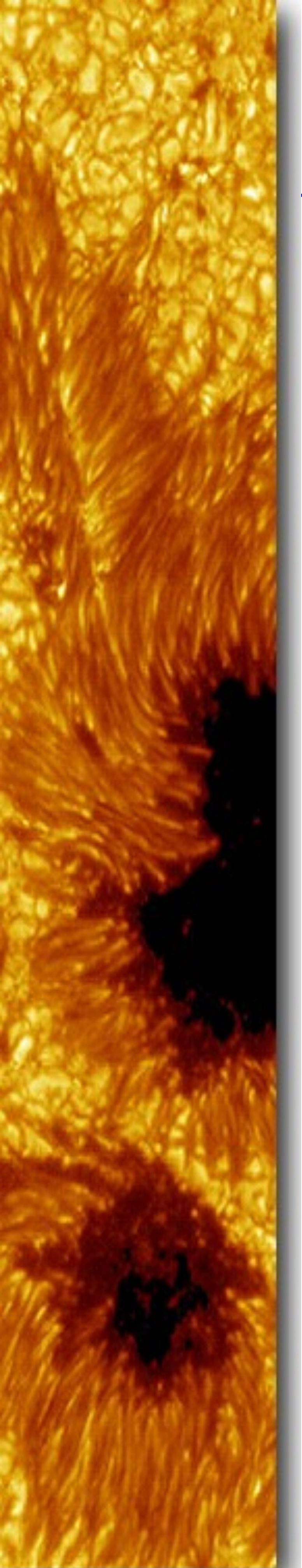
- Apparent surface layer of the sun
- Depth ≈ 500 km
- Temperature ≈ 5800 ^{o}K
- Absorbs and re-emits radiation produced in the solar interior



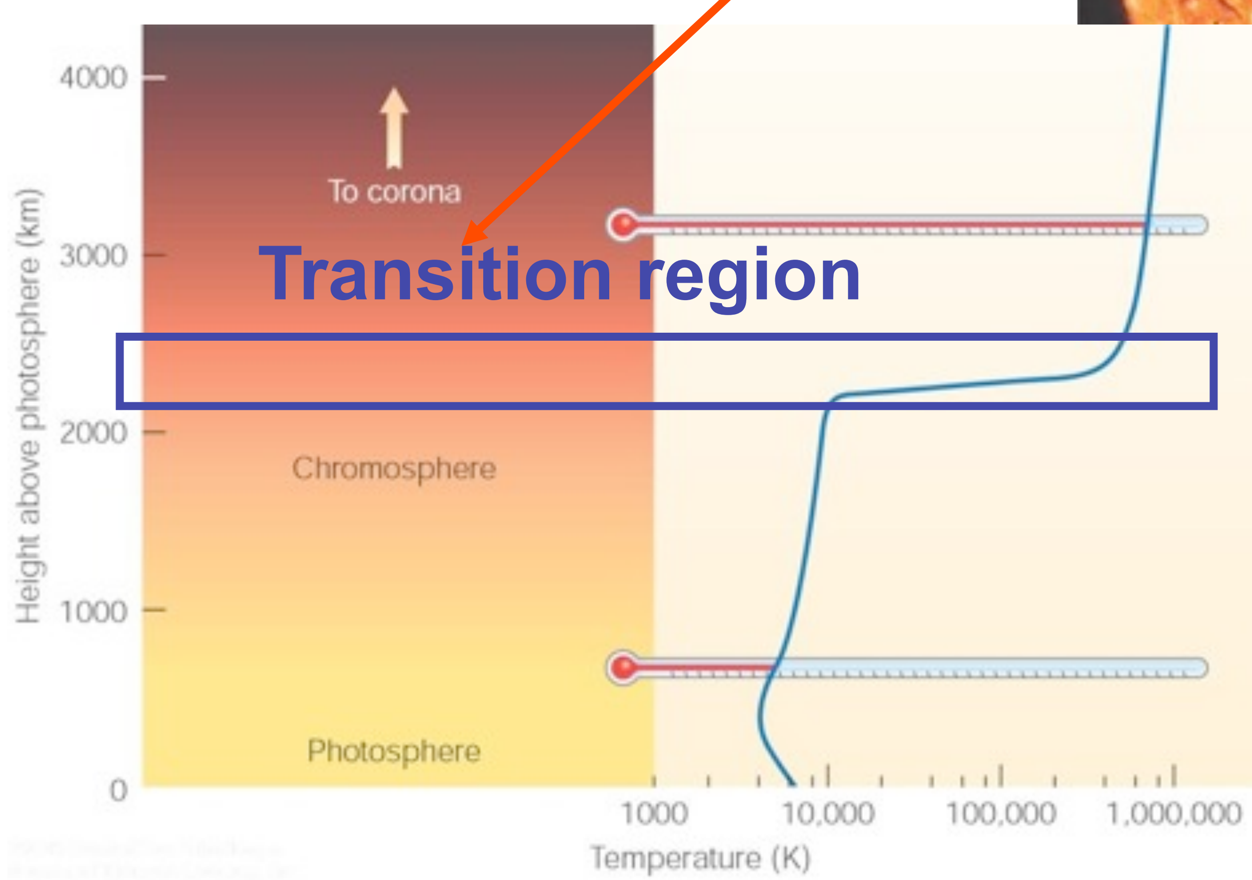


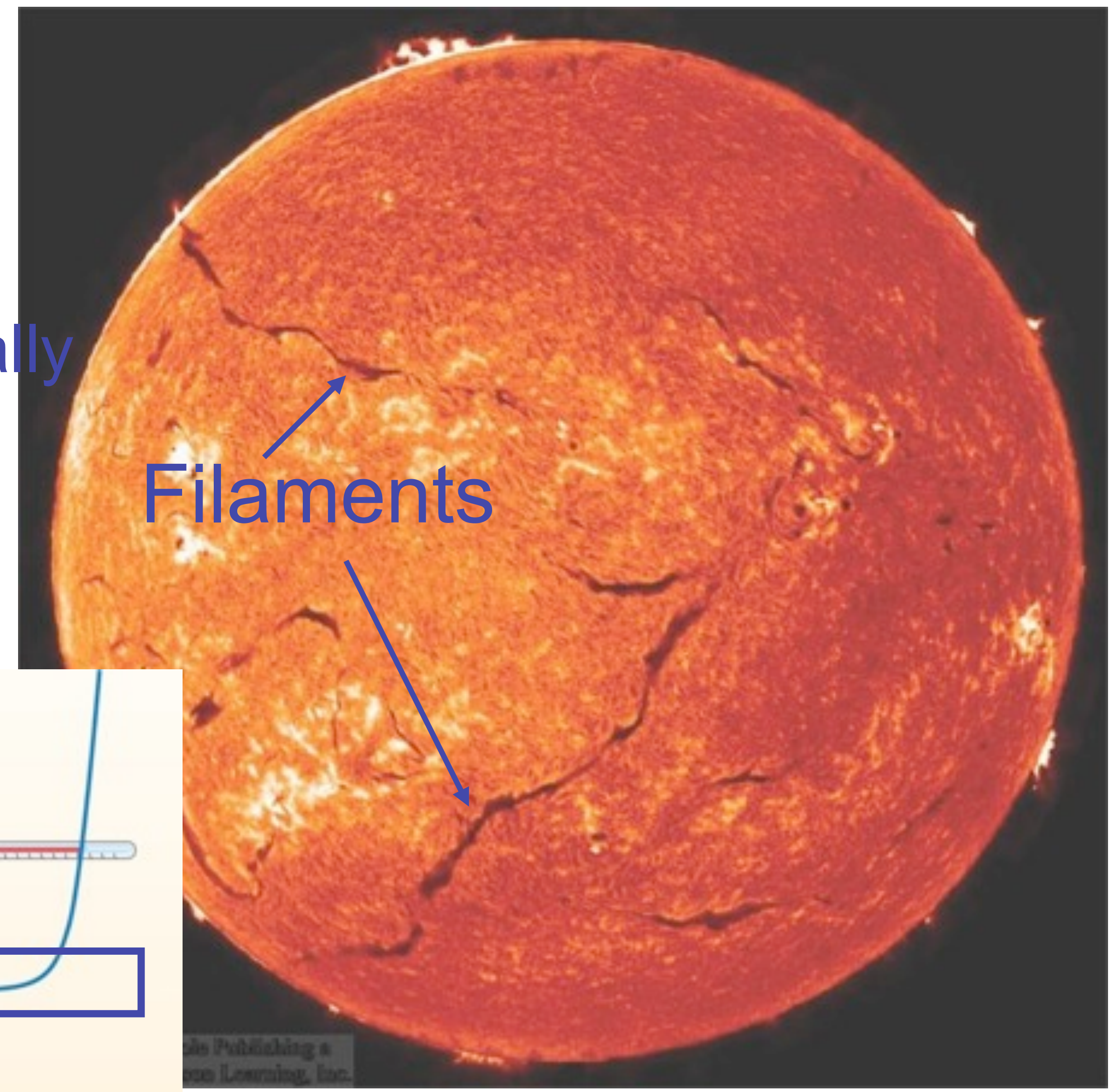
The solar corona

The Chromosphere



- Region of sun’s atmosphere just above the photosphere.
- Visible, UV, and X-ray lines from highly ionized gases
- Temperature increases gradually from ≈ 4500 ^{o}K to ≈ 10,000 ^{o}K, then jumps to ≈ 1 million ^{o}K





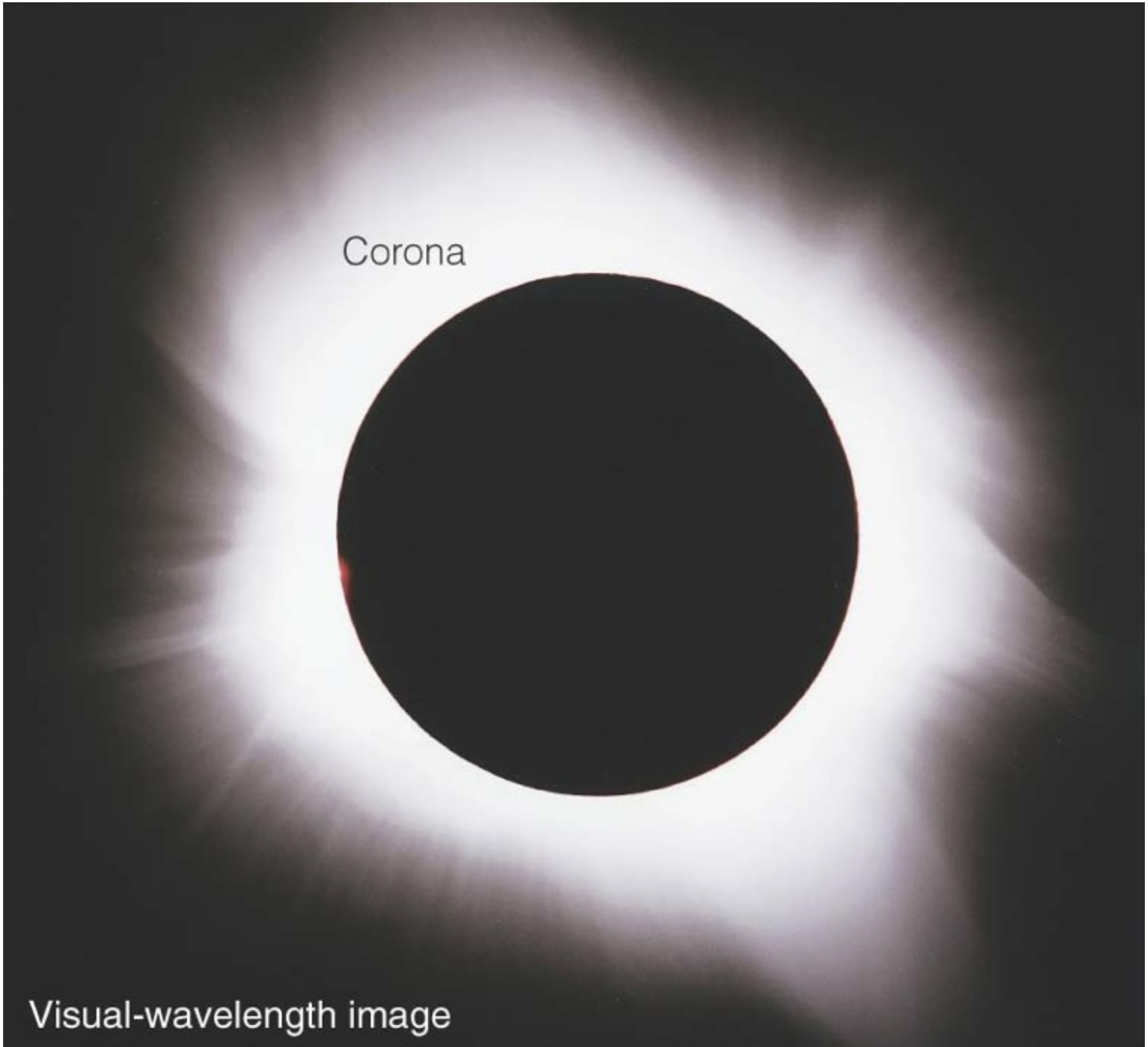
Chromospheric structures visible in Hα emission (filtergram)

The Chromosphere (2)

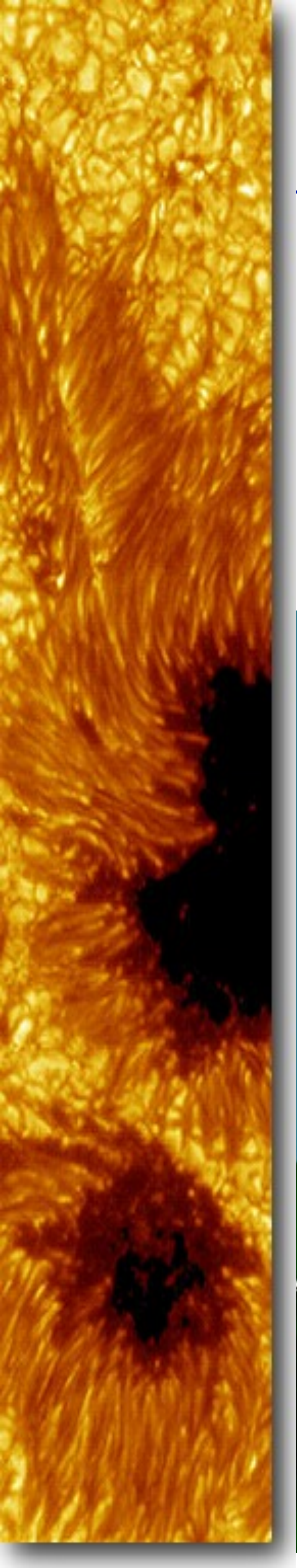
Spicules: Filaments of cooler gas from the photosphere, rising up into the chromosphere.

Visible in Hα emission.

Each one lasting about 5 – 15 min.

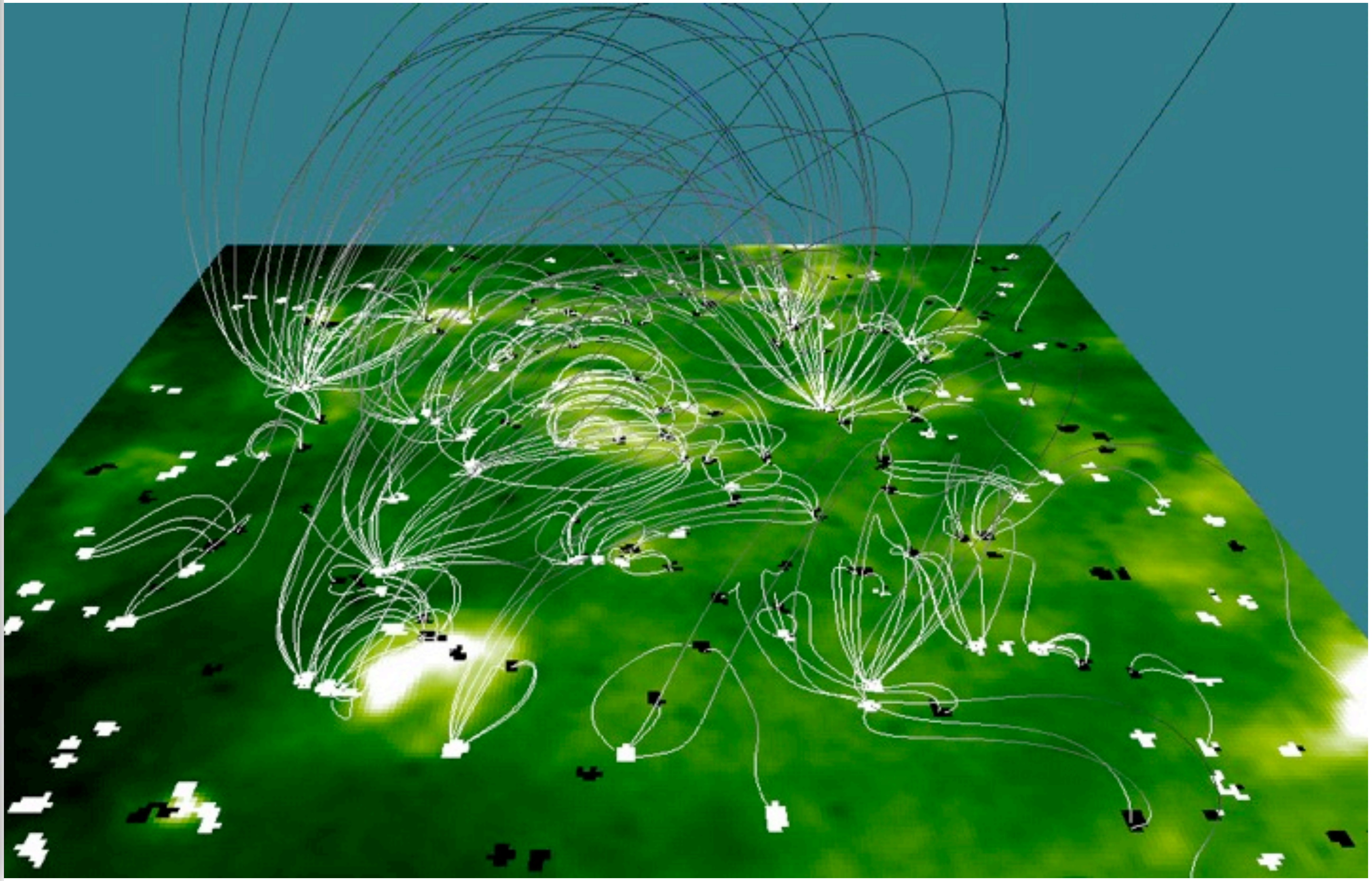


Slide



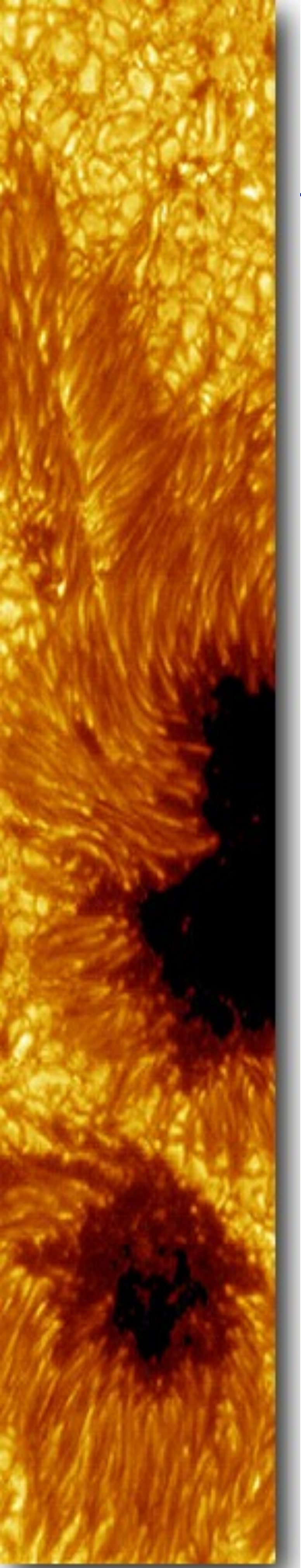
The Magnetic Carpet of the Corona

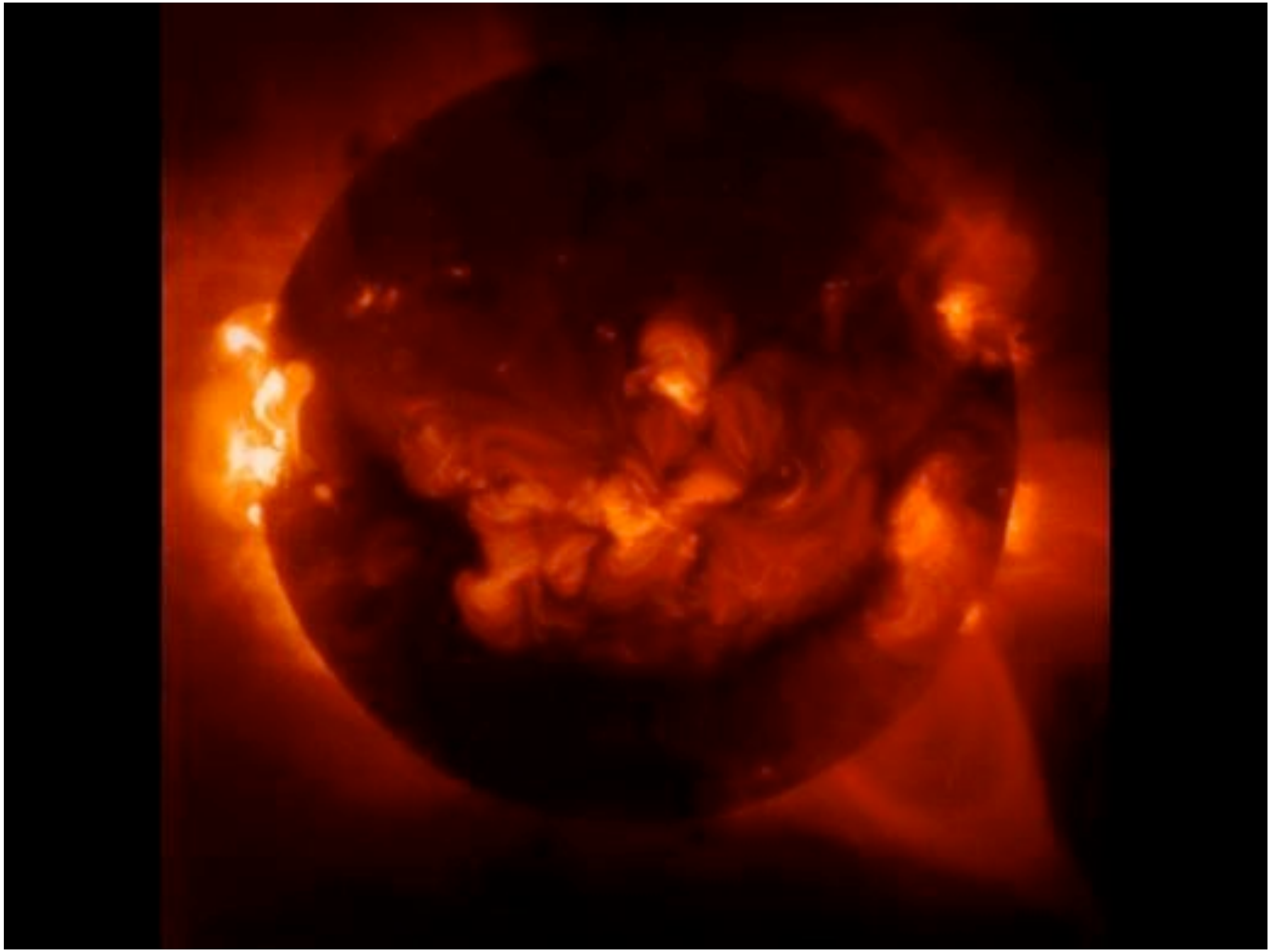
- Corona contains very low-density, very hot (1 million ^{o}K) gas
- Coronal gas is heated through motions of magnetic fields anchored in the photosphere below ("magnetic carpet"). Precise mechanism is unknown.



