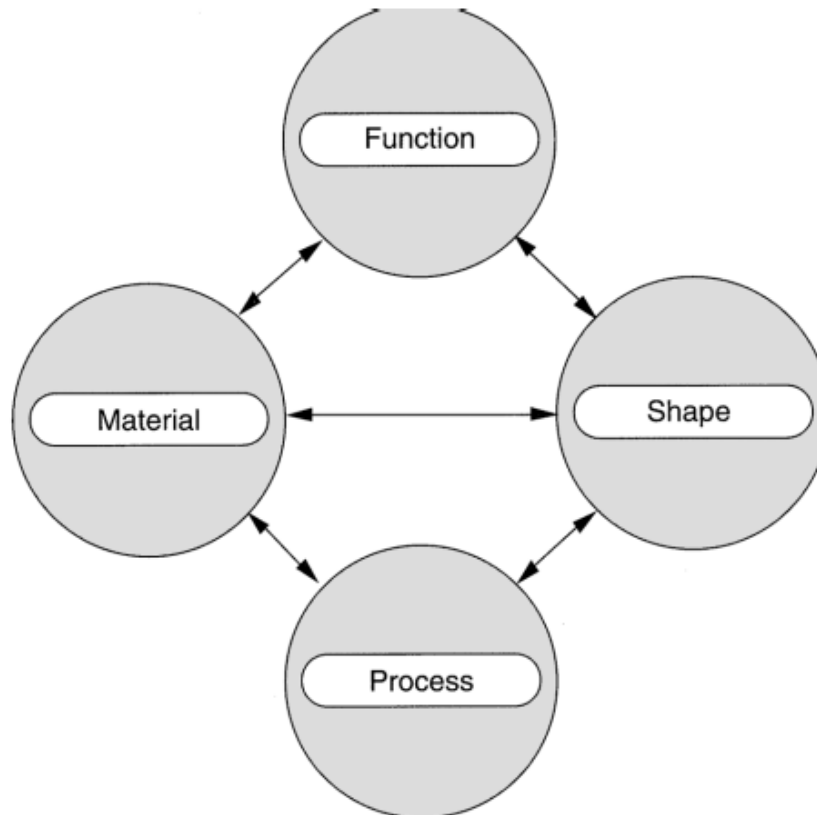# Materials Selection in Mechanical Design Michael Ashby

Chapter 1. Introduction
Mechanical components have mass, they carry loads, they conduct heat and electricity, they are exposed to wear and to corrosion, they are made of one or more materials; they have shape; and they must be manufactured. We need to understand how these activities are related.

Materials have limited design since man first build shelters, made clothes and undertook human conflict. But right now materials and the processes to shape them are developing faster than at any time in history; the challenges and opportunities are therefore greater than ever before. This book and this course are about strategies for exploiting materials for design.

Normally the choice of material is dictated by the design, but sometimes it is the other way around, the development of a new material changes the way something is designed. You can all think of examples in sporting equipment where for example the development of fiber-reinforced composites changed the pole vault or modified the fishing pole or the golf club. Today the number of materials available to the designer is vast, more than 50,000 are available. While some materials are standardized, removing some close options, new materials are developed that expands the list. The process for the selection

of the material depends on the stage of the design, with initial design suggesting the consideration of a wide range of material, but final design requiring more accurate information to choose between a few materials and to make the final accurate design.

The choice cannot be made independently of the forming and finishing processes for the materials for these significantly impact on properties and cost. Another factor that must be considered is the aesthetics of the final product. Here function overlaps marketing.

Evolution of engineering materials

Throughout history materials have limited design. The prehistoric ages were named for the chief materials that man employed: the stone age, the bronze age and the iron age. The evolution is shown schematically in Figure 2 in which the time scale is not linear.
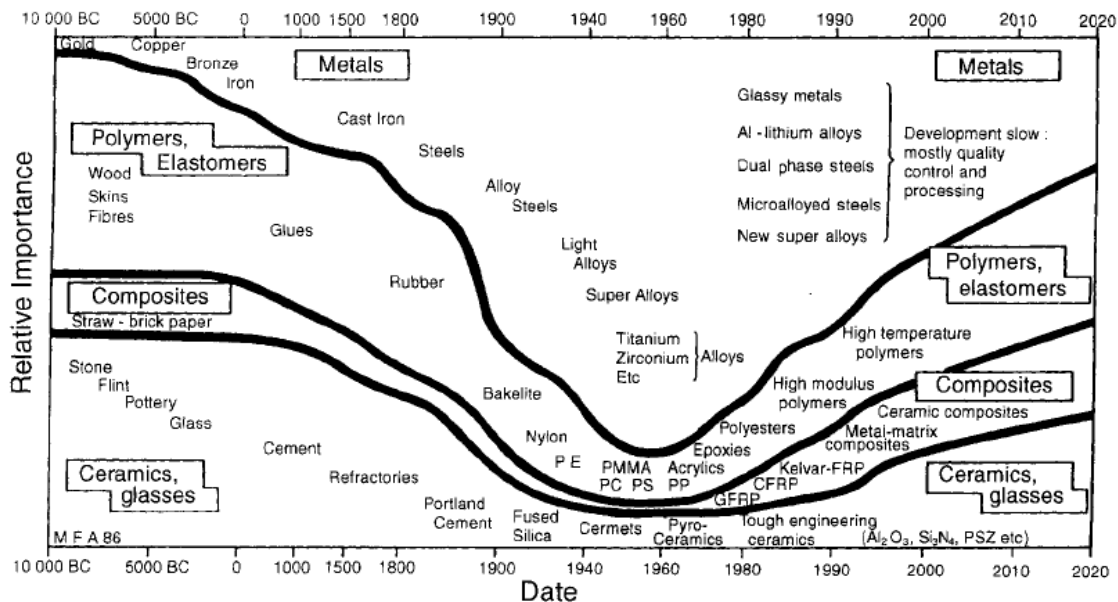

Fig. 1.2 The evolution of engineering materials with time. 'Relative Importance' in the stone and bronze ages is based on assessments of archaeologists; that in 1960 is based on allocated teaching hours in UK and US universities; that in 2020 on predictions of material usage in automobiles by manufacturers. The time scale is non-linear. The rate of change is far faster today than at any previous time in history.

