**Problem 14.** A trapezoid and a triangle are inscribed in circle. One side of the trapezoid is a diameter of the circle. Each side of the triangle is parallel to a side of the trapezoid. Prove that the trapezoid and triangle have the same area.

**Solution.** We may assume the circle has radius 1. An inscribed trapezoid with one side a diameter must isosceles. It follows that the triangle is also isosceles with base parallel to the parallel sides of the trapezoid and congruent sides parallel to the congruent sides of the trapezoid. See Figure 1.

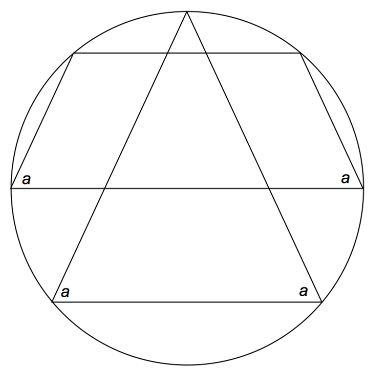
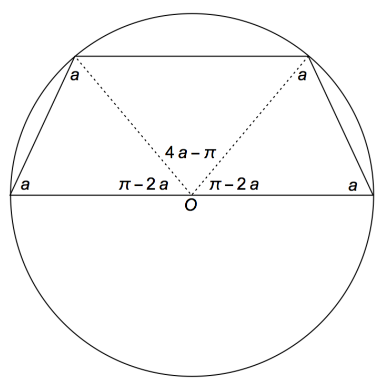


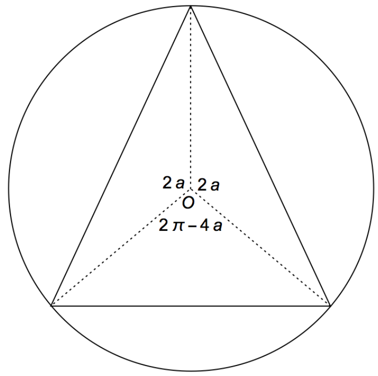

Figure 1: An isosceles trapezoid and an isosceles triangle.

Let $O$ be the center of the circle, and partition each figure into triangles with radial segments to the vertices. The area of each figure is then the sum of the areas of the three triangles into which it is partitioned. For the trapezoid,




$$\text{Area} = \frac{1}{2}\sin(\pi - 2a) + \frac{1}{2}\sin(4a - \pi) + \frac{1}{2}\sin(\pi - 2a) = \sin(2a) - \frac{1}{2}\sin(4a).$$

For the traingle




$$\text{Area} = \frac{1}{2}\sin(2a) + \frac{1}{2}\sin(2\pi - 4a) + \frac{1}{2}\sin(2a) = \sin(2a) - \frac{1}{2}\sin(4a).$$

The area of the trapezoid is equal to the area of the triangle.