Computer model of the magnetic carpet

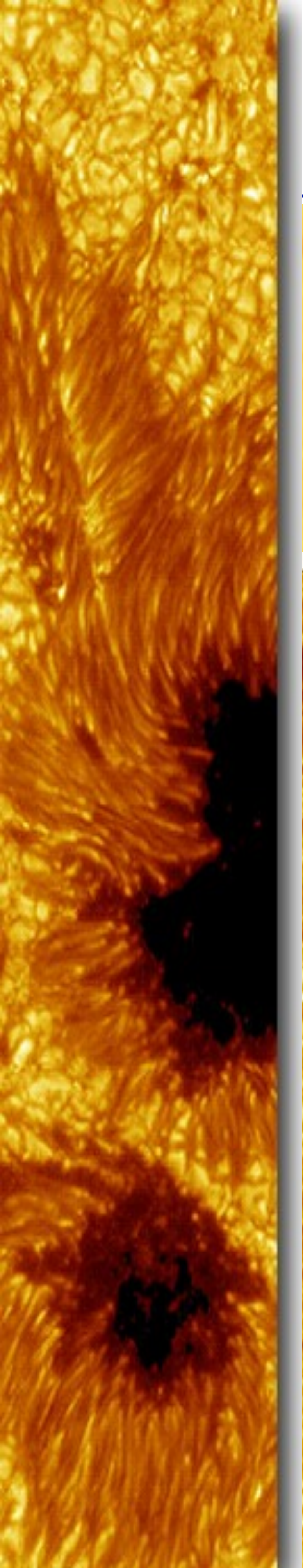
Changing Face of the Sun

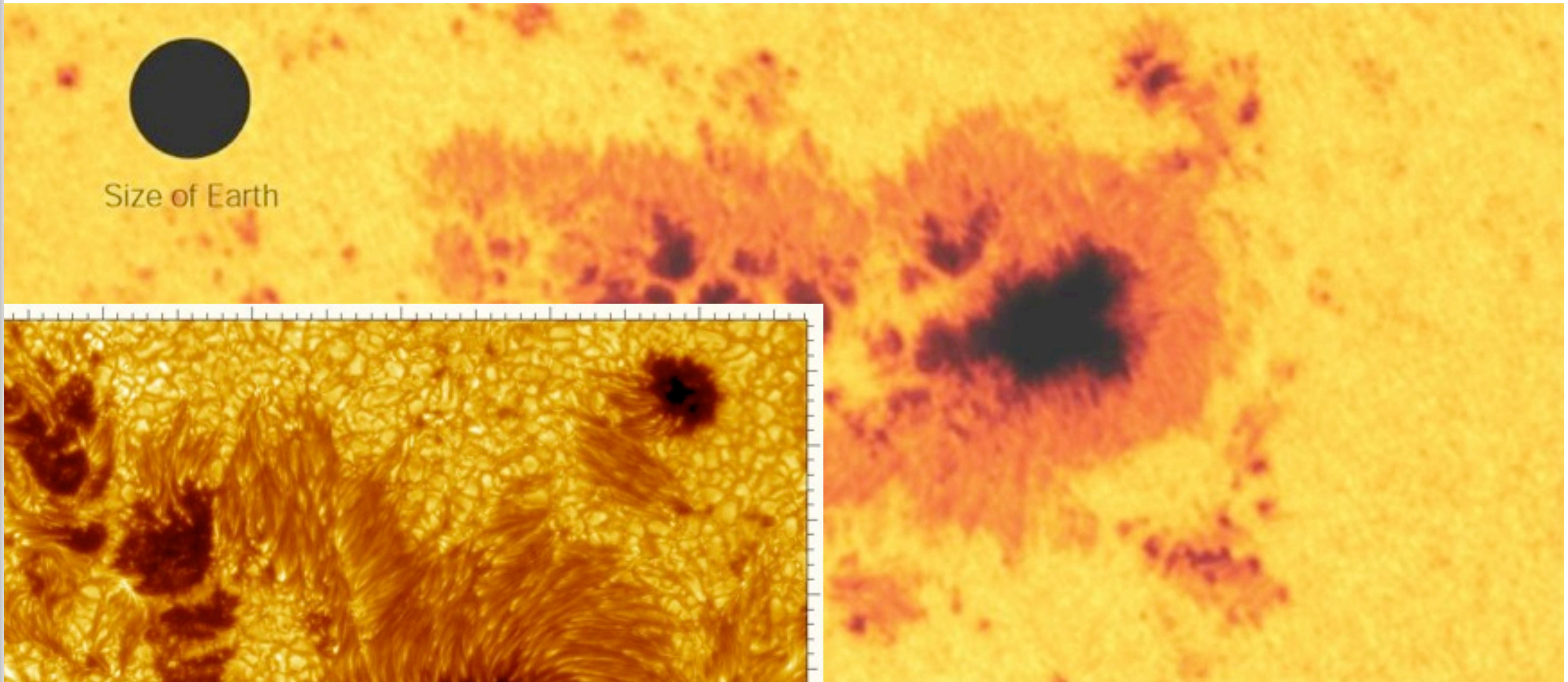


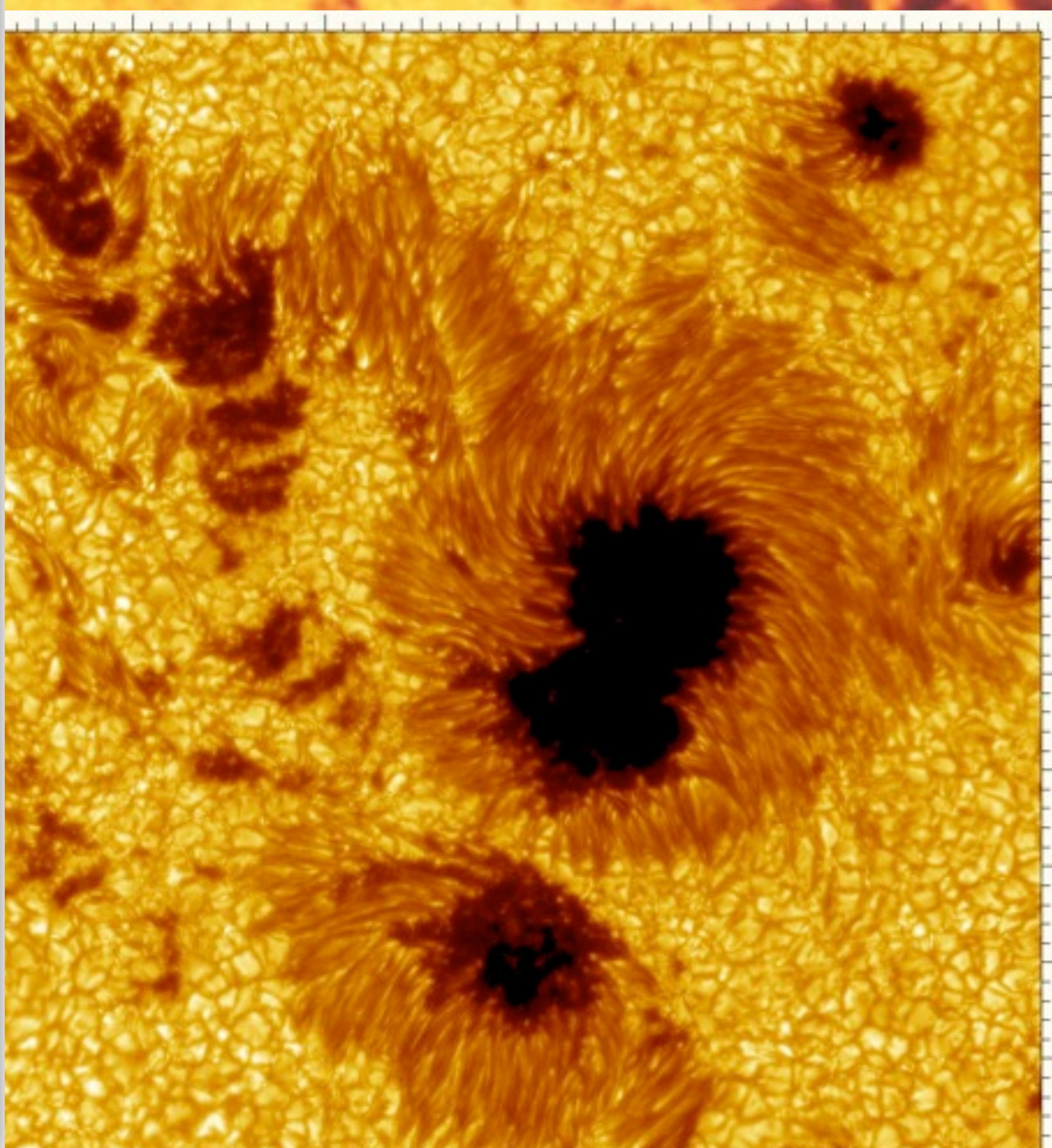


Solar Activity, seen in soft X-rays

Sun Spots



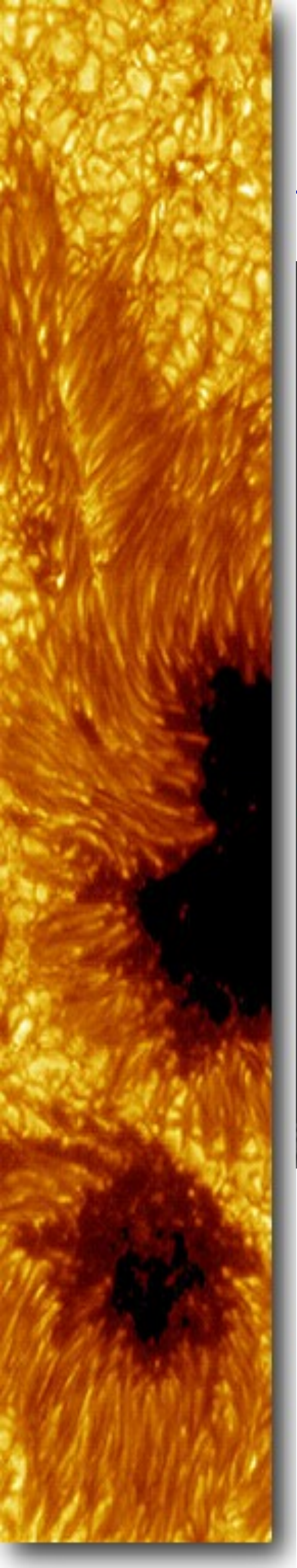


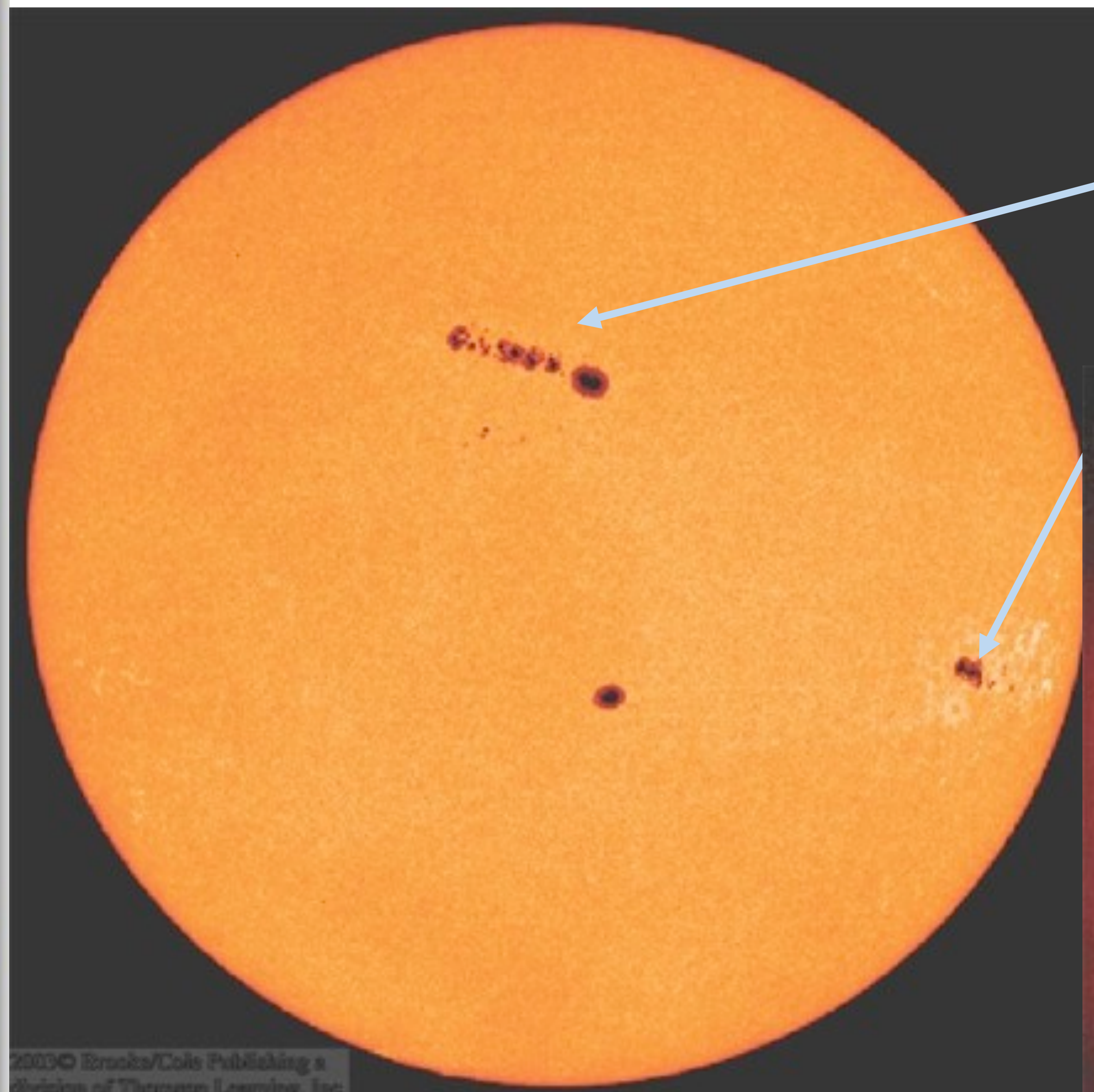


Cooler regions of the photosphere ($T \approx 4200$ K).

Only appear dark against the bright sun. Would still be brighter than the full moon when placed on the night sky!

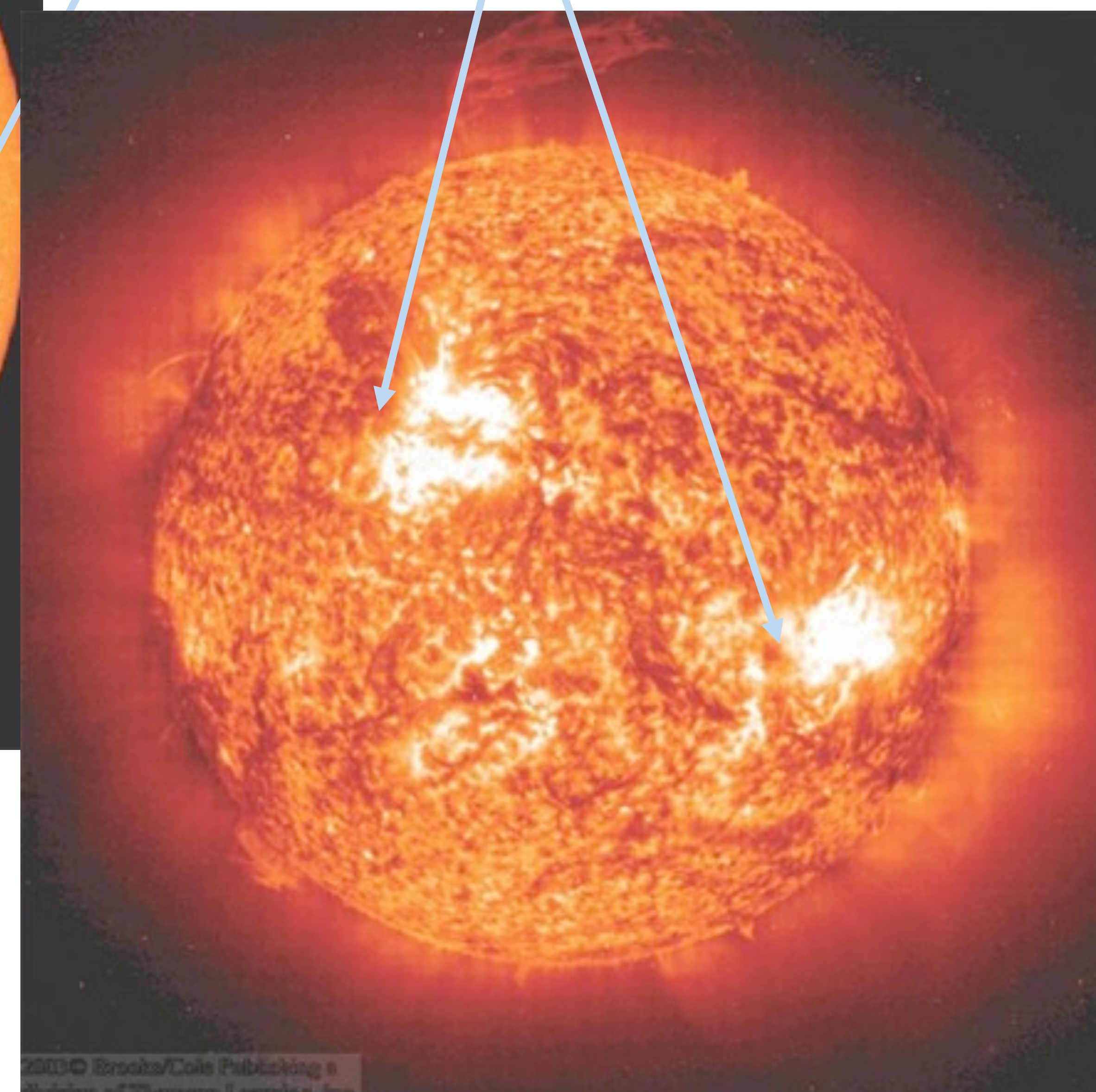
Sun Spots (2)