We see the early history of materials was dominated by the development of metals – usually produced by metallurgical processes, which replaced the naturally occurring materials like wood, skins and flint. Laterally, a range of high performance polymers, composites and ceramics has been reversing the process away from metals and now metals are just one of the options. Although the expansion of the steel industry has slowed or even started to contract, it should never be forgotten that 90% of all metal used in technology is iron based and it would be impossible to replace this versatile, inexpensive, strong and stiff material in most of its applications.

Evolution of Vacuum Cleaners

Early cleaning practices, sometime, put as much dust in the air as they removed from the floor, hence the invention of the vacuum cleaner. The early design used a hand driven wooden bellows required two persons to operate, moved about 1 litre of air per second and was made mainly of wood, canvas, leather and rubber (natural polymers). The first electric vacuum cleaner invented by Hoover came in 1908. We see the development as new materials are brought in and the product improved in the figure and table.

(a) 1905   (b) 1950
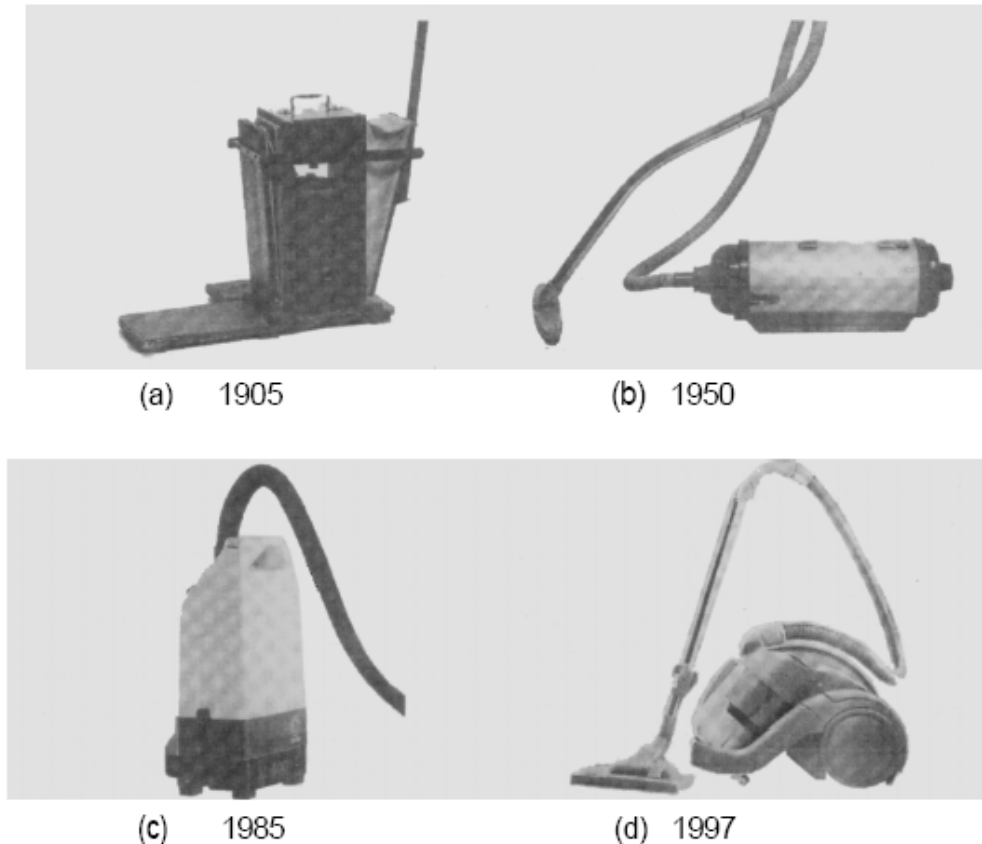
(c) 1985   (d) 1997

Fig. 1.3 Vacuum cleaners: (a) The hand-powered bellows cleaner of 1900, largely made of wood and leather. (b) The cylinder cleaner of 1950. (c) The lightweight cleaner of 1985, almost entirely polymer. (d) A centrifugal dust-extraction cleaner of 1997.

Table 1.1 Comparison of cost, power and weight of vacuum cleaners

Cleaner and Date	Dominant materials	Power (W)	Weight (kg)	Cost*
Hand powered, 1900	Wood, canvas, leather	50	10	£240/$380
Cylinder, 1950	Mild Steel	300	6	£96/$150
Cylinder, 1985	Moulded ABS and polypropylene	800	4	£60/$95
Dyson, 1995	Polypropylene, polycarbonate, ABS	1200	6.3	£190/$300

*Costs have been adjusted to 1998 values, allowing for inflation.

It is important to consider all materials in design, not just the familiar or traditional ones.

Chapter 2. The Design Process

The process of design we are concerned with here is the mechanical design, dealing with physical principles and proper functioning and manufacture of the designed object. The following process of industrial design, dealing with pattern, colour, texture, and consumer appear then follows.

Design is perhaps best understood through examples or case studies, and we will look at several examples. Ashby tries to get the student to look at design in a number of ways three of which are shown in the following figures.
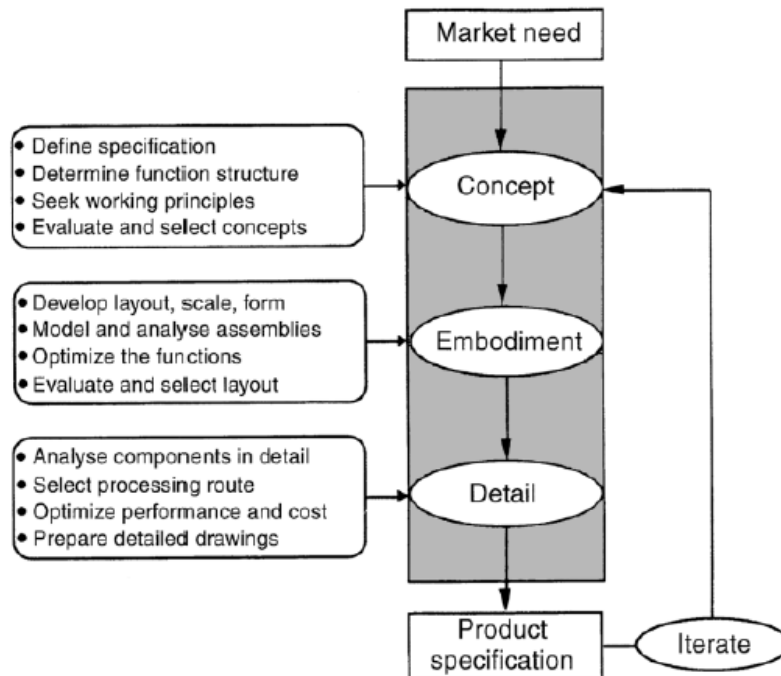

Fig. 2.1 The design flow chart. The design proceeds from an identification and clarification of task through concept, embodiment and detailed analysis to a product specification.

