Algebra 2 Trig Lesson Page

# Angle Sum and Difference, Double Angle and Half Angle Formulas

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Hipparchus, considered to be the most eminent of Greek astronomers (born 160 B.C.), derived the formulas for $\sin(A \pm B)$ and $\cos(A \pm B)$.
The following formulas (or formulae, in Latin) are trigonometric identities.

## Sum and Difference Formulas:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Double Angle Formulas:

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$
$$= 2\cos^2 A - 1$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

## Half Angle Formulas:

$$\sin\left(\frac{1}{2}A\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$
$$\cos\left(\frac{1}{2}A\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$
$$\tan\left(\frac{1}{2}A\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

Or ...

$$\tan\left(\frac{1}{2}A\right) = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

To find these types of formulas
for the reciprocal trig functions,
(sec, csc, cot),
utilize the relationship between
the functions given and the
function you need.

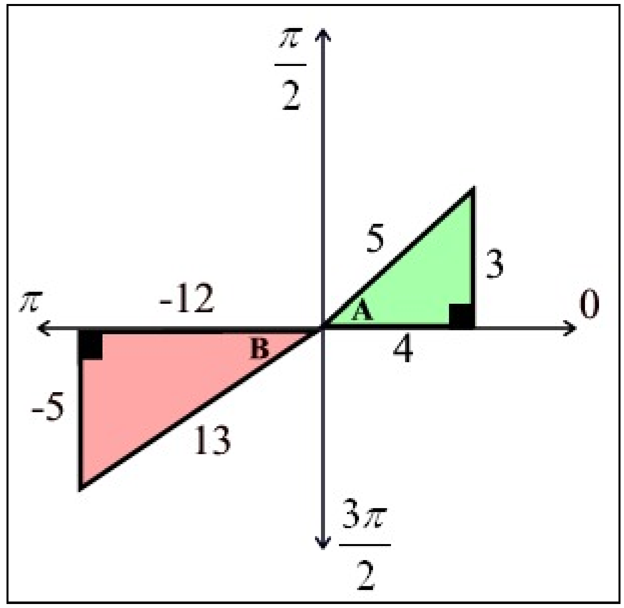
## Example 1:

If $\sin A = \frac{3}{5}$ and $\cos B = \frac{-12}{13}$ where $0 \le A \le \frac{\pi}{2}$ and $\pi \le B \le \frac{3\pi}{2}$

find *a.)* $\cos(A+B)$

*b.)* $\tan(A-B)$

**Solution:** The given information produces the triangles shown at the right. The Pythagorean Theorem, or a Pythagorean triple, is used to find the missing sides. Using the information from the triangles, find the answers to parts a and b.




a.)

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A+B) = \frac{4}{5} \bullet \frac{-12}{13} - \frac{3}{5} \bullet \frac{-5}{13}$$

$$\cos(A+B) = \frac{-48}{65} - \frac{-15}{65} = \frac{-33}{65}$$

b.)

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \bullet \frac{5}{12}} = \frac{16}{63}$$

If you did not know the formula for tan(*A* - *B*), the relationship between tangent, sine and cosine could be used to solve this problem.

$$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$$

$$\tan(A-B) = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

$$\tan(A-B) = \frac{\frac{3}{5} \bullet \frac{-12}{13} - \frac{4}{5} \bullet \frac{-5}{13}}{\frac{4}{5} \bullet \frac{-12}{13} + \frac{3}{5} \bullet \frac{-5}{13}}$$

$$\tan(A-B) = \frac{\frac{-16}{65}}{\frac{-63}{65}} = \frac{16}{63}$$

## Example 2:

Using the half angle formula, find the exact value of cos 15º.

**Solution:**

The positive square root is chosen because cos 15º lies in Quadrant I.

$$\cos 15^\circ = \cos\left(\frac{1}{2}30^\circ\right) = +\sqrt{\frac{1+\cos 30^\circ}{2}}$$

$$= +\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = +\sqrt{\frac{2+\sqrt{3}}{4}} = +\frac{\sqrt{2+\sqrt{3}}}{2}$$

## Example 3:

Given that $\tan x = \frac{a}{b}$, and $\pi \le x \le \frac{3\pi}{2}$

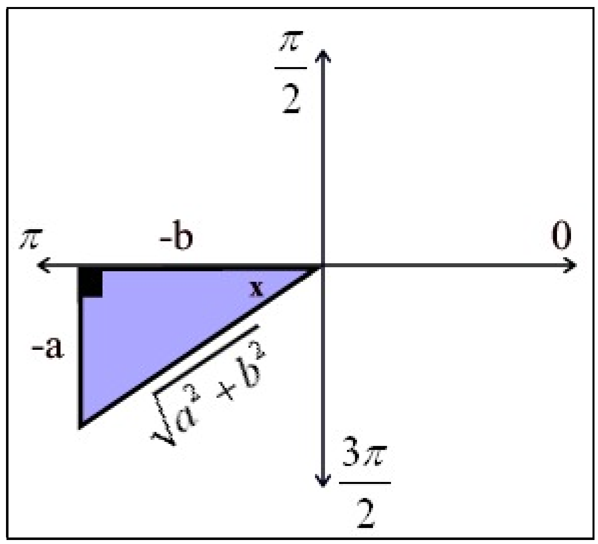
evaluate in terms of $a$ and $b$:

$a.)$ $\sin 2x$

$b.)$ $\tan 2x$

*Tricky one!!*

**Solution:** The given information produces the triangle shown at the right. Note the signs associated with *a* and *b*. The Pythagorean Theorem is used to find the hypotenuse. Using the information from the triangle, find the answers to parts a and b.




a.)

$$\sin 2x = 2\sin x\cos x$$

$$= 2\cdot\frac{-a}{\sqrt{a^2+b^2}}\cdot\frac{-b}{\sqrt{a^2+b^2}}$$

$$= \frac{2ab}{a^2+b^2}$$

b.)

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\frac{a}{b}}{1-\left(\frac{a}{b}\right)^2}$$

$$= \frac{\frac{2a}{b}}{\frac{b^2-a^2}{b^2}} = \frac{2a}{b}\cdot\frac{b^2}{b^2-a^2} = \frac{2ab}{b^2-a^2}$$

If you did not know the formula for tan 2*x*, you could use the relationship between tangent, sine and cosine to find the answer. This solution will utilize the answer from part a for the numerator.

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{\dfrac{2ab}{a^2+b^2}}{\left(\dfrac{-b}{\sqrt{a^2+b^2}}\right)^2 - \left(\dfrac{-a}{\sqrt{a^2+b^2}}\right)^2} = \frac{\dfrac{2ab}{a^2+b^2}}{\dfrac{b^2-a^2}{a^2+b^2}} = \frac{2ab}{b^2-a^2}$$

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