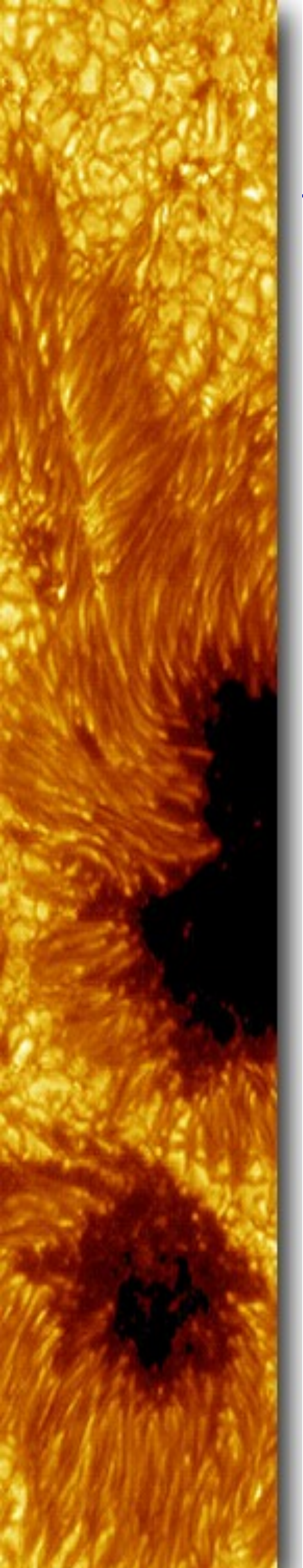


Active Regions

Visible

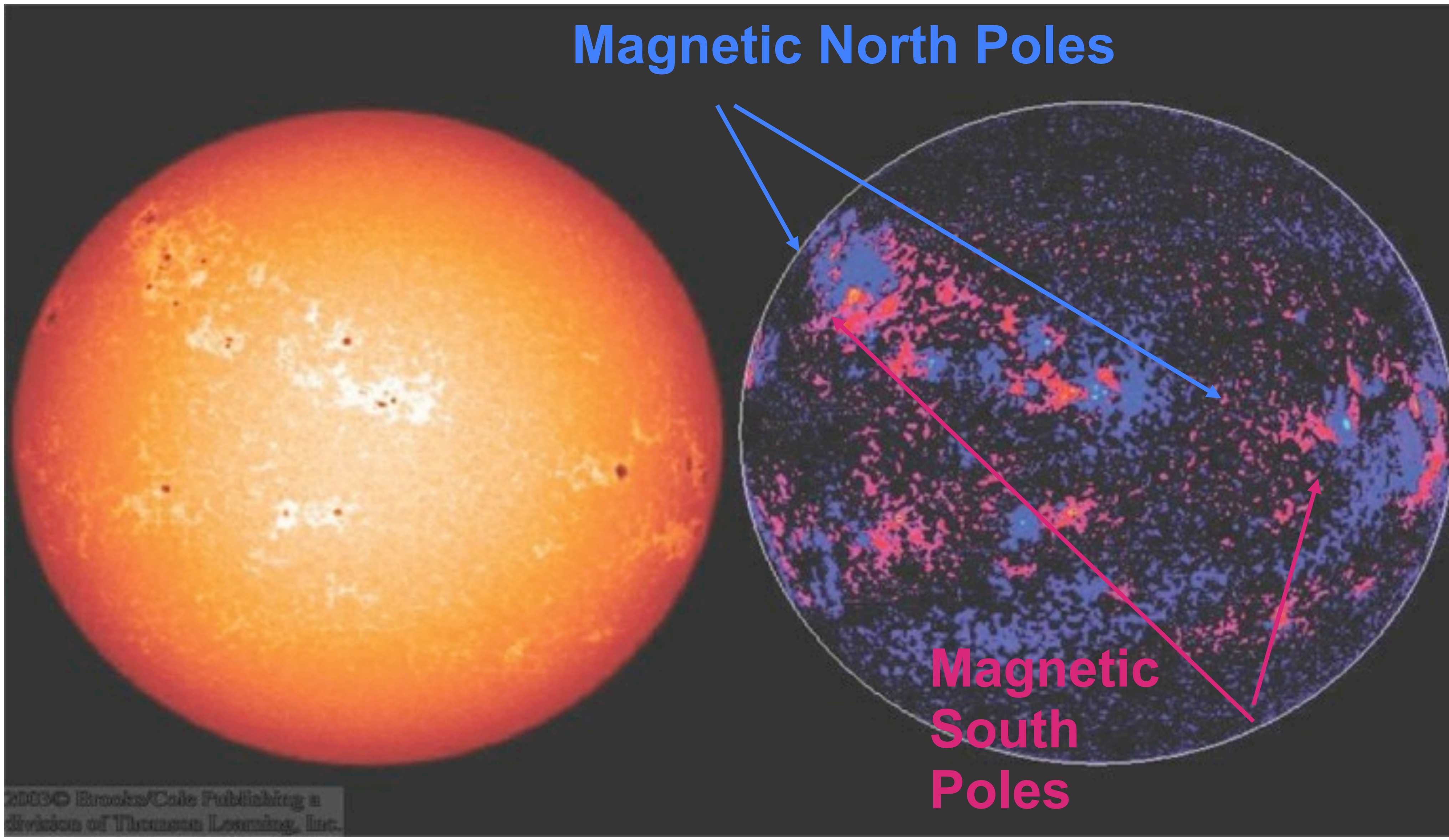


Ultraviolet



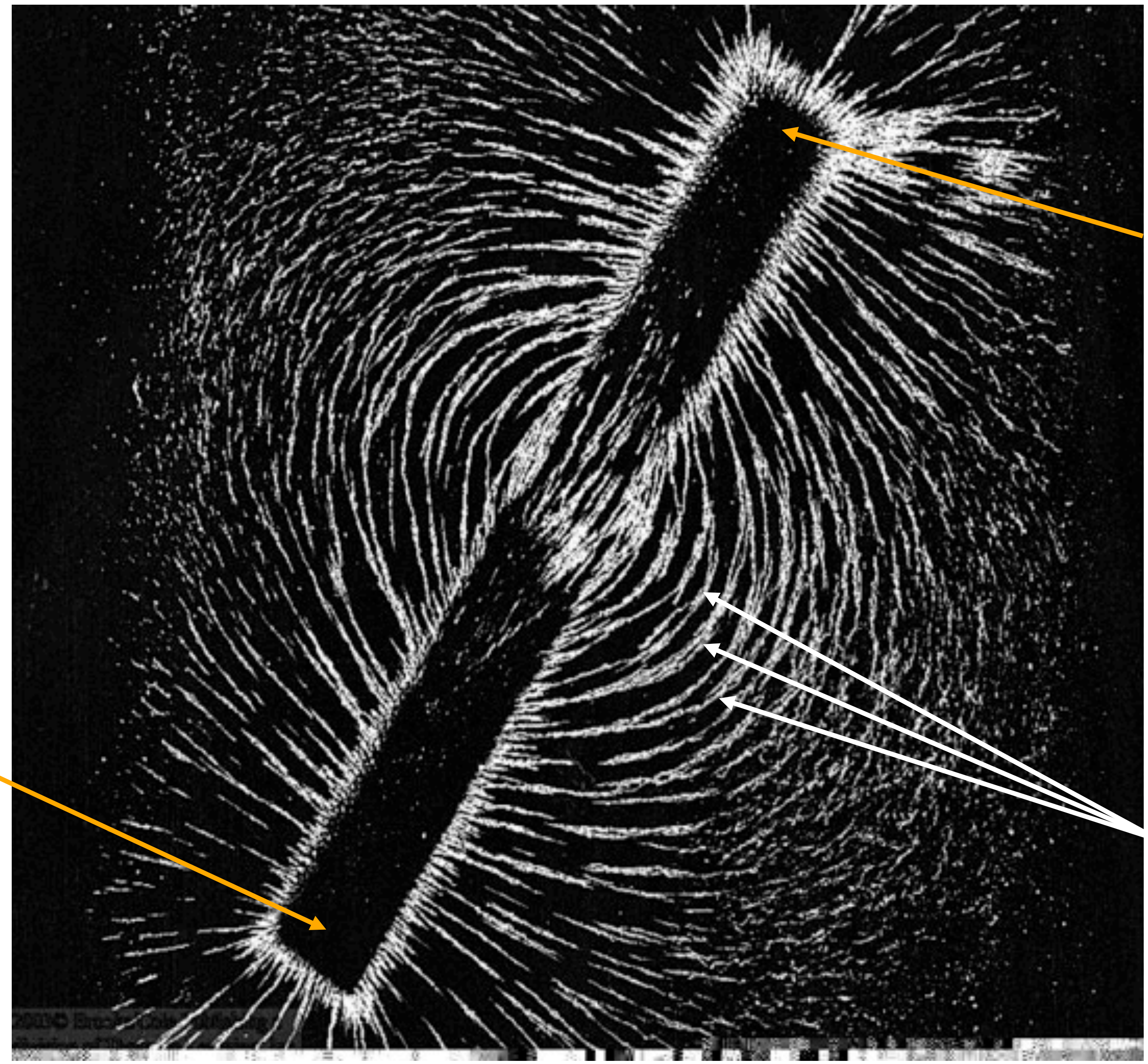
Sun Spots (3)

Magnetic field in sun spots is about 1000 times stronger than average.