The process can be thought of as a solution to a market need or a solution to some kind of problem. It could be anything from the need to cut material (done in the kitchen with a paring knife, but industrially by everything from a hacksaw, to a laser to a waterjet). The process requires an iterative process that is refined in several steps. In your design project, you may want to take the whole process or just part of the design of the overall concept, for example the concept could be wind energy conversion and you may just look at the design of the airfoil and the use of materials.

The product itself is a technical system, it may be as simple as a paring knife or as complex as a nuclear power plant. The process of design all the sub-assemblies can be visualized as in the next figure.
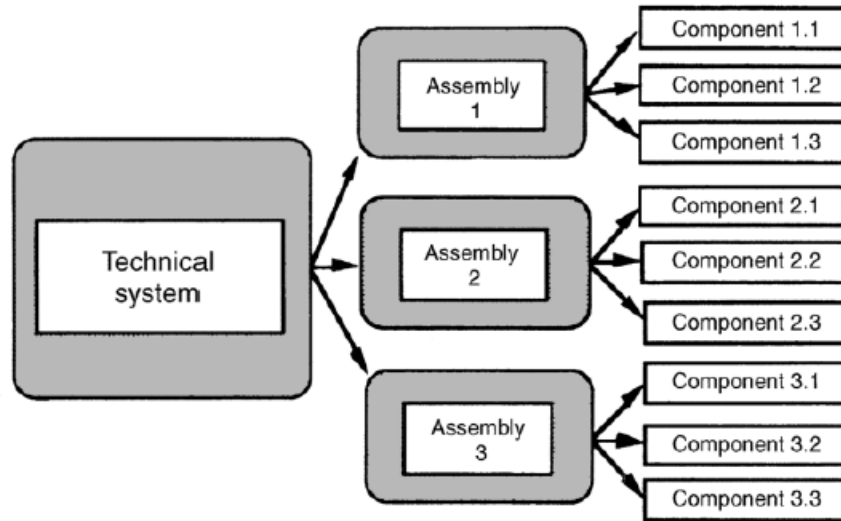
Fig. 2.2 The analysis of a technical system as a breakdown into assemblies and components. Material and process selection is at the component level.

This way of thinking is most useful in analysing an existing design and perhaps refining it by better materials selection. Ashby suggests that a systems approach may be more useful in innovative design as suggested in the next figure.
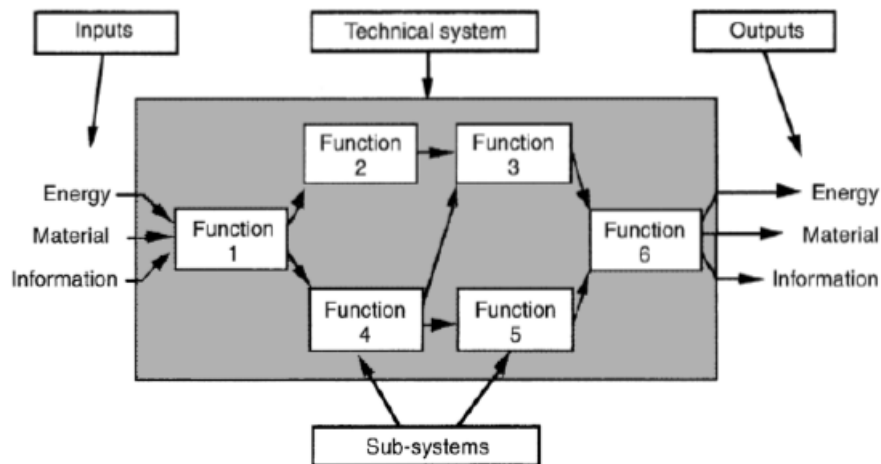

Fig. 2.3 The systems approach to the analysis of a technical system, seen as transformation of energy, materials and information (signals). This approach, when elaborated, helps structure thinking about alternative designs.

The process is to consider what has to be accomplished and to look for different ways of doing it. In the design of a car such traditional thinks as the motor, the transmission, the braking system etc., could be looked at by considering a different ordering and structuring of components, so the braking system can be hooked up to use that energy to help drive the car forward – an innovative design that is used in some energy efficient cars.

Types of Design
Innovative design (a new approach to the solution of the need) such as the change from a record player for sound to the tape recorder or the CD player are the most complete design steps. New materials like semiconductors or high purity glass may permit breakthrough original designs.
Adaptive or development design takes an existing design and improves it through better materials selection. We see this in new sports equipment for example.
The final variation is so called 'variant design' where a change of scale or dimension necessitates a change of materials. Small boats can be made of fibreglass most efficiently, but large tankers can only be made from steel.

At each stage of design from conceptual to detailed, materials data is needed, but the type of data is different and gets more precise and less broad as the final design is approached as shown in the next figure. Final design is often done with manufacturer's specifications and may require in-house testing of critical properties.
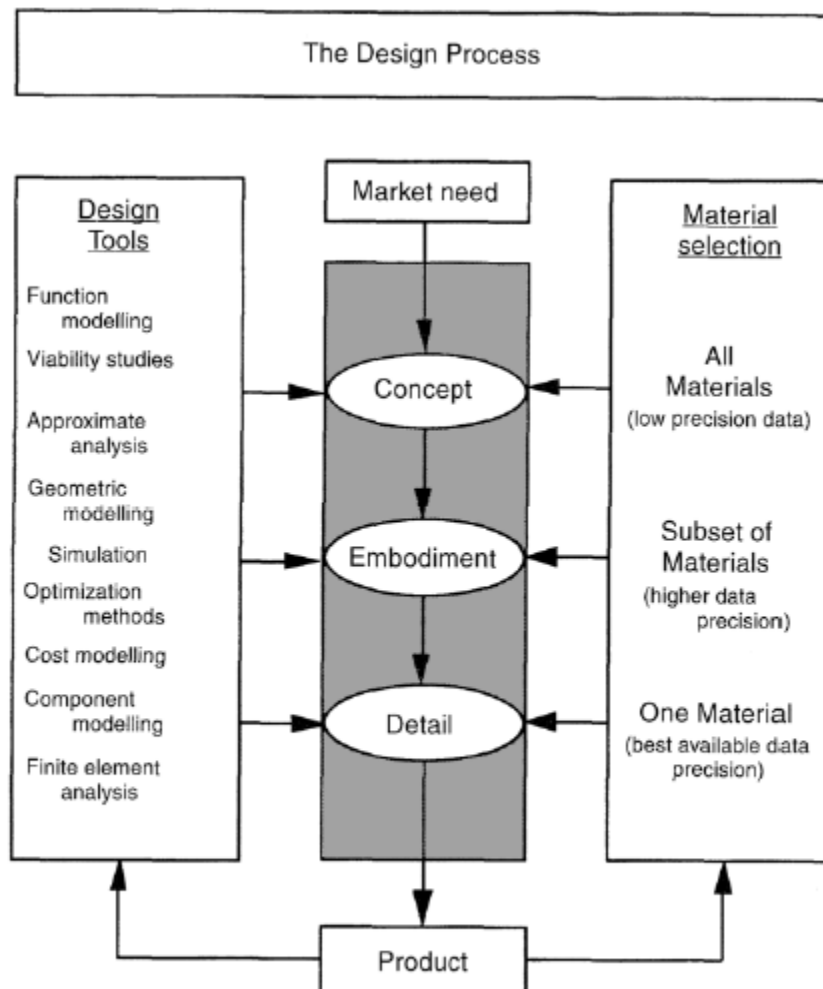