→ Sun Spots are related to magnetic activity on the photosphere

Magnetic Field Lines

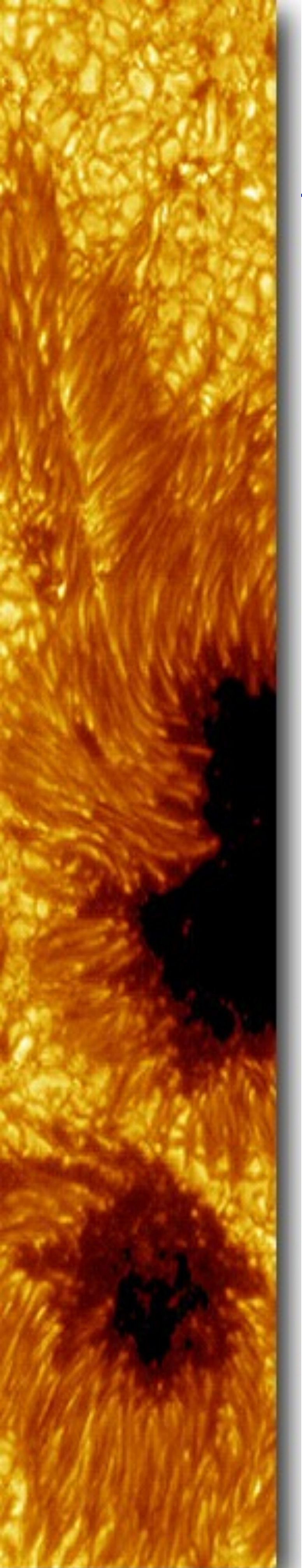


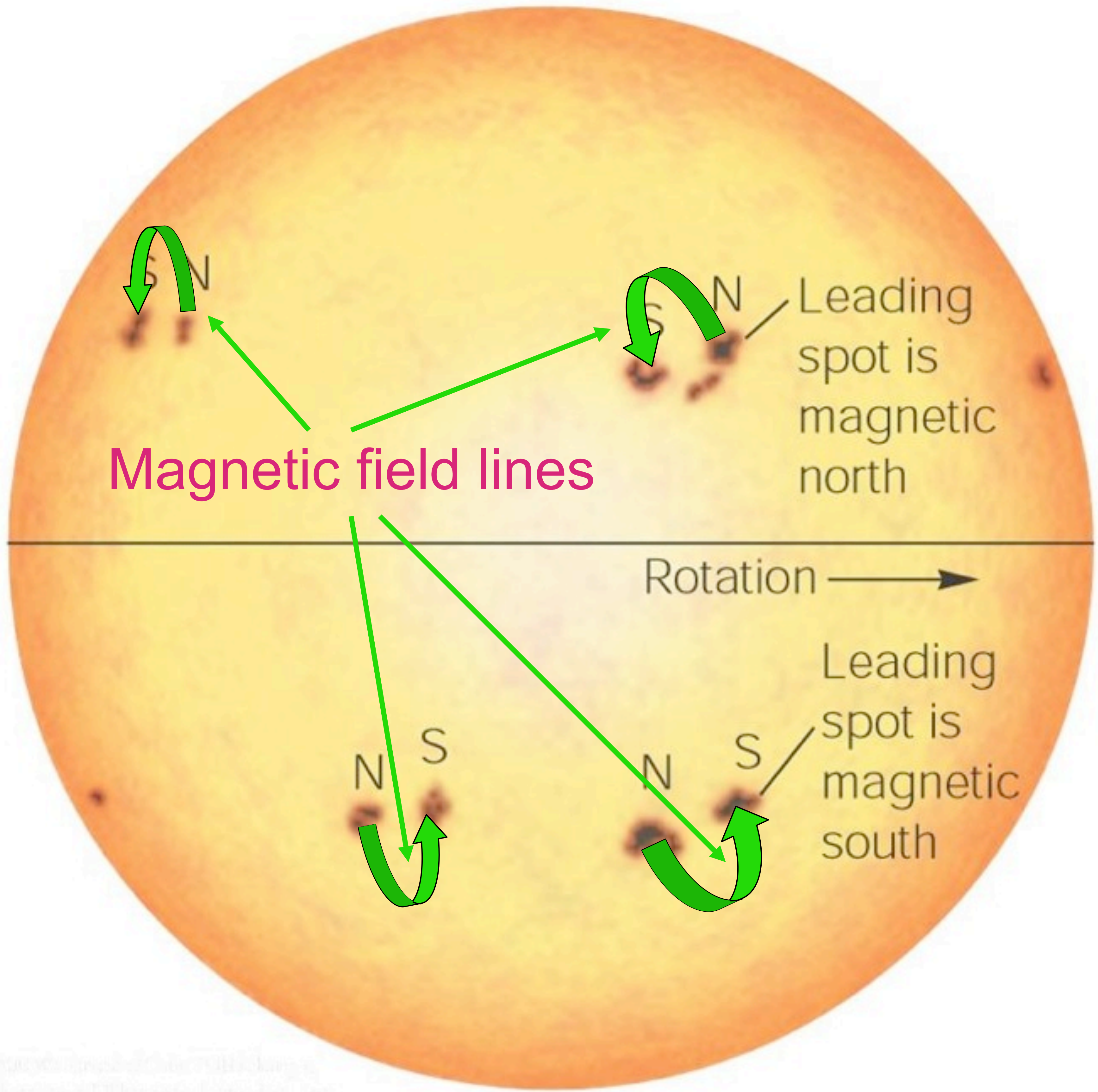
Magnetic North Pole

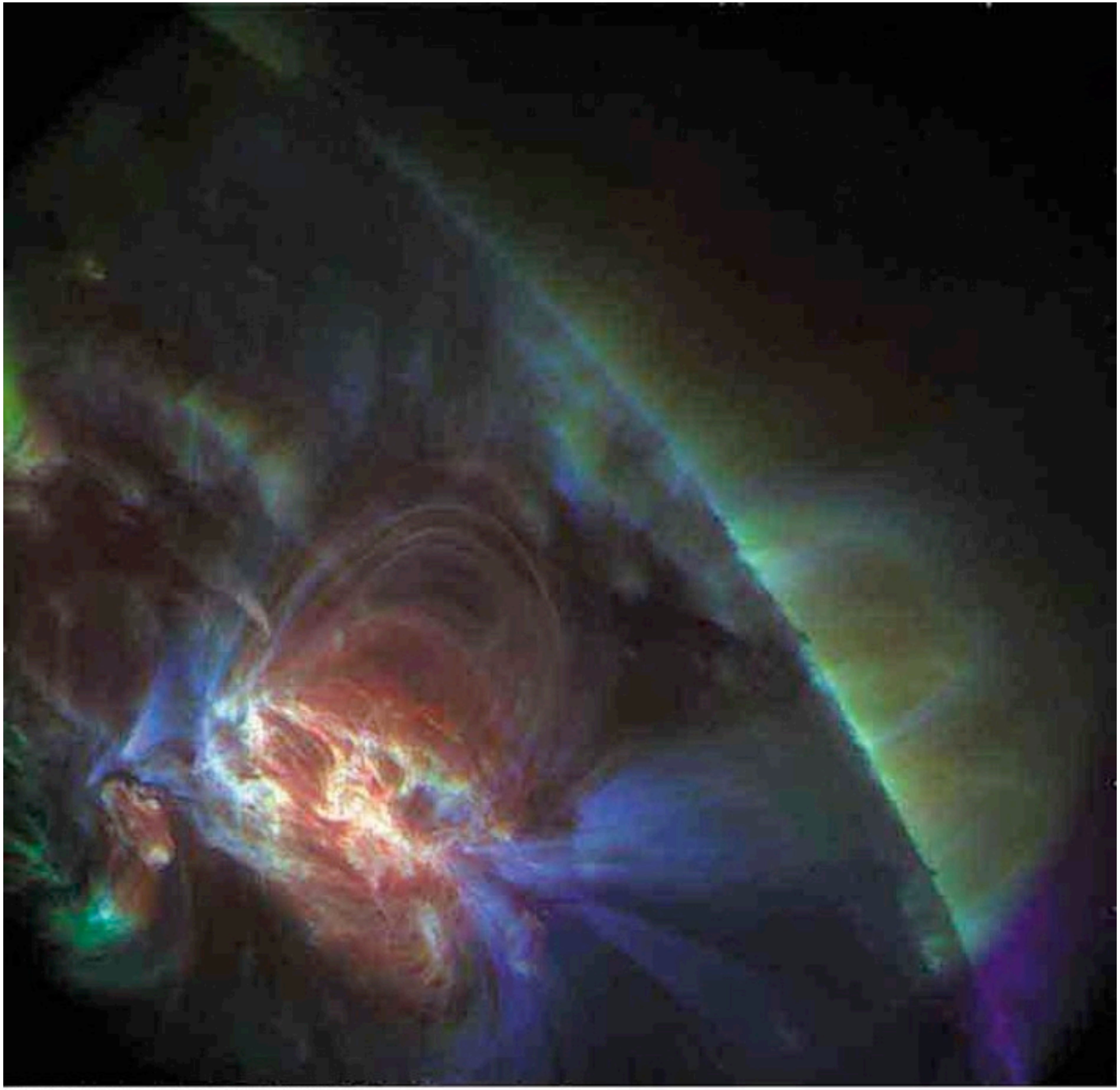
Magnetic South Pole

Magnetic Field Lines

Magnetic Loops

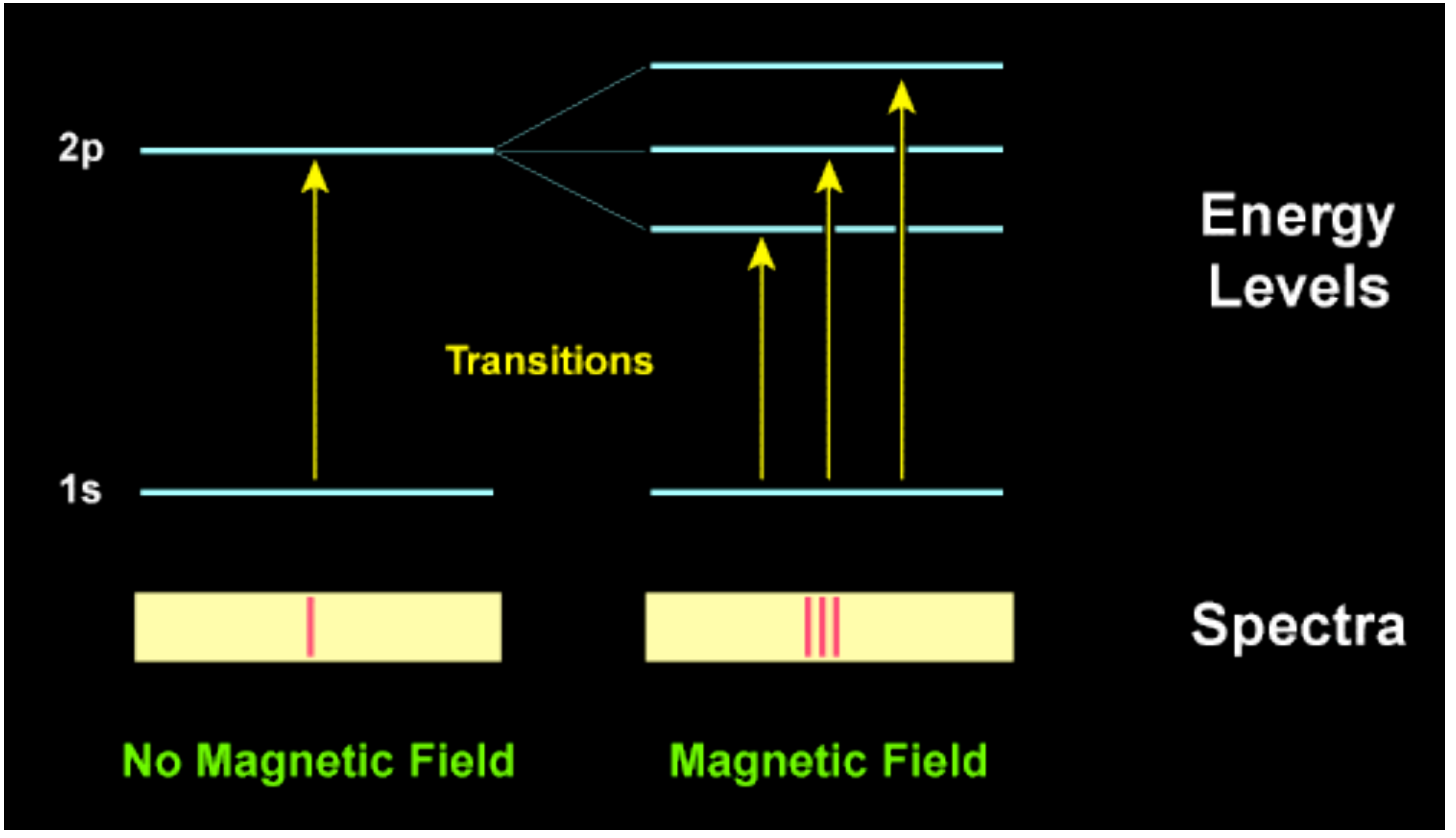






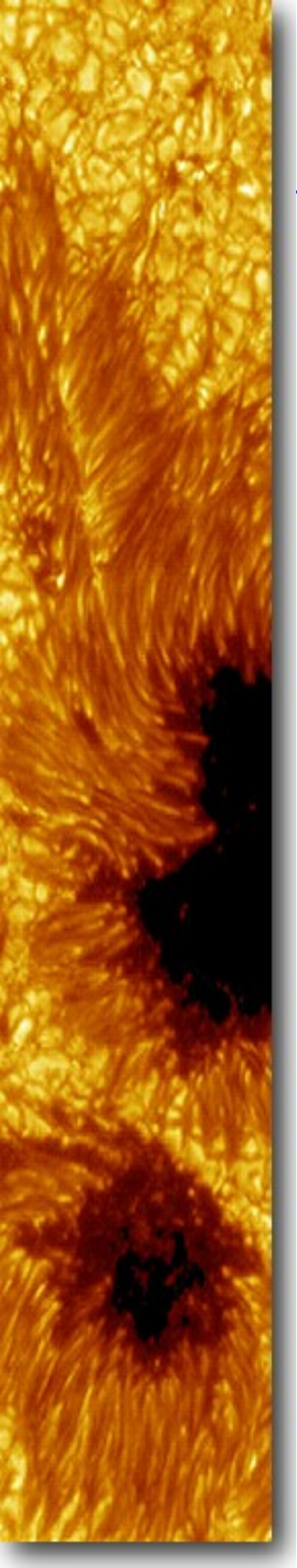
Slide

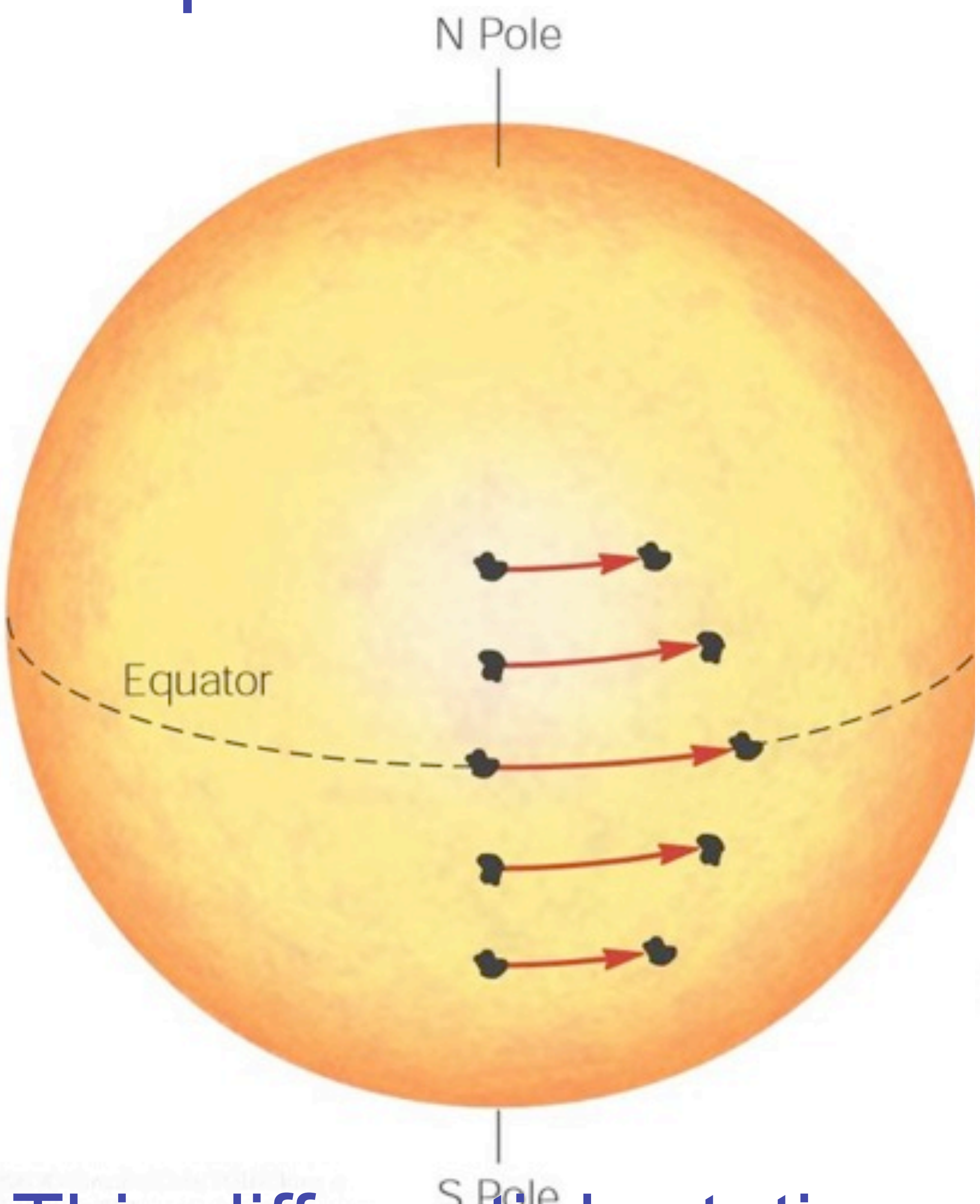
Zeeman effect

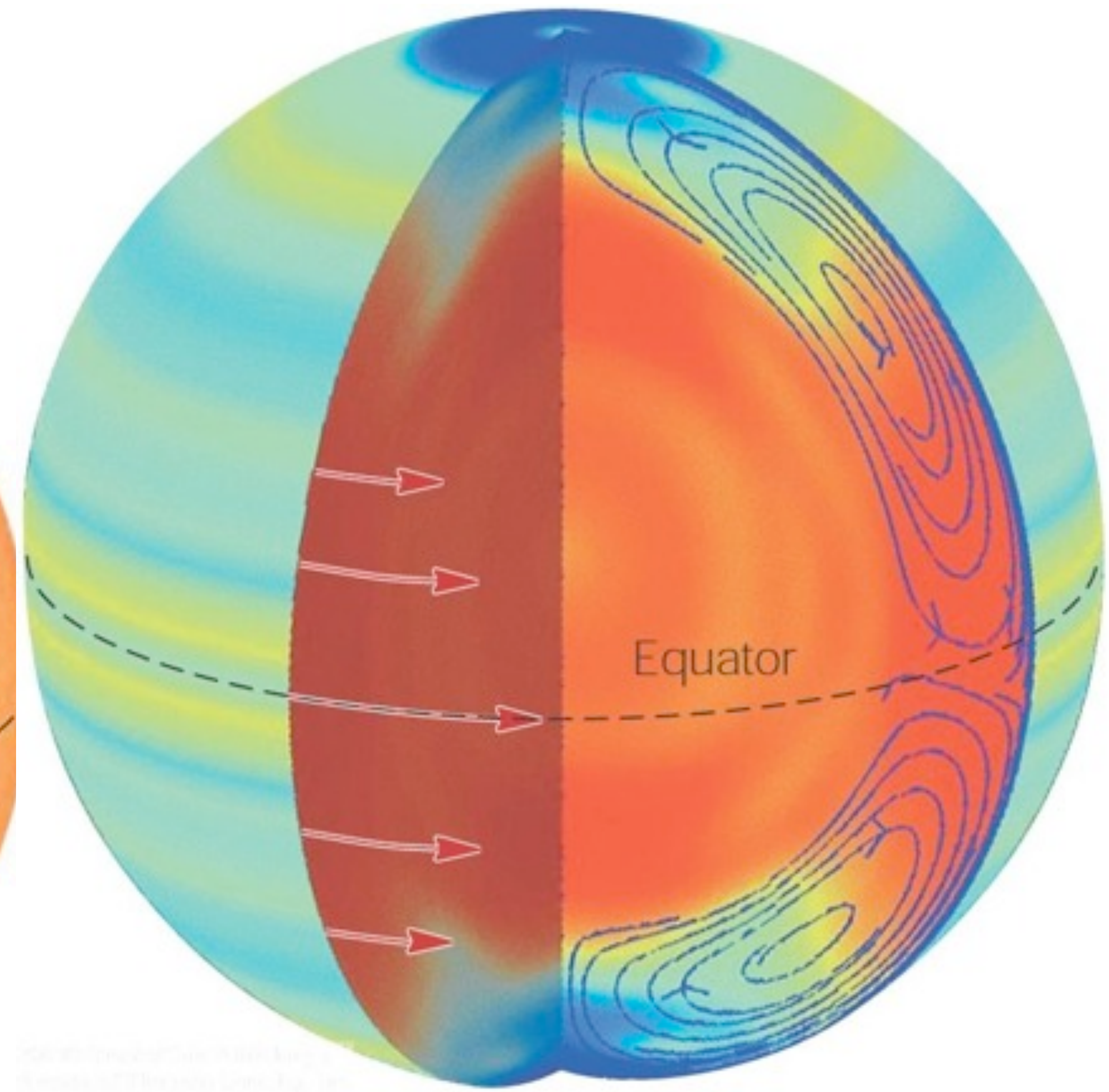


The Sun’s Magnetic Dynamo

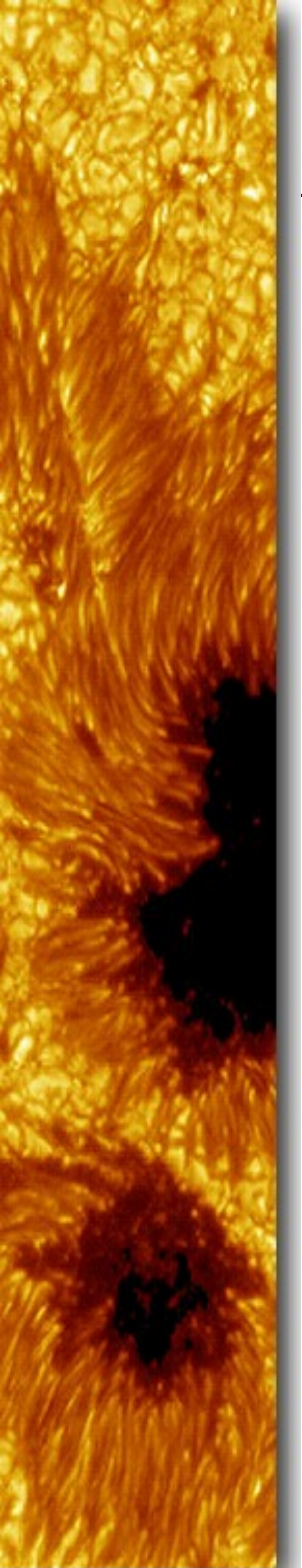
The sun rotates faster at the equator than near the poles.







This differential rotation might be responsible for magnetic activity of the sun.

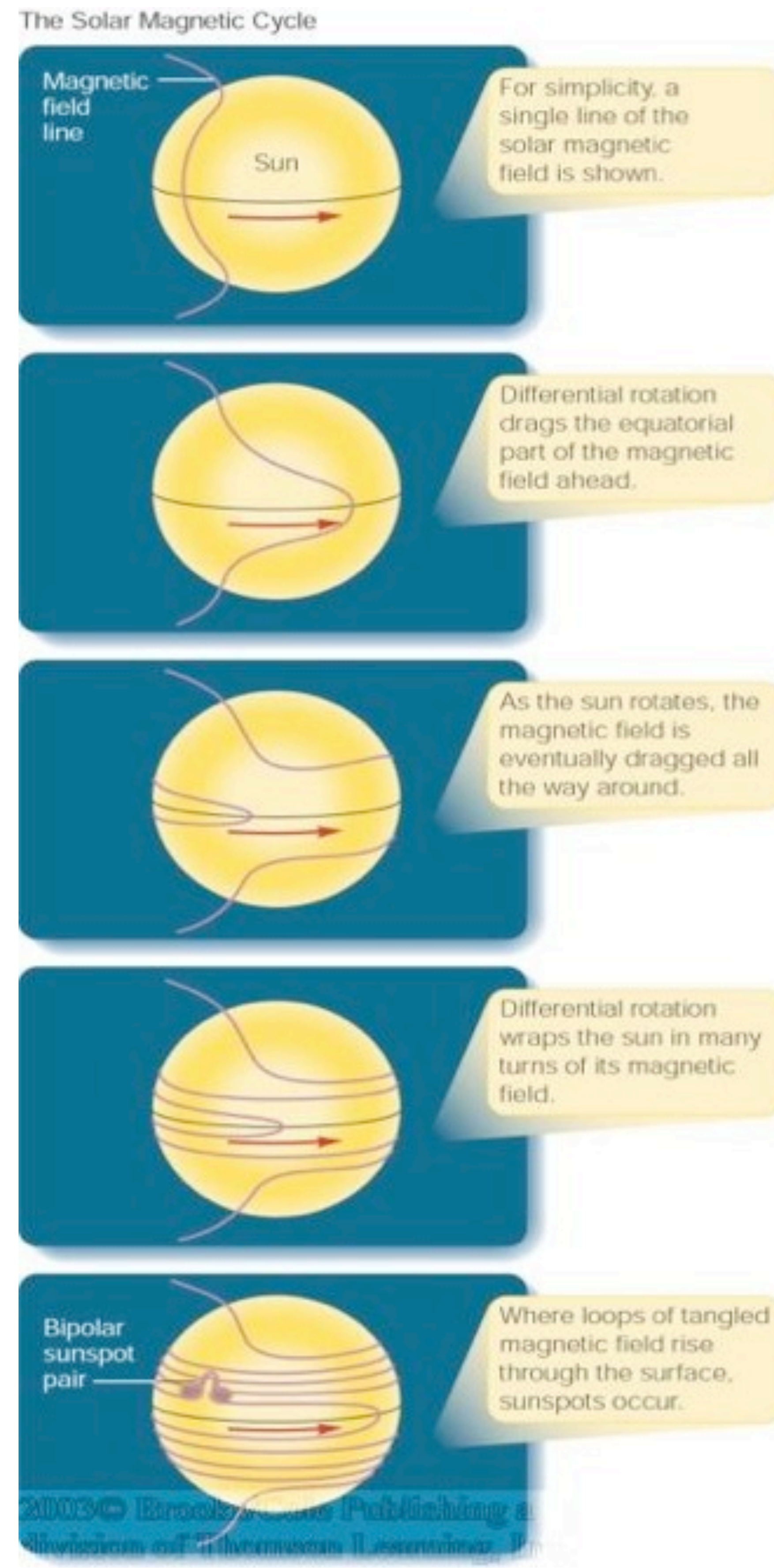


The Sun’s Magnetic Cycle

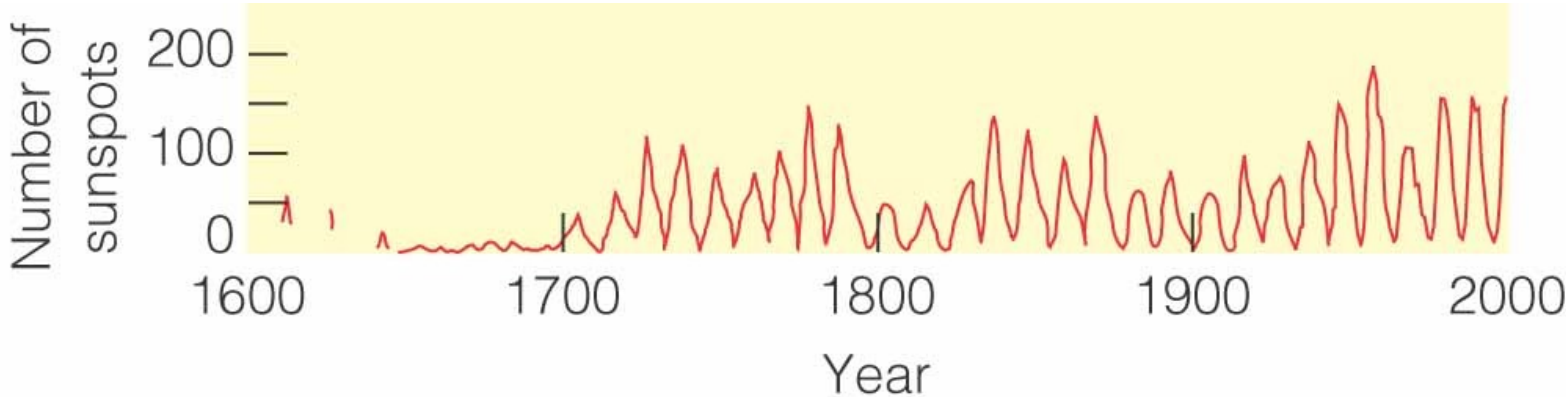
After 11 years, the magnetic field pattern becomes so complex that the field structure is re-arranged.

→ New magnetic field structure is similar to the original one, but reversed!

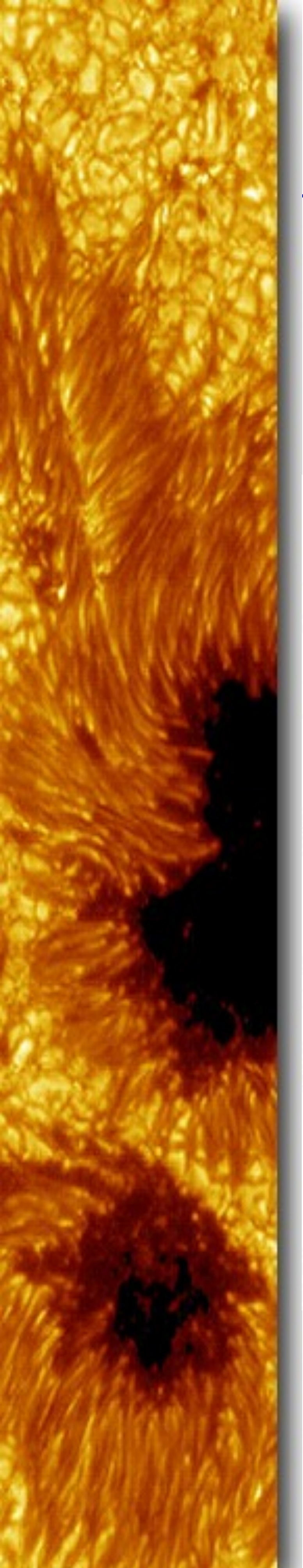
→ New 11-year cycle starts with reversed magnetic-field orientation



11-year period of solar activity

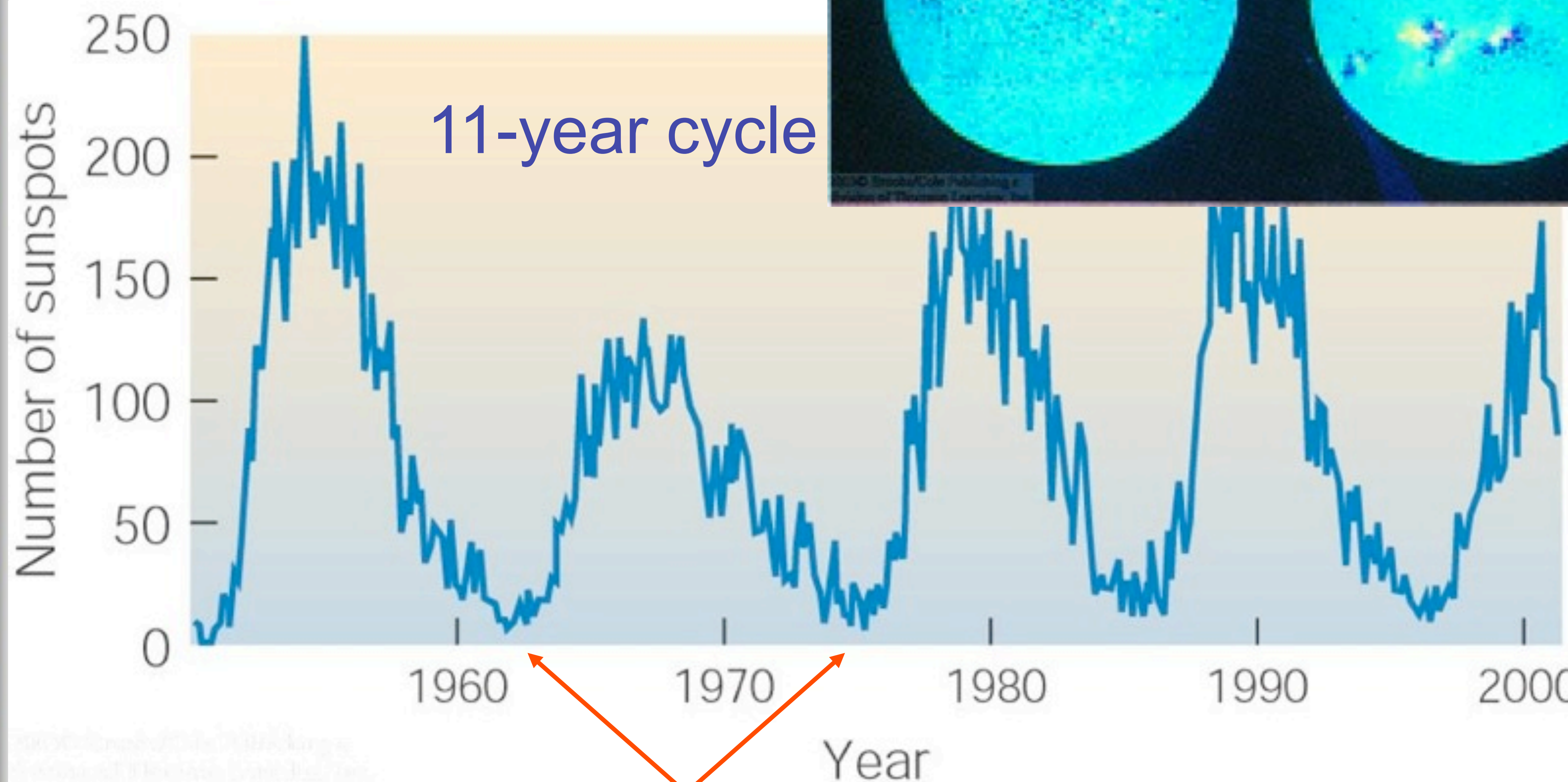


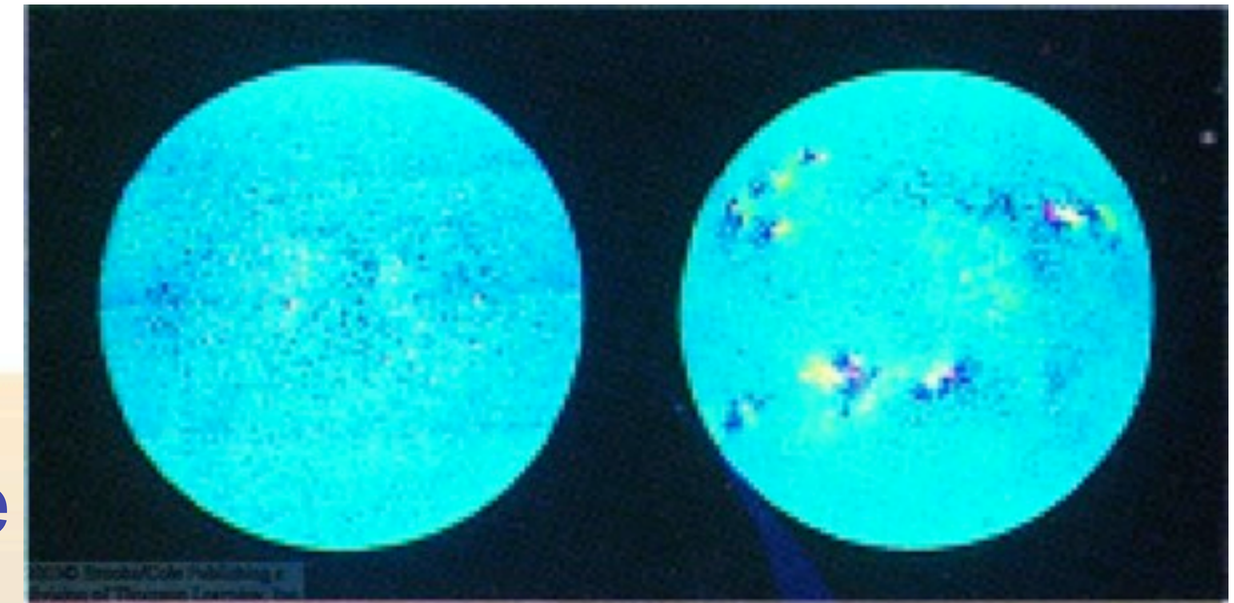
Slide



The Solar Cycle

After 11 years, North/South order of leading/trailing sun spots is reversed

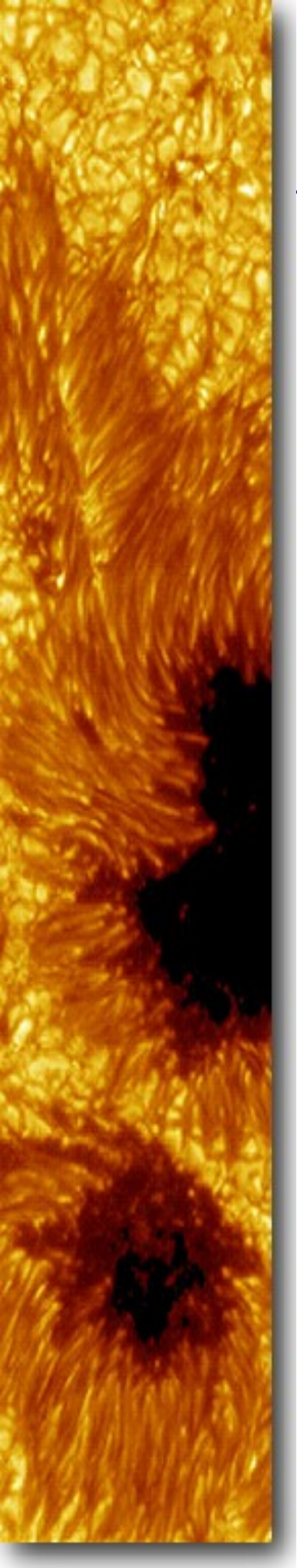




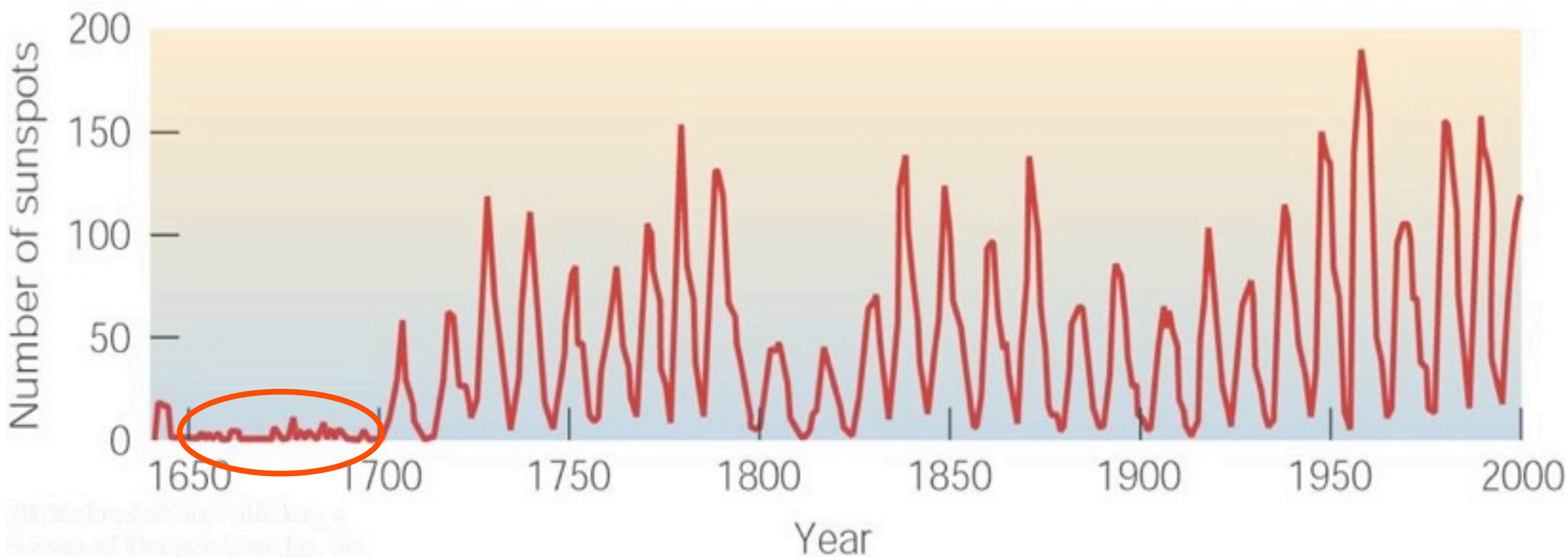
Reversal of magnetic polarity

=> Total solar cycle = 22 years

The Maunder Minimum



The sun spot number also fluctuates
on much longer time scales:

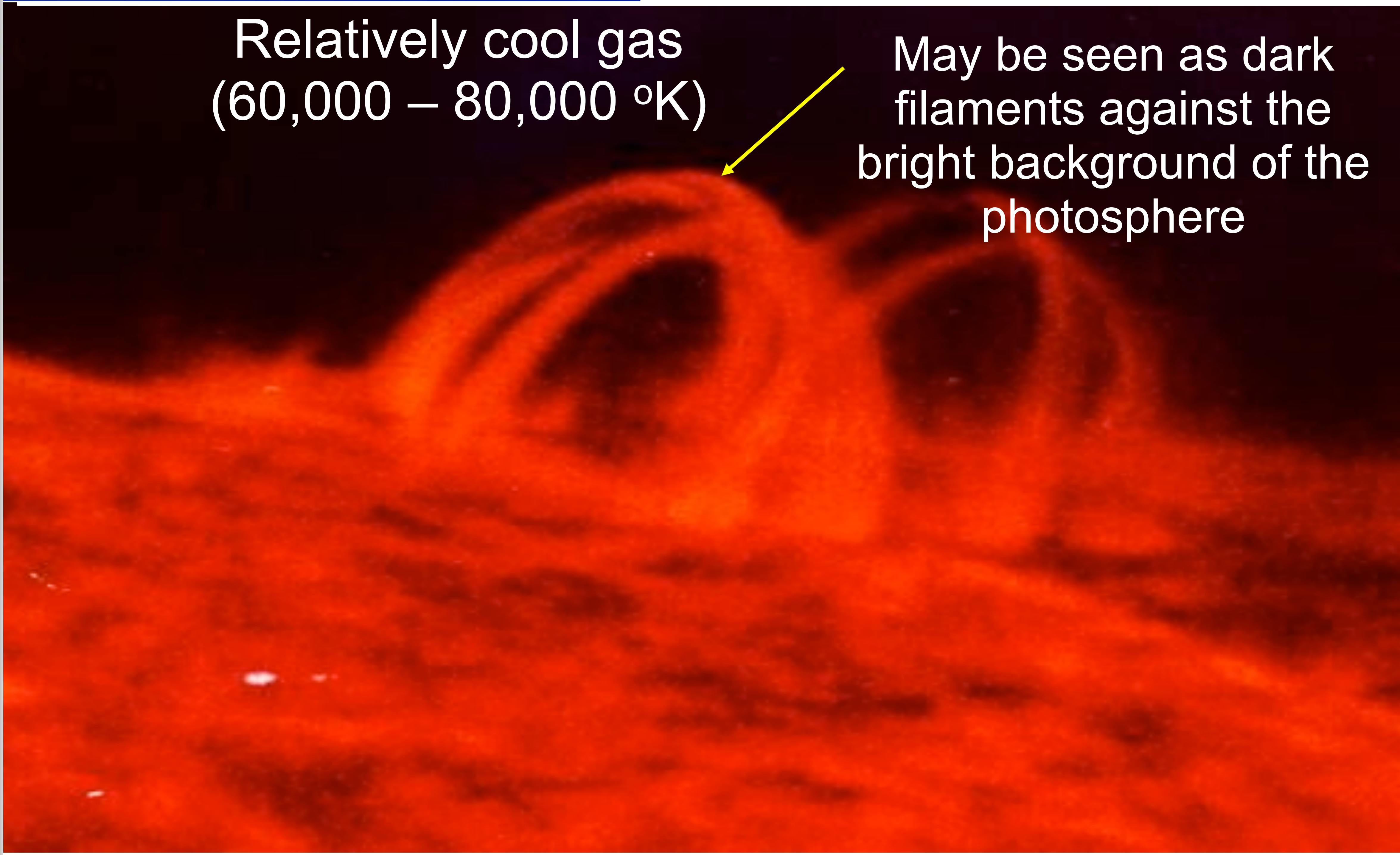


Historical data indicate a very quiet phase of the sun, ~ 1650 – 1700: The **Maunder Minimum**

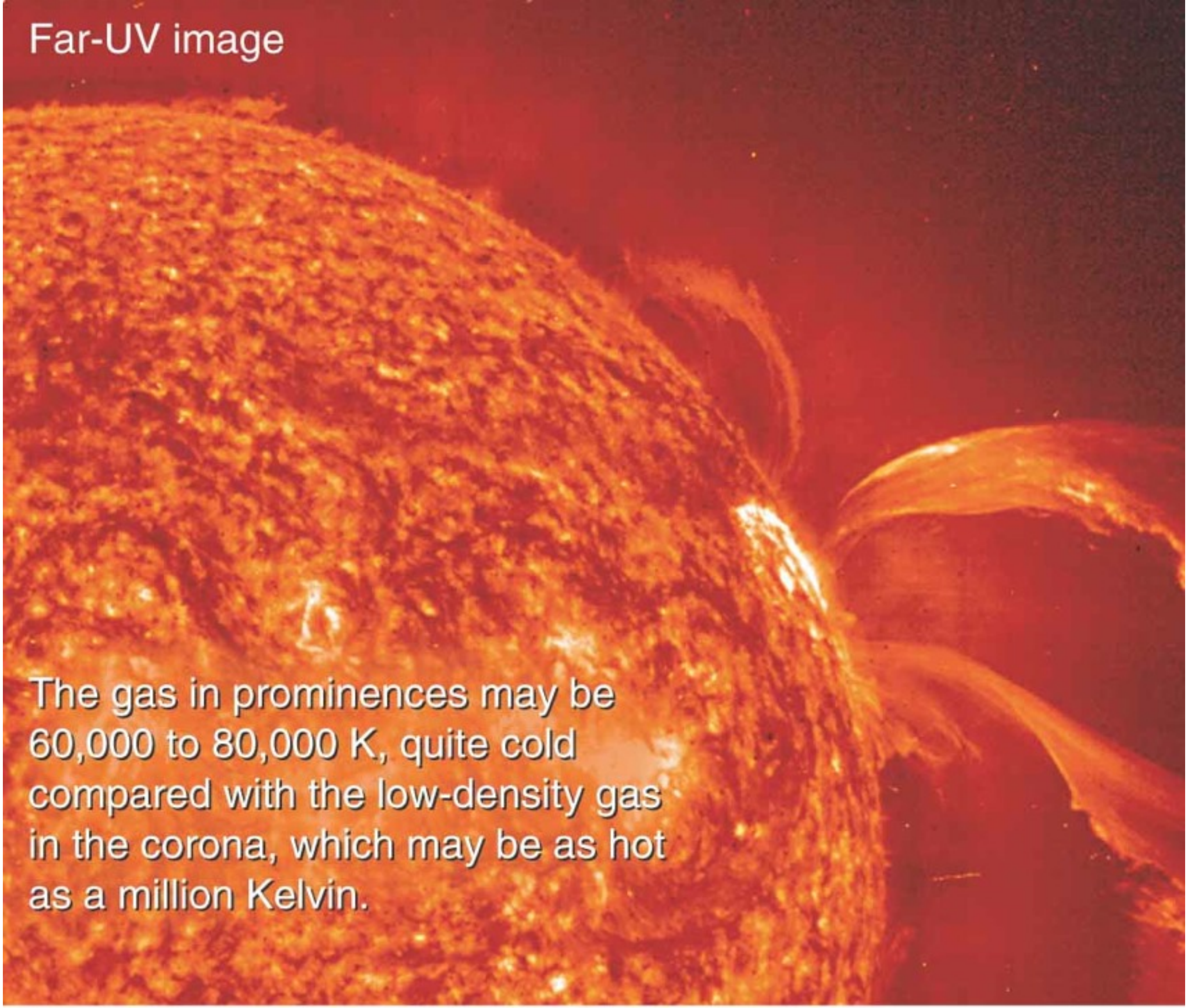
The Little Ice Age: unusually cold winters in the XVII-XVIII cen.

Slide

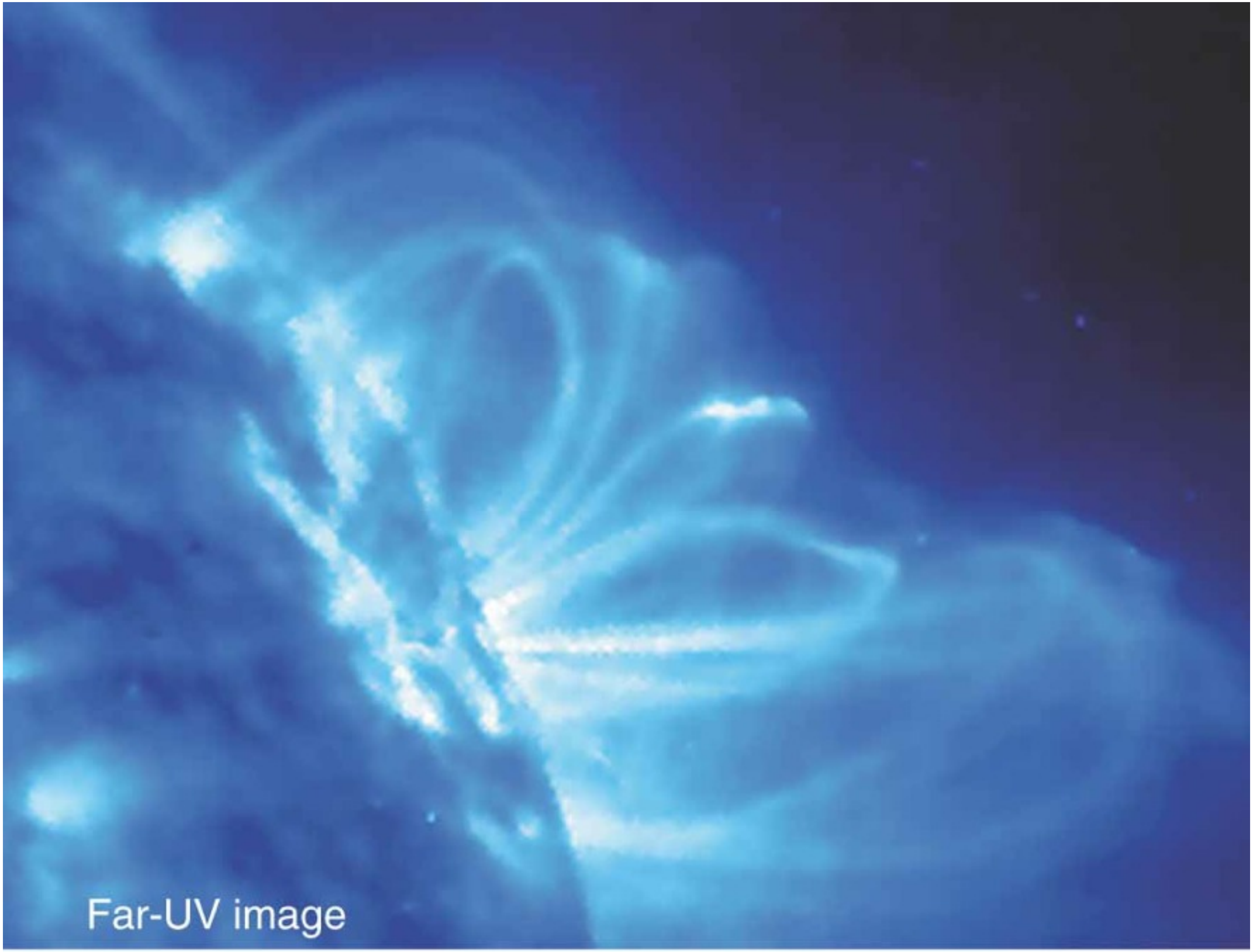
Prominences



Looped Prominences: gas ejected from the sun's photosphere, flowing along magnetic loops

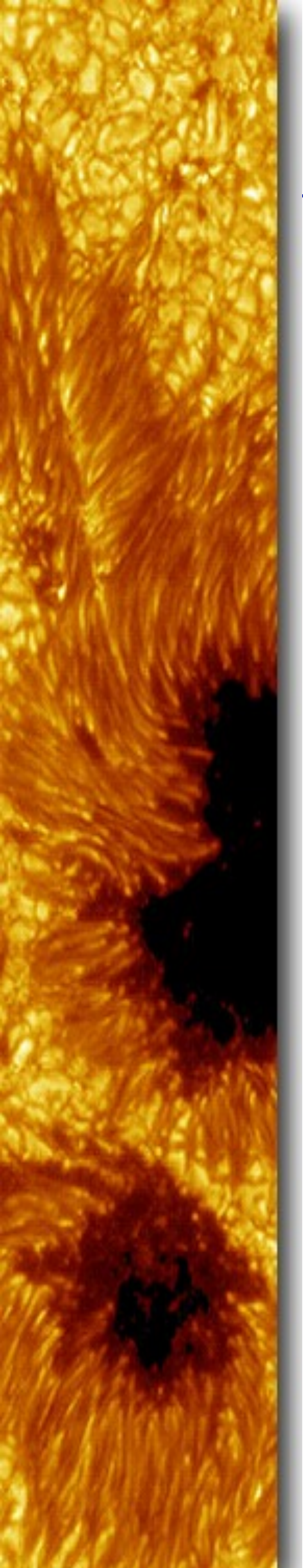

Far-UV image
The gas in prominences may be 60,000 to 80,000 K, quite cold compared with the low-density gas in the corona, which may be as hot as a million Kelvin.