Fig. 2.4 The design flow chart, showing how design tools and materials selection enter the procedure. Information about materials is needed at each stage, but at very different levels of breadth and precision.

The final insight is the material selection is three fold as suggested in the following diagram.
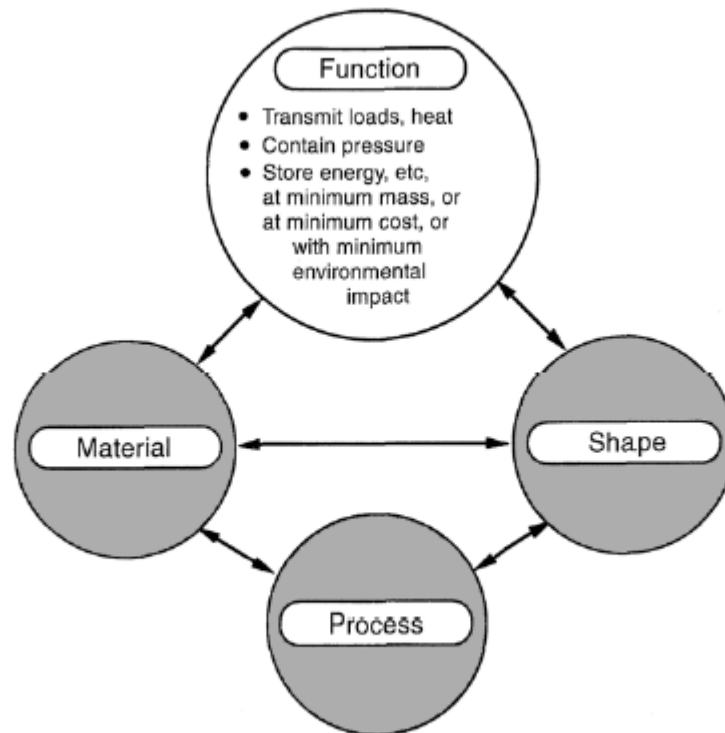

Fig. 2.5 The central problem of materials selection in mechanical design: the interaction between function, material, process and shape.

Specifically if we want a beam to support a load in one direction, then the I beam is an ideal shape and becomes part of the design. But while both steel and wood have similar properties per unit weight as beams, steel is readily shaped by hot rolling into an I beam, but it is not so simple to make an I beam out of wood in an efficient manner, so steel may become the material of choice.

There is a more complicated example of wine cork removing devices in the text in Chapter 2 to which you are referred.

Chapter 3. Engineering Materials and Their Properties

This chapter reviews the important classes and properties of materials. The material classes are shown in the figure 3.1. You are likely already familiar with these materials. The materials properties used in design are covered in Table 3.1, again, most of these properties you will have covered. If in your design you come up against a property that you are not totally familiar, this chapter is a place to start. But in design we need to escape the blindfold of thinking only of one class of materials in a given context and think instead of a materials as a certain property profile. In the next chapter we will look at an innovative way of considering these property profiles, which will aid us in thinking about design.
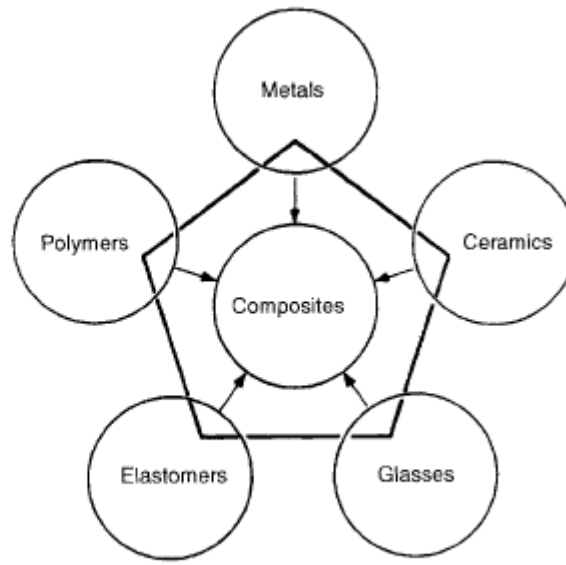
Fig. 3.1 The menu of engineering materials.

Table 3.1 Design-limiting material properties and their usual SI units*

Class	Property	Symbol	and units
General	Cost	C_m	($/kg)
	Density	ρ	(kg/m^3)
Mechanical	Elastic moduli (Young's, shear, bulk)	E, G, K	(GPa)
	Strength (yield, ultimate, fracture)	σ_f	(MPa)
	Toughness	G_c	(kJ/m^2)
	Fracture toughness	K_{Ic}	(MPa m$^{1/2}$)
	Damping capacity	η	(—)
	Fatigue endurance limit	σ_e	(MPa)
Thermal	Thermal conductivity	λ	(W/mK)
	Thermal diffusivity	a	(m^2/s)
	Specific heat	C_p	(J/kg K)
	Melting point	T_m	(K)
	Glass temperature	T_g	(K)
	Thermal expansion coefficient	α	(°K^{-1})
	Thermal shock resistance	ΔT	(°K)
	Creep resistance	-	(—)
Wear	Archard wear constant	k_A	(MPa^{-1})
Corrosion/	Corrosion rate	K	(mm/year)
Oxidation	Parabolic rate constant	k_P	(m^2/s)

*Conversion factors to imperial and cgs units appear inside the back and front covers of this book.