Slide

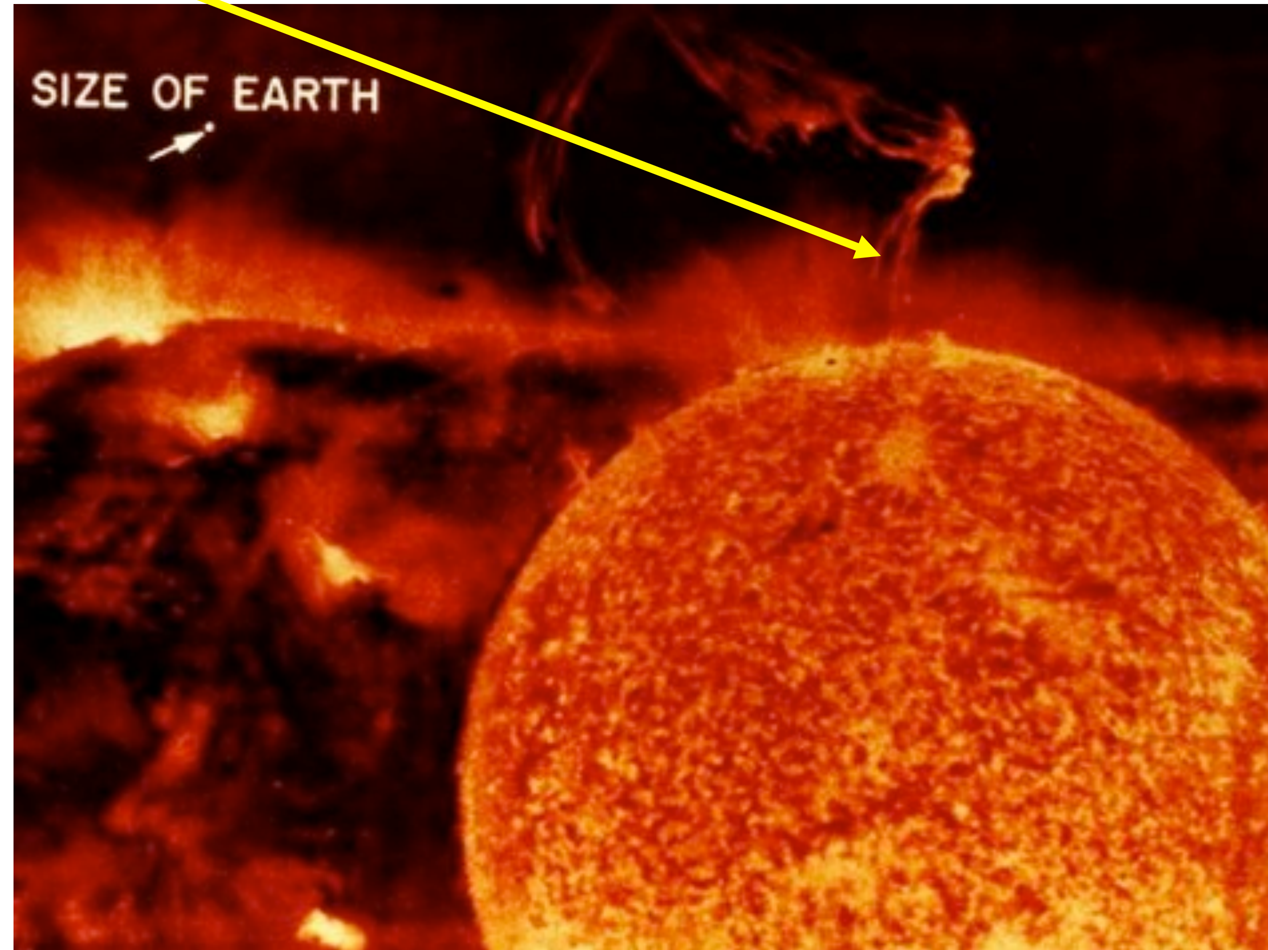


Slide

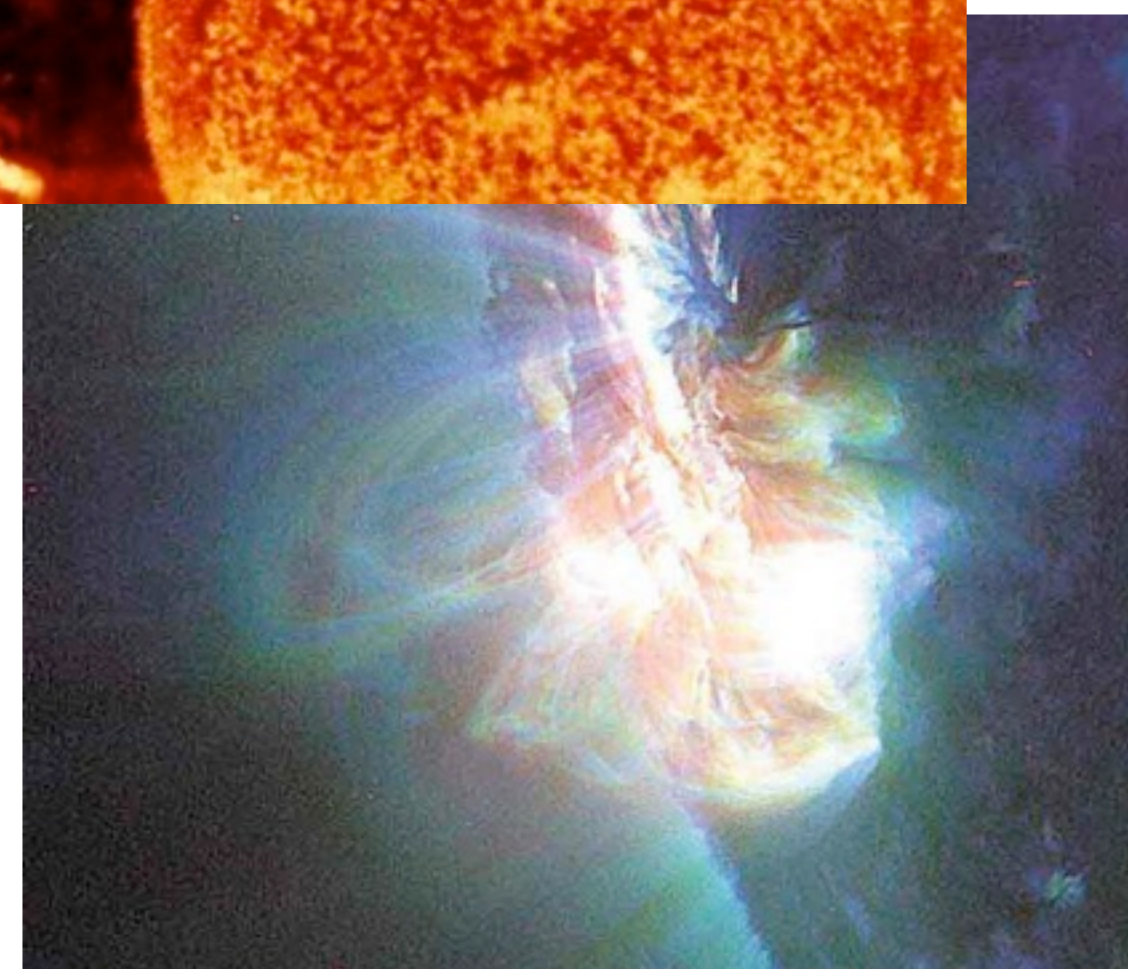
Eruptive Prominences

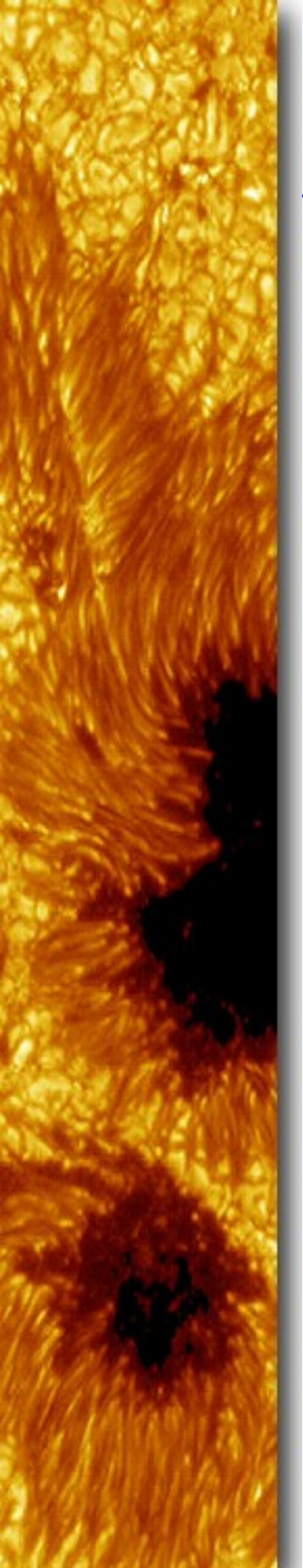


(Ultraviolet images)



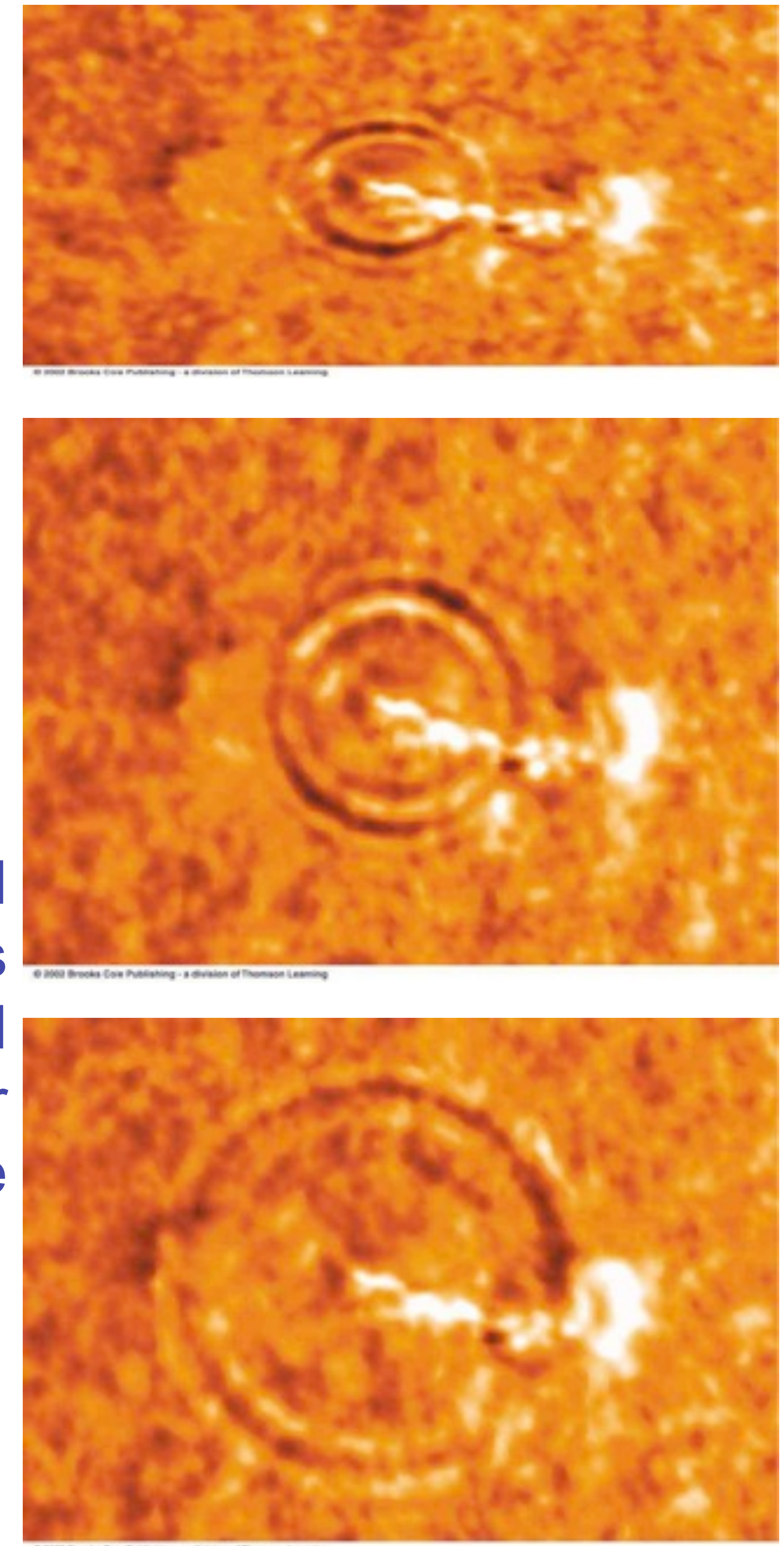
Extreme events (**solar flares**) can significantly influence Earth's magnetic field structure and cause northern lights (*aurora borealis*).





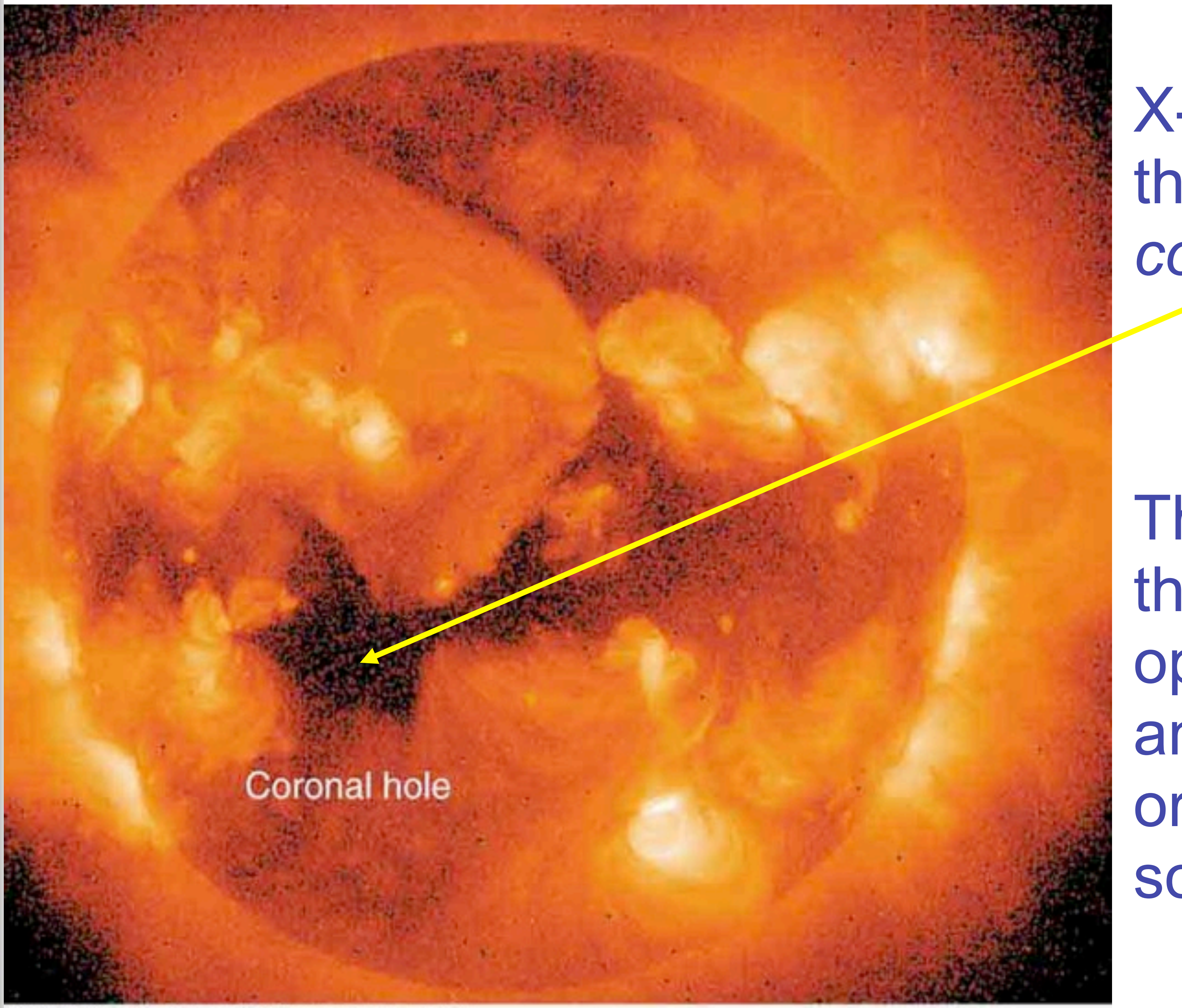
Solar Flares

~ 5 minutes



Sound waves produced by a solar flare

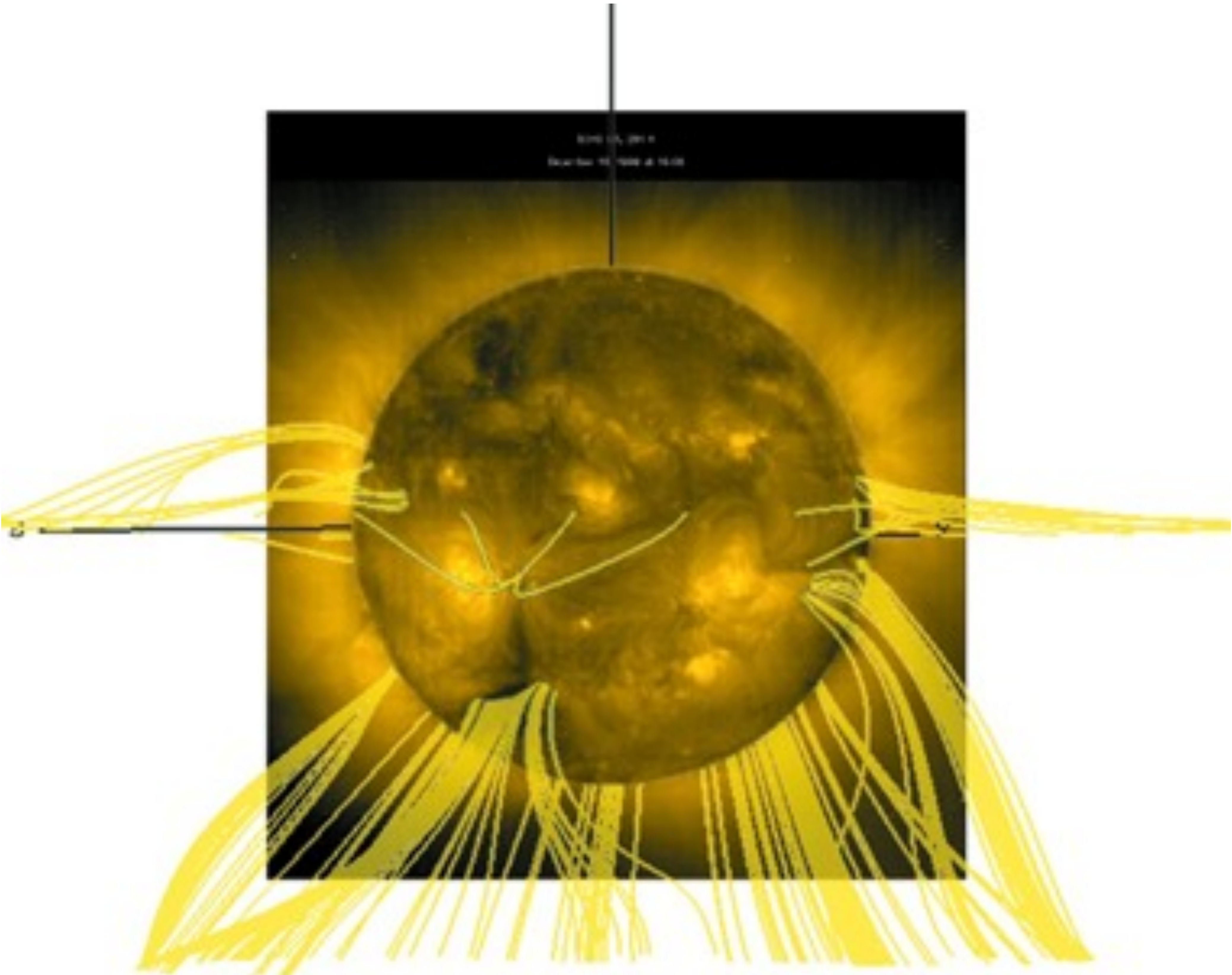
Coronal Holes



X-ray images of the sun reveal *coronal holes*.

These arise at the foot points of open field lines and are the origin of the solar wind.

Coronal holes

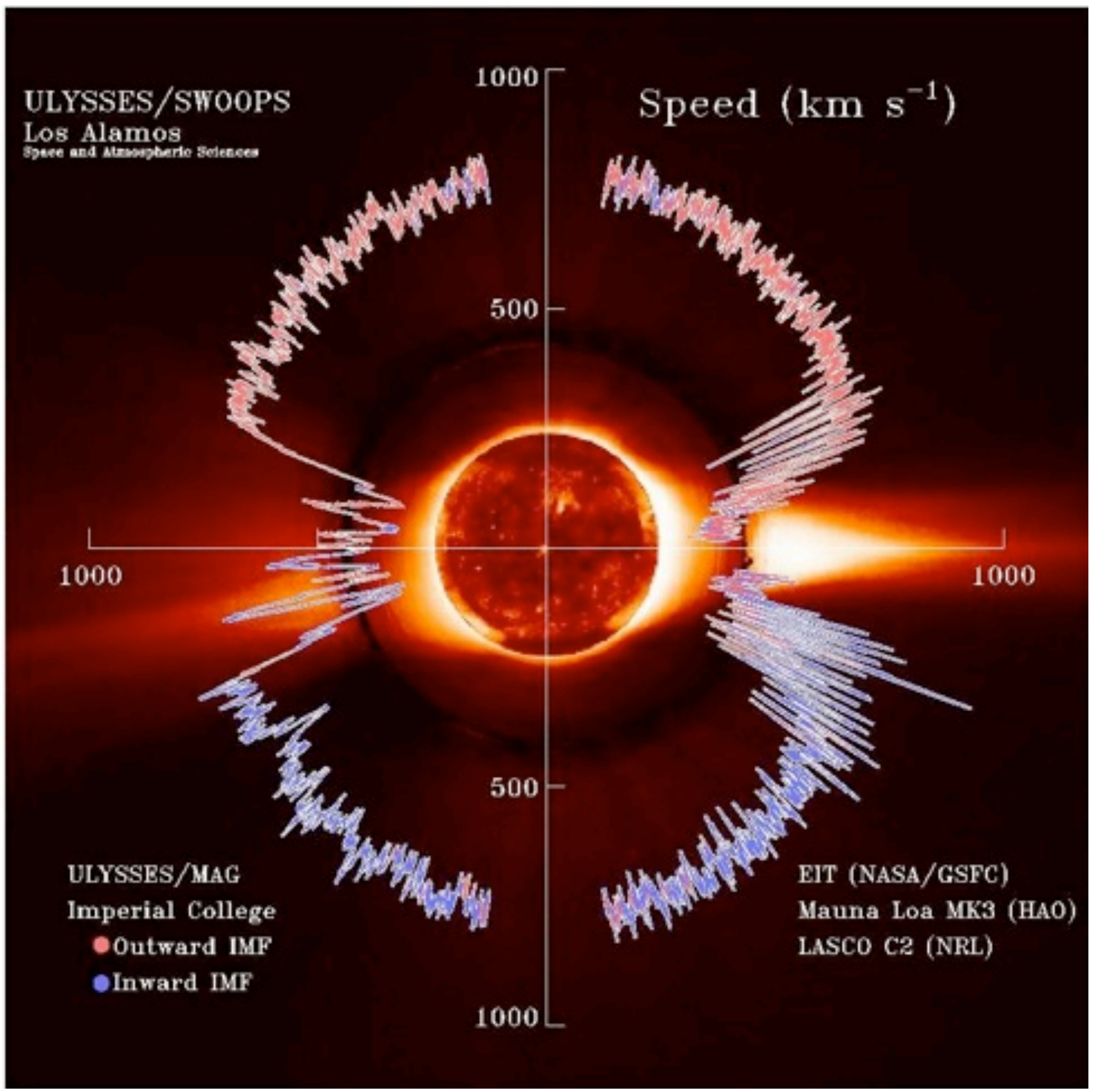


Slide

The Solar Wind

Constant flow of particles from the sun.