Chapter 4. Materials Selection Charts

Since material properties are what limits design (e.g. a crane's lift strength is limited by its strength), we need to be able to survey properties to help us do design. The simplest way is with bar charts as shown in Figure 4.1.
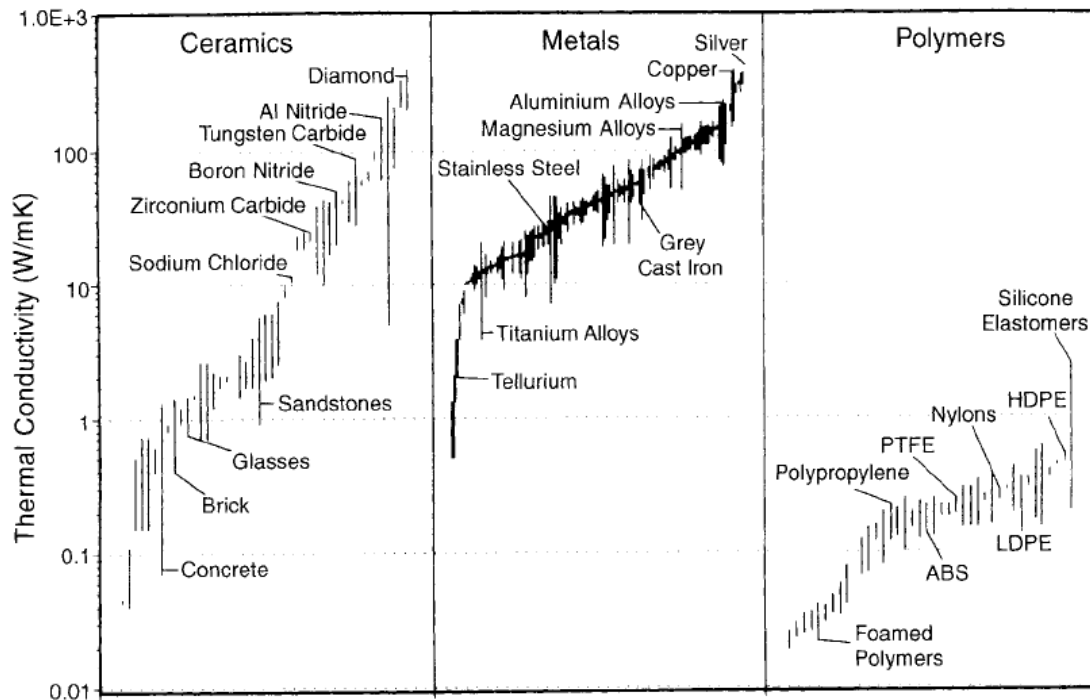
Fig. 4.1 A bar-chart showing thermal conductivity for three classes of solid. Each bar shows the range of conductivity offered by a material, some of which are labelled.

Each bar represents a single material, with the length of the bar showing the variation in conductivity for the various forms of the materials. We see that the metals are higher conductivity, than the polymers, but that ceramics have a very wide variation.

However it is seldom that one property is sufficient for design – more often we want high Stiffness to weight E/ρ or some other favourable combination of properties. What Ashby developed is the chart form exemplified in Fig. 4.2 for this kind of selection. These charts condense a lot of information into a compact but accessible form. They reveal correlations between material properties, which aid in checking and estimating data they lend themselves to the optimization techniques that are introduced in the next chapter, which becomes a basic step in materials selection. We shall look at only one chart here, but if you use one of these charts later you should read the section associated with the chart to be sure of interpreting it correctly. For example the definition of strength is complicated when one leaves the realm of metals and starts looking at ceramics or composites.

Consider Figure 4.2. The ranges are chosen to include all the data, in the case of Young's modulus four orders of magnitude (factor of 10,000) and in the case of density two orders of magnitude (factor of 100). Here we expand the original six material classes to nine. This is done by adding polymeric foams to polymers, and by breaking ceramics into engineering ceramics, glasses and porous ceramics. The list of materials in each group is summarized in Table 4.1. Remarkably we see that the material classes group together naturally with some overlap. The envelope encloses all members of the class. Note that a logarithmic scale is used. For a more detailed chart see Fig. 4.3.
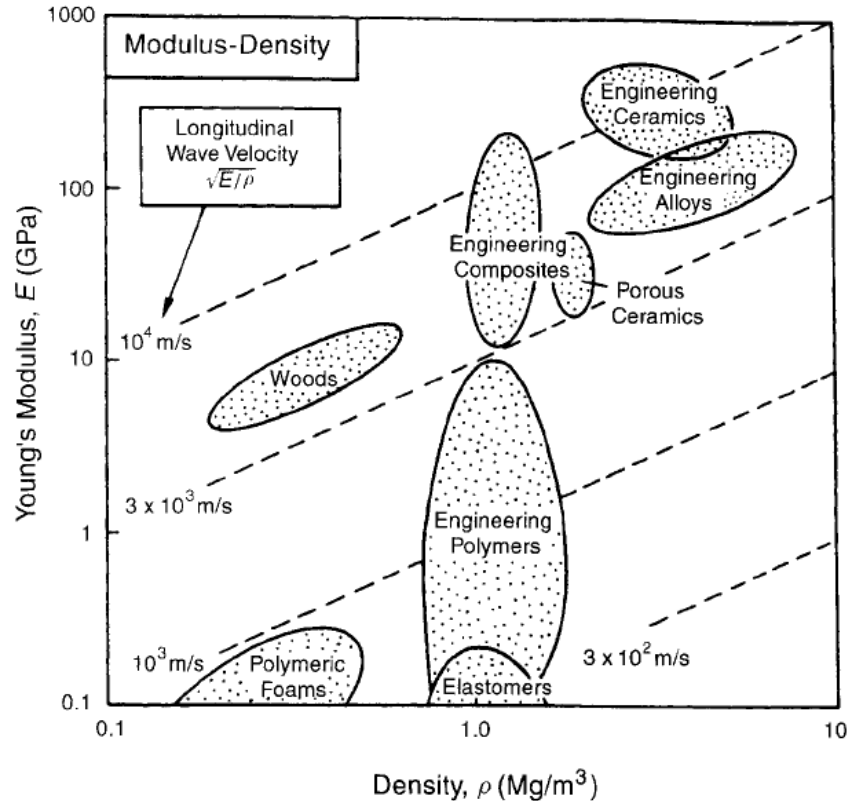
9

Fig. 4.2 The idea of a Materials Property Chart: Young's modulus, E, is plotted against the density, ρ, on log scales. Each class of material occupies a characteristic part of the chart. The log scales allow the longitudinal elastic wave velocity $v = (E/\rho)^{1/2}$ to be plotted as a set of parallel contours.

An example of the application of this chart is to find material with high elastic wave velocity. It is known that $V = \left(E / \rho \right)^{0.5}$. So log V = 0.5 log E – 0.5 log ρ or log E = log ρ +2 log V. Thus constant wave velocity will occur when E and ρ change by equal amounts (i.e. a line of slope 1 on a log log plot). Thus highest wave velocity material types are engineering composites, glasses and engineering ceramics. (see Fig. 4.3 for details). In polymers and elastomers the elastic wave velocity is as much as a factor of 30 lower.

The engineering properties of materials are usefully displayed as materials selection charts. The charts are accessible and summarize the range of properties and the appropriate classes of materials for many different applications of materials. It is most striking how material classes are so clearly clustered on the charts. The first order difference in most material properties are controlled by the nature of the atomic bonding, the mass of the atoms involved and the nature of the packing. Microstructural differences that are stressed in metallurgy while significant are clearly second order in the overall context of differences. Charts can serve to estimate and validate material data. They provide hints as to the potential application of new materials. Most significantly as we'll see they serve as the basis for a very useful procedure for selecting materials for design.
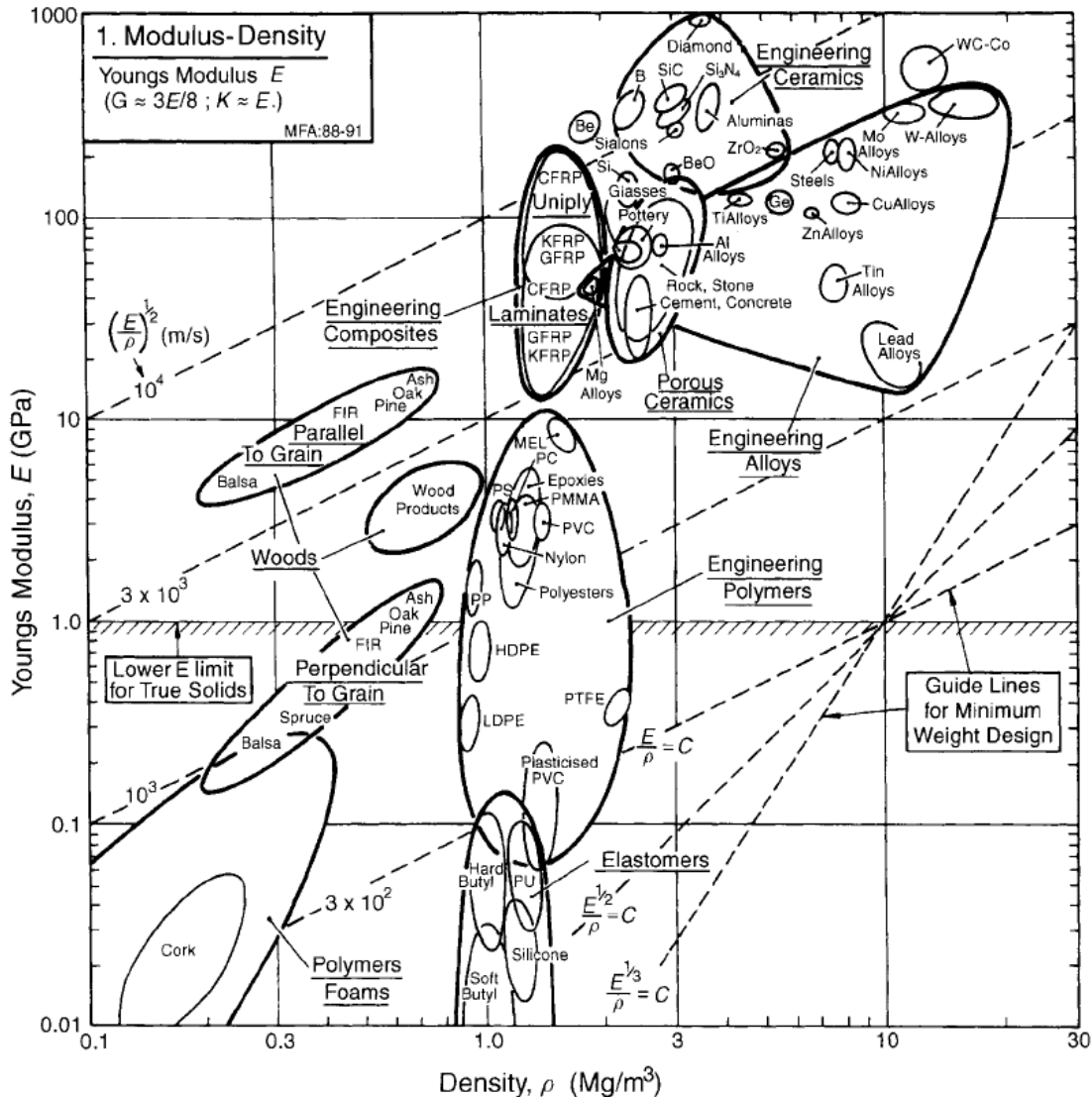
1. Modulus-Density

Youngs Modulus E
($G \approx 3E/8$; $K \approx E$.)

MFA:88-91

Youngs Modulus, E (GPa)

Density, ρ (Mg/m^3)

$\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$ (m/s)

Engineering Ceramics

Diamond SiC Si$_3$N$_4$ B

WC-Co

Be Sialons Aluminas Mo Alloys W-Alloys

Si ZrO$_2$ NiAlloys

CFRP Uniply Glasses BeO Steels

Pottery Ge CuAlloys

KFRP GFRP TiAlloys ZnAlloys

CFRP Al Alloys

Engineering Composites Laminates Rock, Stone Cement, Concrete Tin Alloys

GFRP KFRP

Lead Alloys

Mg Alloys

Porous Ceramics

Engineering Alloys

Ash Oak Pine FIR MEL PC

Parallel To Grain Epoxies

Balsa PS PMMA

Wood Products PVC

Woods Nylon

Ash Oak Pine PP Polyesters

FIR Engineering Polymers

Lower E limit for True Solids

Perpendicular To Grain HDPE

Spruce

Balsa LDPE PTFE

$\frac{E}{\rho} = C$

Guide Lines for Minimum Weight Design

Plasticised PVC

Cork

Elastomers

Hard Butyl PU

$\frac{E^{1/2}}{\rho} = C$

Polymers Foams

Soft Butyl Silicone

$\frac{E^{1/3}}{\rho} = C$

10^4 3×10^3 10^3 3×10^2

Fig. 4.3 Chart 1: Young's modulus, E, plotted against density, ρ. The heavy envelopes enclose data for a given class of material. The diagonal contours show the longitudinal wave velocity. The guide lines of constant E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ allow selection of materials for minimum weight, deflection-limited, design.