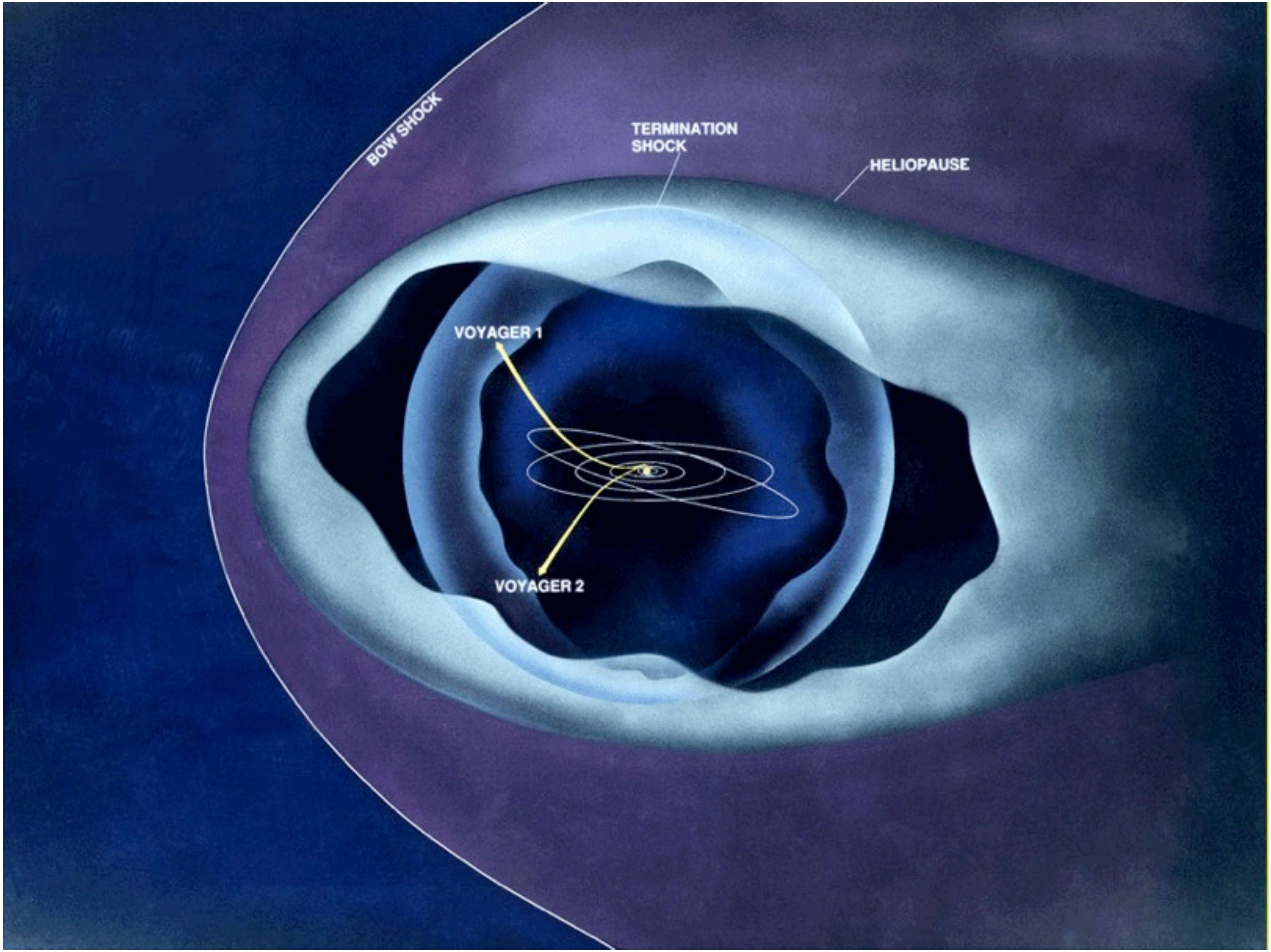
Velocity ≈ 300 – 800 km/s



⇒ **Sun is constantly losing mass:**

10^7 tons/year

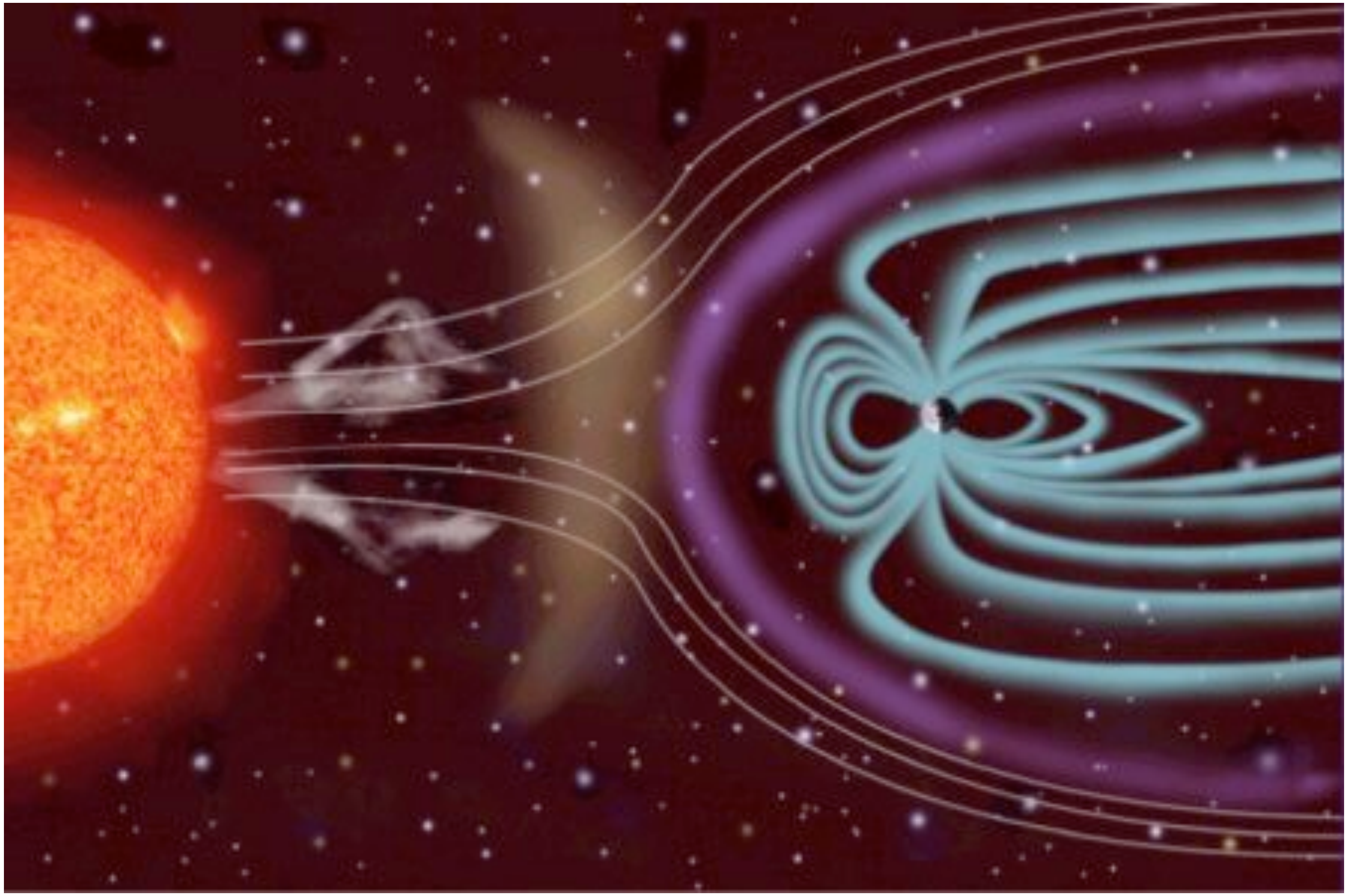
(≈ 10^{-14} of its mass per year)



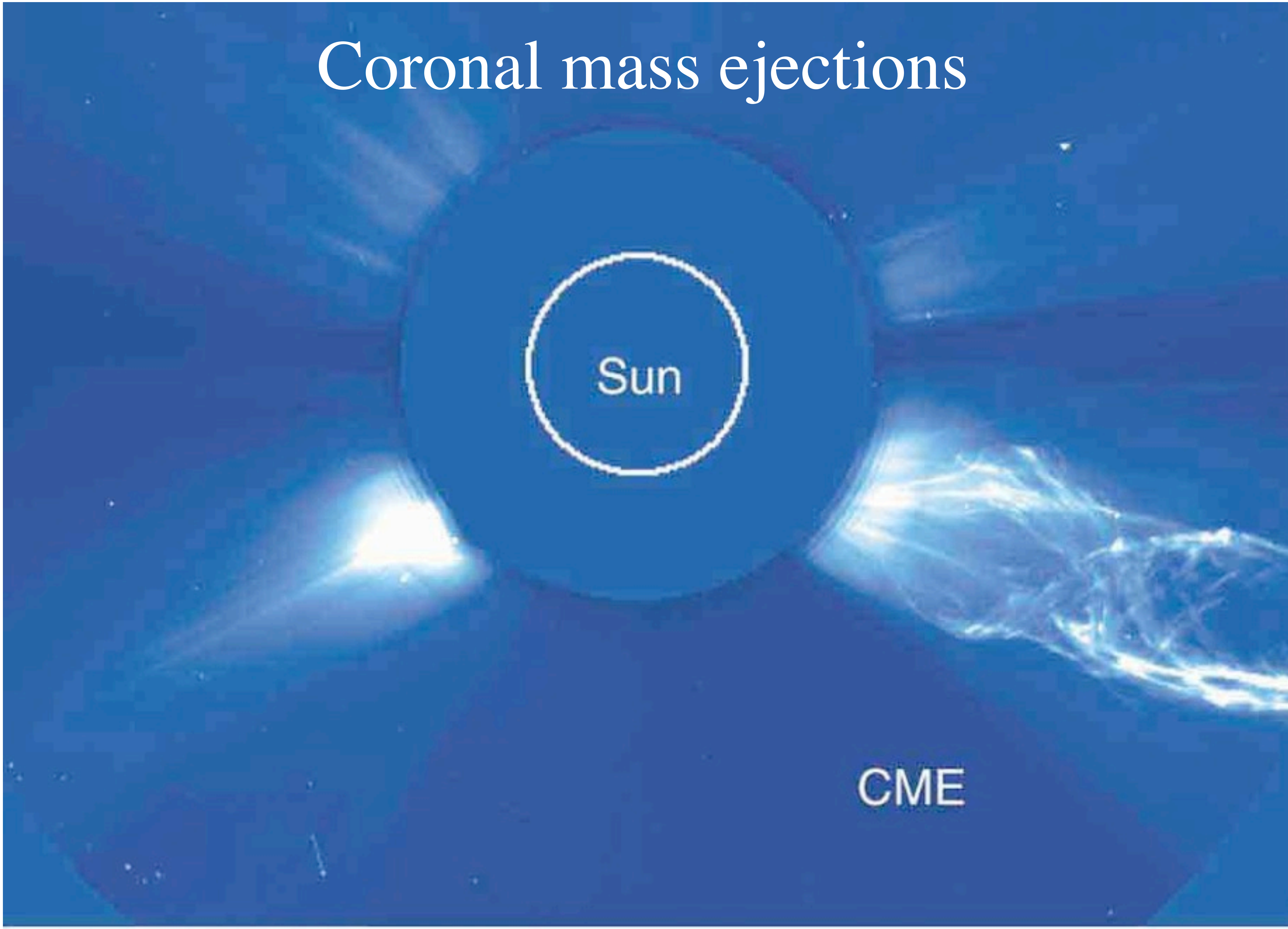
Solar wind creates a hot, rarefied plasma bubble in space

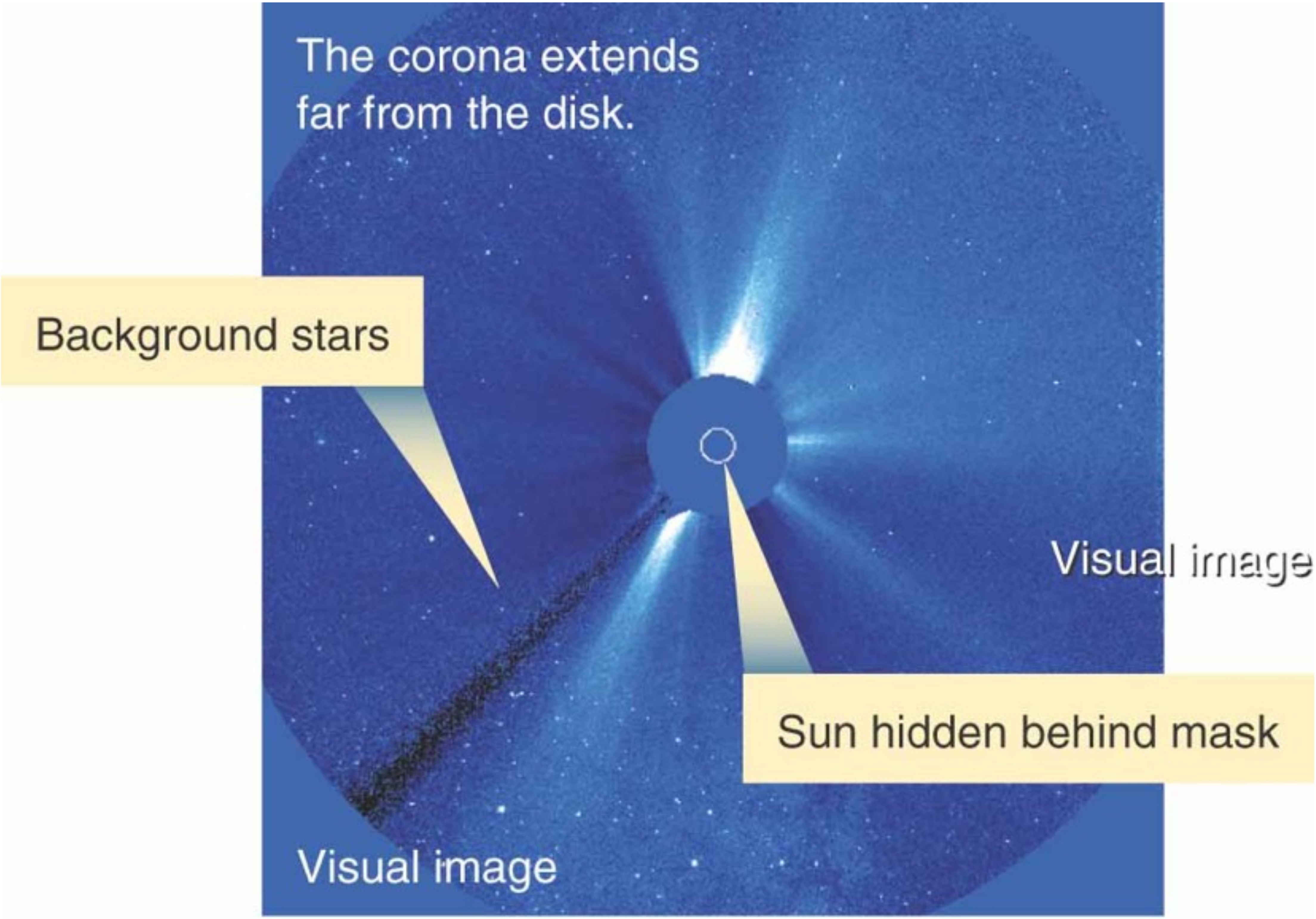
Slide

Solar wind hits the Earth’s magnetosphere

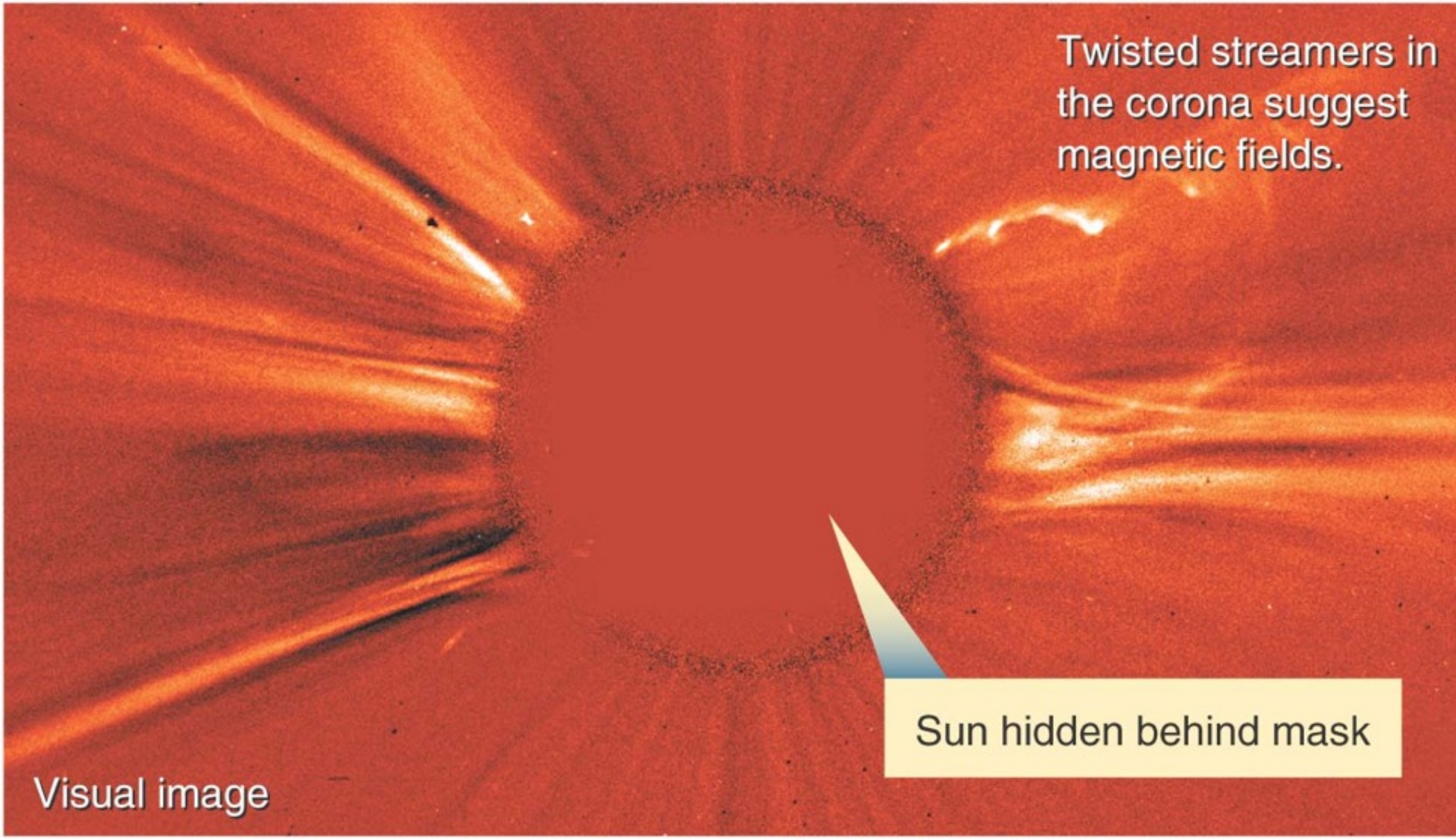


Coronal mass ejections




The corona extends
far from the disk.
Background stars
Visual image
Sun hidden behind mask
Visual image

Slide


Twisted streamers in the corona suggest magnetic fields.
Sun hidden behind mask
Visual image

Slide

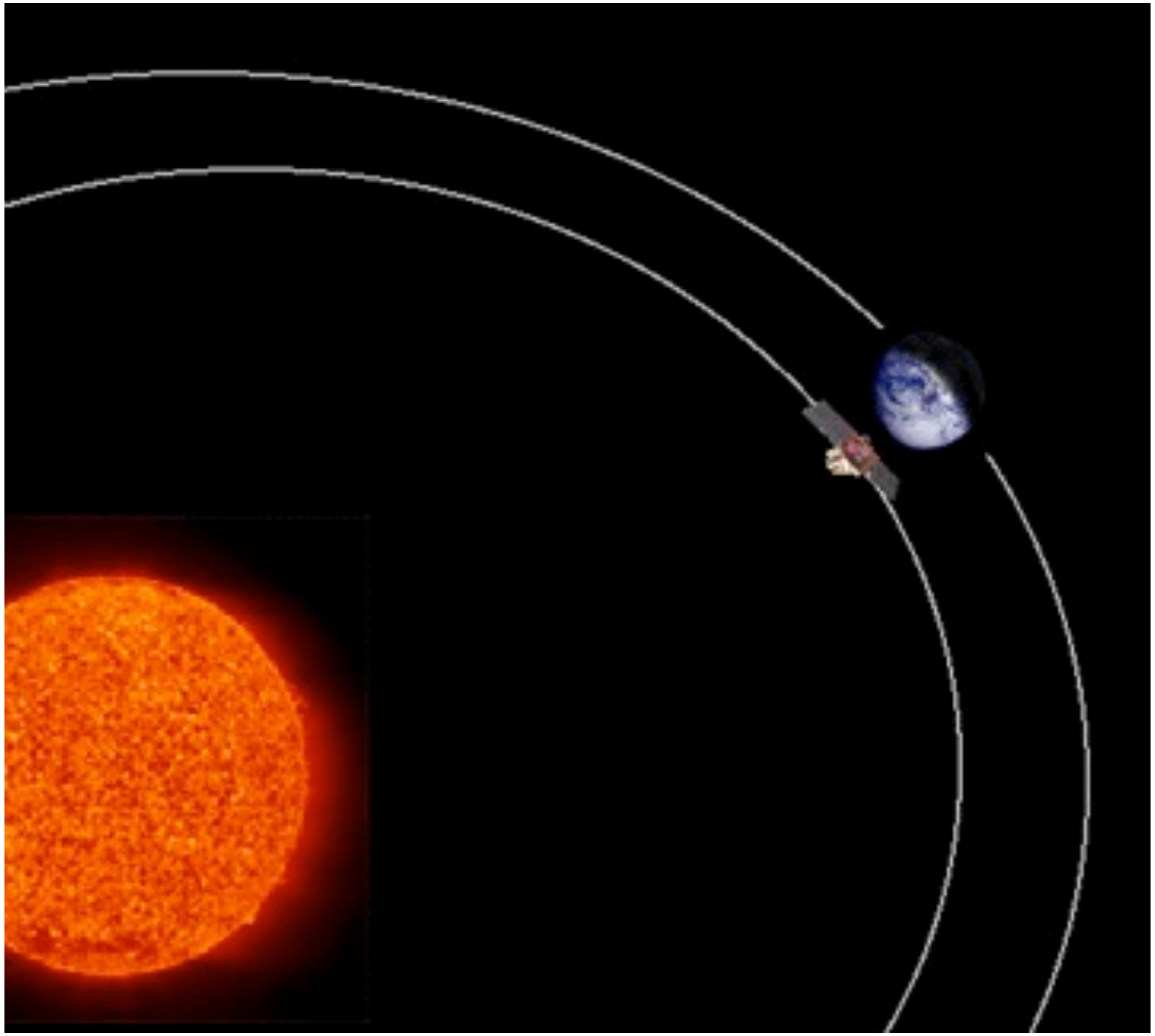
http://www.youtube.com/watch?v=JeAmKKrIVlc

http://www.youtube.com/watch?v=YJBrMXSn-hU

Summary – solar activity

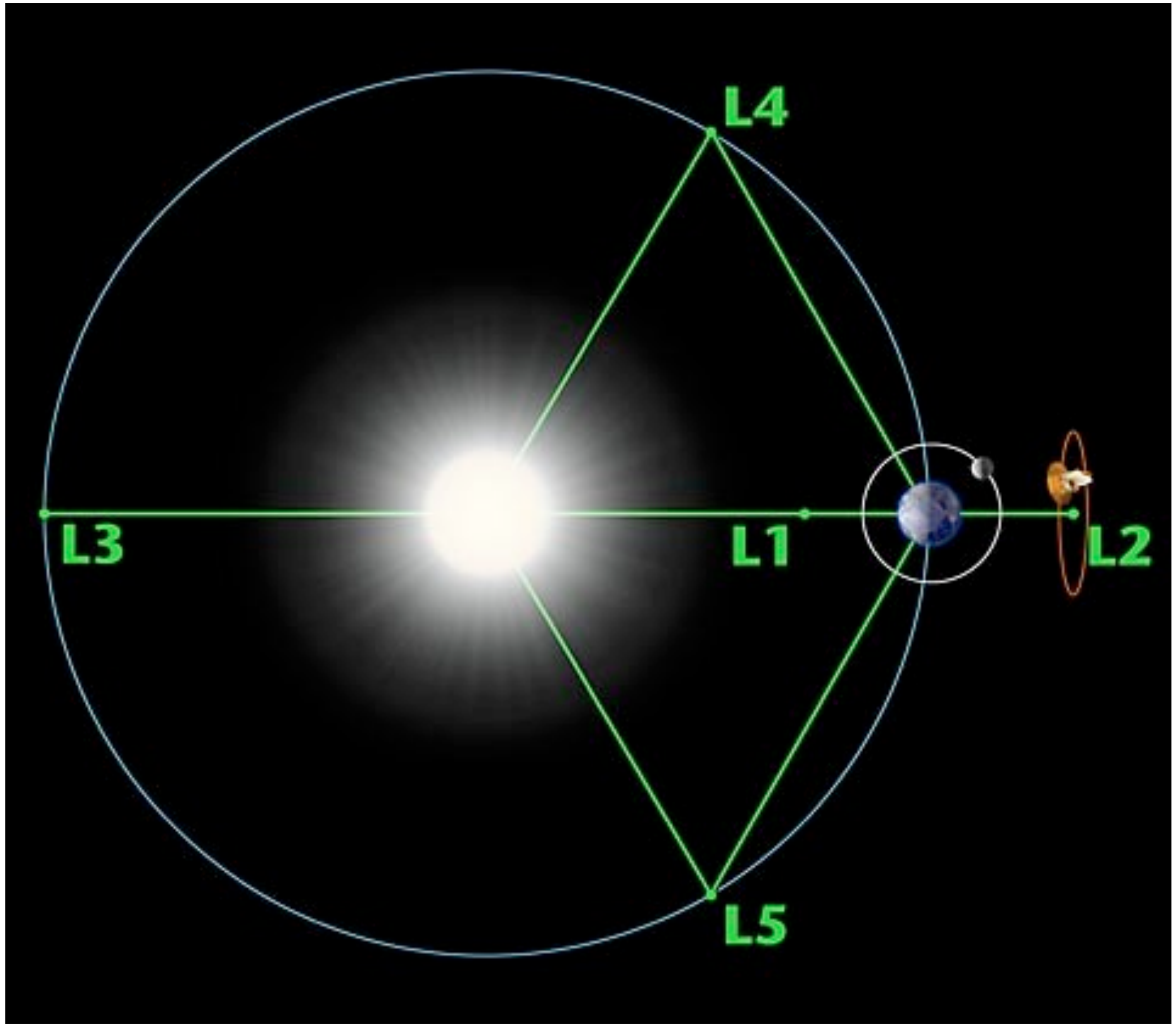
- It is driven by the Sun's magnetic field
- The magnetic field is generated by the differential rotation of the Sun
- Active regions are associated with sunspots that carry a strong magnetic field (~ 1000 G)
- Prominence is a solar plasma trapped in magnetic field arches above the active region
- Violent eruptive phenomena: solar flares (equivalent to billions of H-bombs!) and coronal mass ejections
- Ejected plasma shakes the Earth's magnetic field and causes the magnetic storm