Chapter 5 Materials Selection - the Basics

From the material properties and the properties required in the application, we will derive a scheme for materials selection following the Ashby method. At the beginning we only need the kind of property information that is available in handbooks. In this course we will use the materials database CES, with which you have had some experience already in MATE 452. Using this program we have access to a large amount of detailed and authenticated information about materials in all classes. As will be made clear, the selection of material will actually involve a bit more than the property profile. Shape and the ability to shape the material in desired form also can have a significant impact in

some applications. Thus the full package in selecting a material can be illustrated as in Figure 5.1.
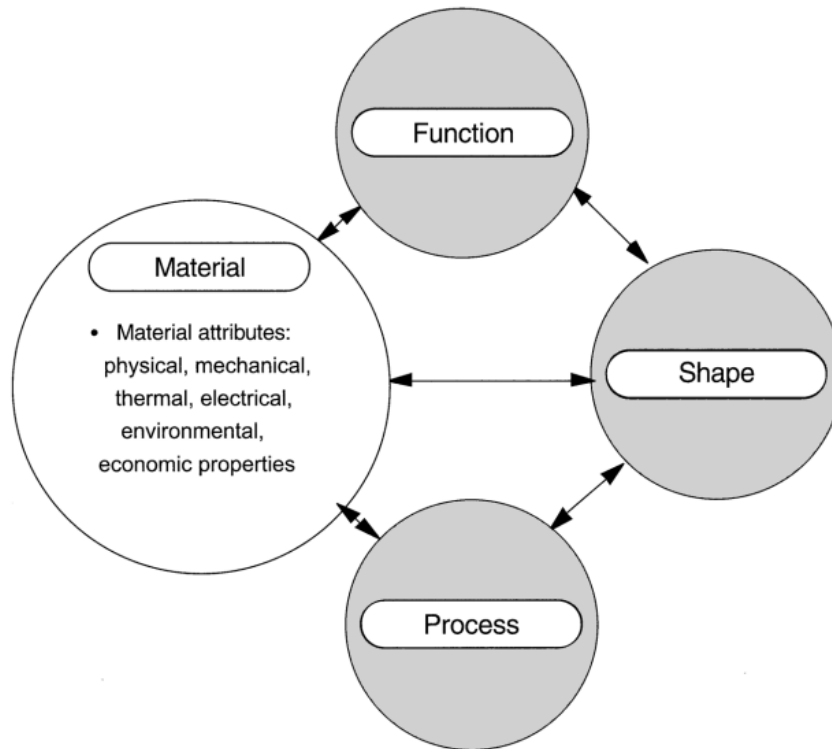

Fig. 5.1 Material selection is determined by function. Shape sometimes influences the selection. This chapter and the next deal with materials selection when this is independent of shape.

A simple example of the importance of shape and processing can be seen in the selection of materials for constructing the structural members of a bicycle. One of the important attributes that these members must possess is the ability to resist bending (a stiffness property) and at the same time be light. When it comes to bending the resistance to bending of different shapes can be found using the second moment of inertia. A shape that has a high second moment of inertial for bending in all directions is the cylindrical tube. (The I beam is good in resisting bending in a gravitational field). So if we want to use a material for the members of a bicycle it is advantageous if that the material can be readily shaped into tubes. Thus, steel and aluminium and titanium can be considered, but wood, which has some very good specific properties, cannot be readily shaped into a tube (it can be drilled out, but this weakens it and is expensive). So wood becomes much less attractive in this application. The development of better techniques for making tubes out of composites and of joining them would contribute to making these materials more desirable for this application.

Thus we must consider a combination of the material properties, how it can be processed, what the function of the material is in the application and what the desirable shape of the material is to perform that function. In the CES database there is information about material processing as well as information about all its properties.