SOHO: The Solar and Heliospheric Observatory



1.5 million km from the Earth at the L1 point

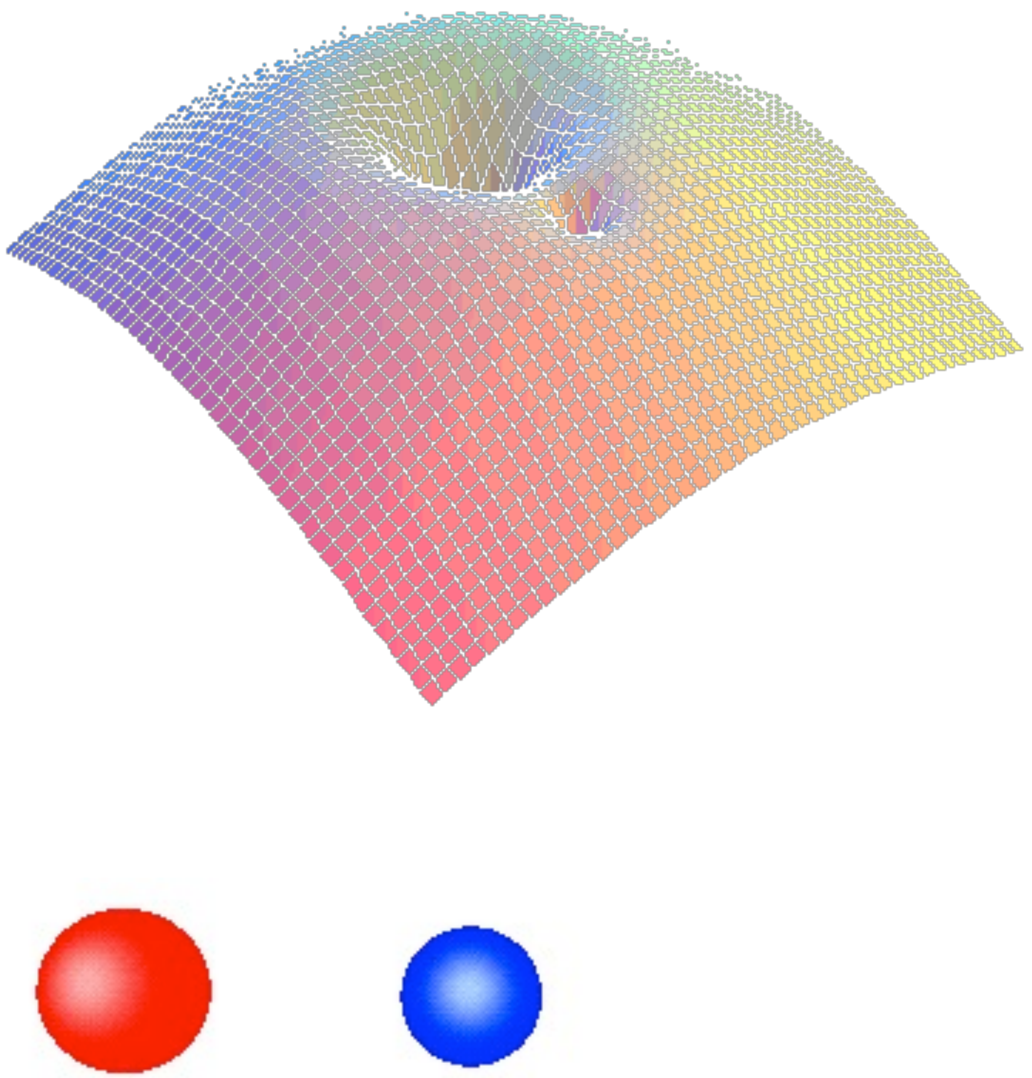
The Lagrange Points

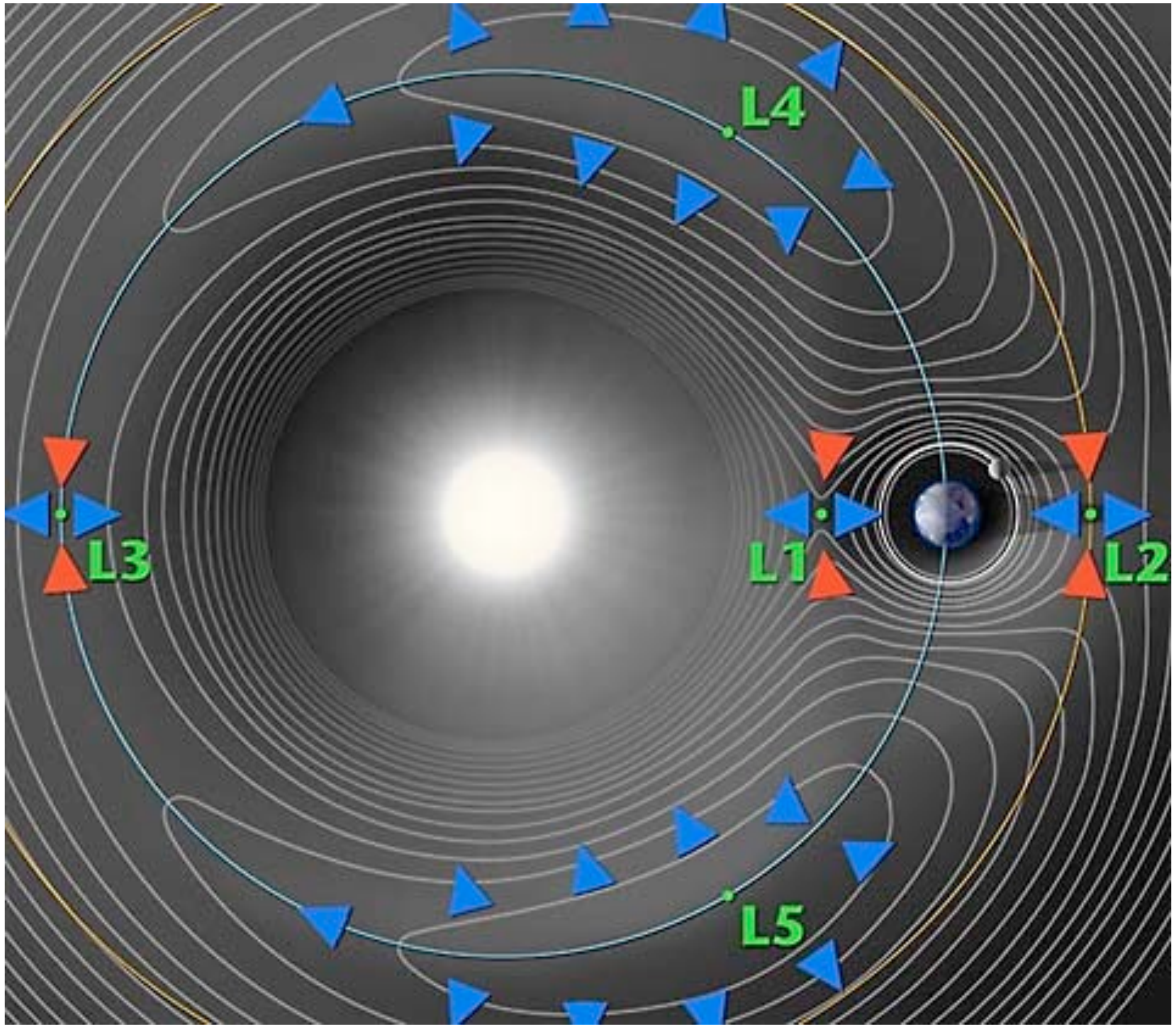


L1: SOHO
L2: WMAP (unstable points)
L3: empty

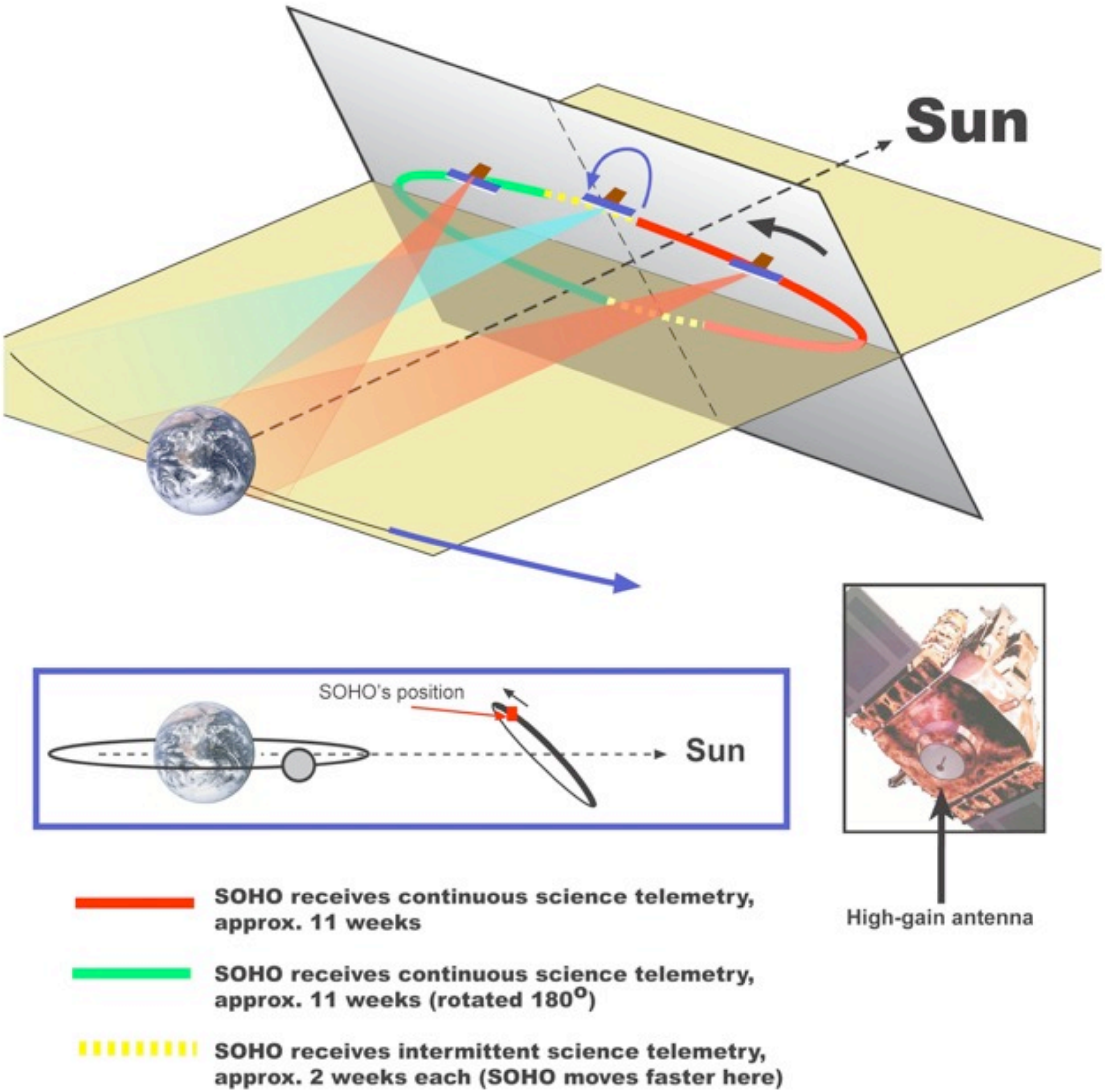
L4,5: Trojans (stable points)

Gravitational potential in the corotating frame





SOHO: The Solar and Heliospheric Observatory



Slide

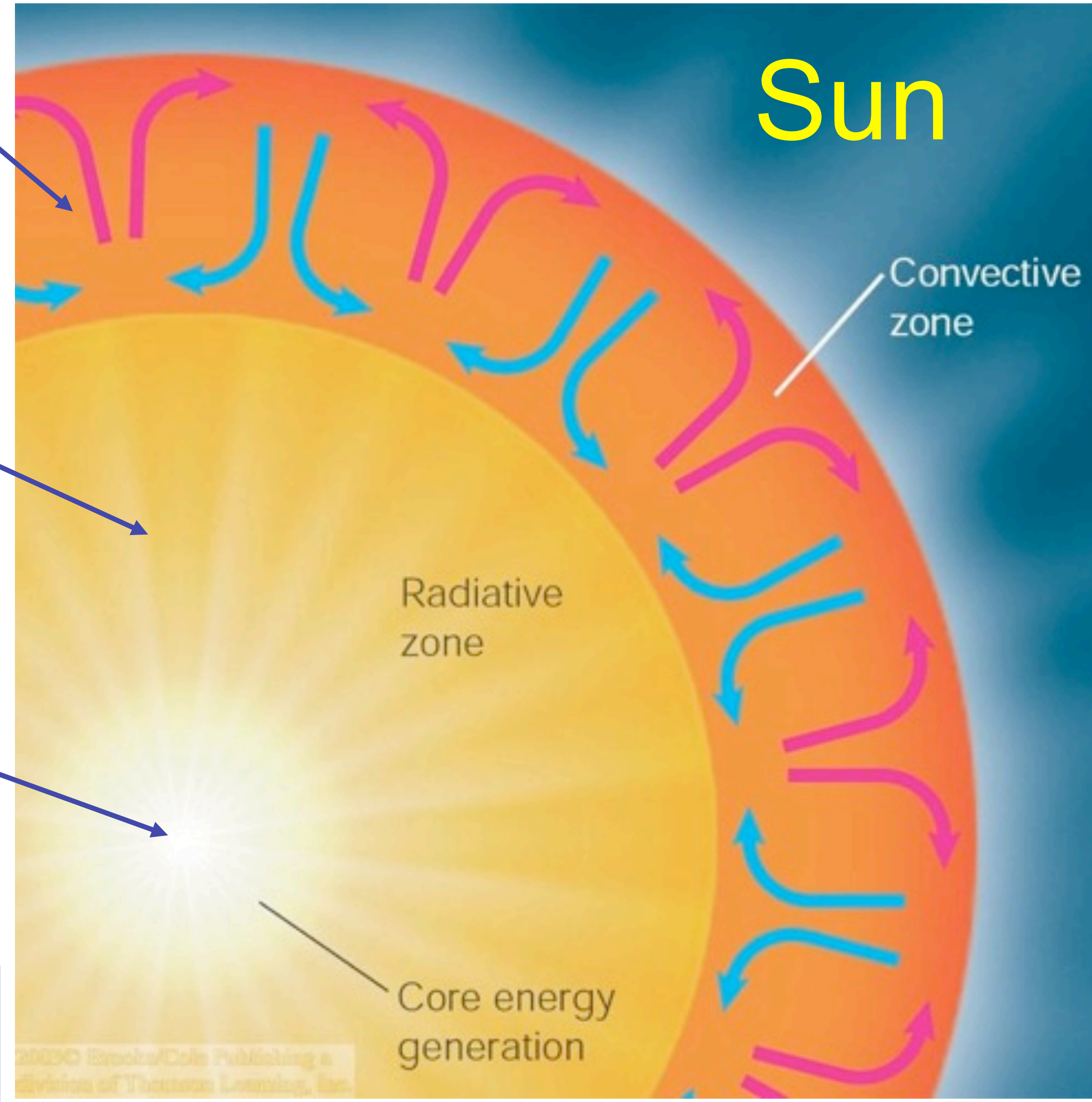
Structure: sun-like stars

Flow of energy

Energy transport via convection

Energy transport via radiation

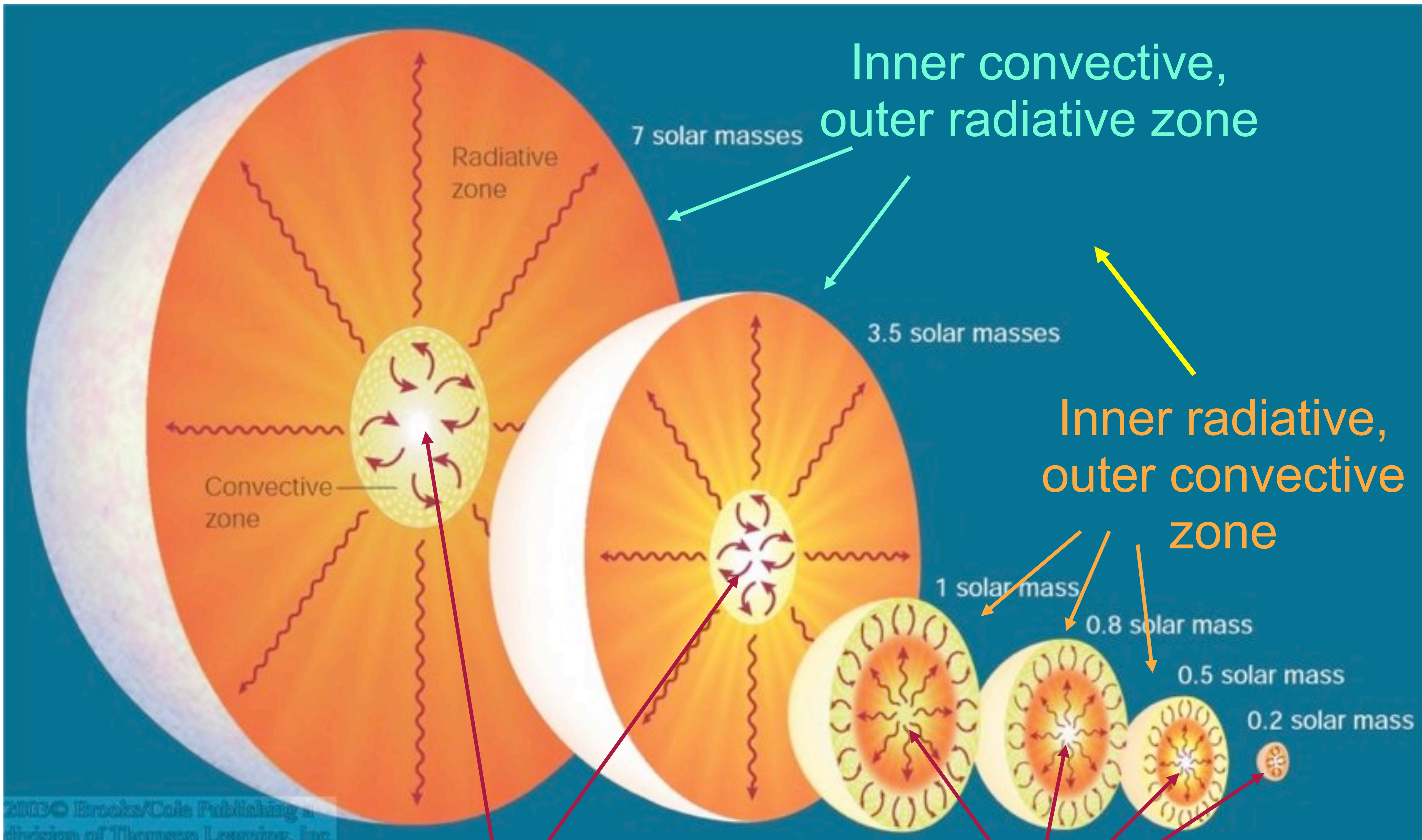
Energy generation via nuclear fusion



Temperature, density and pressure decreasing

Basically the same structure for all stars with approx. 1 solar mass or less.

Structure: all stars



CNO cycle dominant PP chain dominant

Summary: Stellar Structure

Convective Core, radiative envelope;

Energy generation through CNO Cycle

Radiative Core, convective envelope;

Energy generation through PP Cycle