Ashby creates a useful analogy for material selection when he compares it to selecting a candidate for a job. The steps can be visualized as in Fig. 5.3.
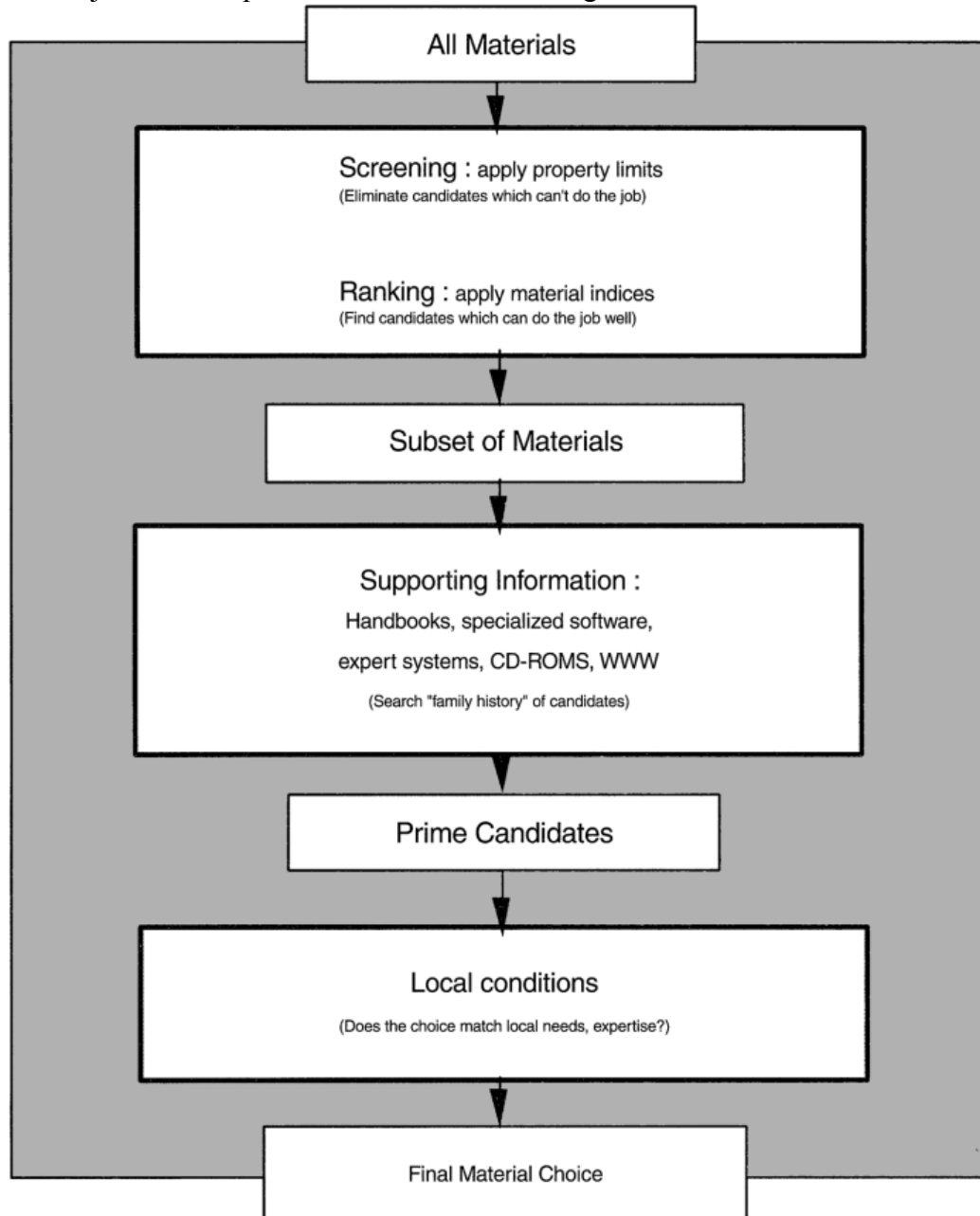
All Materials

Screening : apply property limits
(Eliminate candidates which can't do the job)

Ranking : apply material indices
(Find candidates which can do the job well)

Subset of Materials

Supporting Information :

Handbooks, specialized software,

expert systems, CD-ROMS, WWW

(Search "family history" of candidates)

Prime Candidates

Local conditions

(Does the choice match local needs, expertise?)

Final Material Choice

Fig. 5.3 The strategy for materials selection. The main steps are enclosed in bold boxes.

As in finding a short list of candidates for a job, we get basic information about the materials from the database and eliminate candidates that are not suitable for the job. It is important that all materials be considered as candidates until shown to be unsuitable by a careful analysis if we want to improve our chances of picking the right material. Although material limits such service temperature or the need for transparency can help us in limiting our consideration, they will not help in the final selection. Here we need to find optimization criterion such as E/ρ (specific stiffness), which will allow us to rank the

candidate materials. Some of this detailed information can come from the CES database, but other sources such as manufacturers data sheets are often needed to determine the precise properties, prices and availability of the candidate materials.

Deriving property limits and materials indices.
Since this is a design course, it is important that we consider how to develop our own criteria using the Ashby method. Perhaps the best way to do this is by considering a number of case studies that illustrate the method. The book is a rich source of examples in which you may be able to find examples close to the design problem, which you select for your project. I will start with one of the examples from the book Example 2 on p. 73, which considers a material index for a light, stiff beam, a very common engineering consideration. We will then go to an assignment considering springs. Let's look at the beam problem using Fig. 5.6.
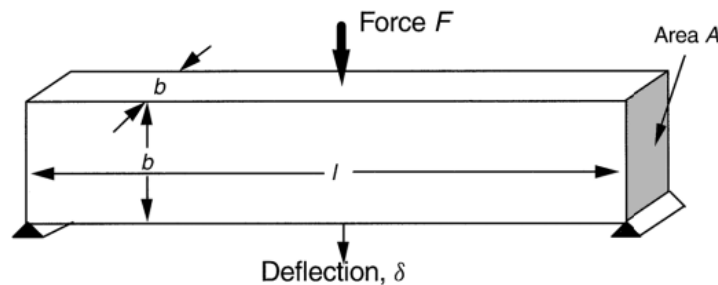

Fig. 5.6 A beam of square section, loaded in bending. Its stiffness is $S = F/\delta$, where F is the load and δ is the deflection. In Example 2, the active constraint is that of stiffness, S; it is this which determines the section area A. In Example 3, the active constraint is that of strength; it now determines the section area A.

In this problem we want to minimize the mass of a beam, subject to the constraint that it must not bend under some force F more than a certain amount characterized by the deflection δ. (Here we have already incorporated our safety factor into our constraint, removing that step from the consideration). The problem is summarized than as:

Table 5.3 Design requirements for the light stiff beam

Function	Beam
Objective	Minimize the mass
Constraints	(a) Length ℓ specified
	(b) Support bending load F without deflecting too much

From the appendix A3 (or CIV E 270) $\dfrac{F}{\delta} \geq \dfrac{C_1 EI}{l^3}$, where C_1 is 48 in this case, and I is the second moment of the area of the section, which for a square beam is $b^4/12$ or

$A^2/12$, so $\dfrac{F}{\delta} \geq \dfrac{4EA^2}{l^3}$ Now, since mass m is given by **m=Alρ**, we now that mass can be reduced by reducing A (the area of the beam given by b^2, until the deflection

criterion is just met. Hence, we can find A in both equations to obtain: $m/l\rho = (Fl^3/4\delta E)^{0.5}$

or $\boldsymbol{m} \geq \left(\dfrac{\boldsymbol{F}}{4\boldsymbol{\delta}}\right)^{0.5} \boldsymbol{l}^{1.5} \dfrac{\boldsymbol{\rho}}{\boldsymbol{E}^{0.5}}$. Here we've arranged the last term to contain the

material properties, so it becomes the materials index. The other terms are the geometric dimensions and the given constraints (the allowable deflection for load F).

Ashby generalizes this by noting that a design requirement can be written as:

$$p = f\left[\left(\begin{array}{c}\text{Functional} \\ \text{requirements, } F\end{array}\right) \cdot \left(\begin{array}{c}\text{Geometric} \\ \text{parameters, } G\end{array}\right) \cdot \left(\begin{array}{c}\text{Material} \\ \text{properties, } M\end{array}\right)\right] \qquad (5.1)$$

or $\qquad p = f(F, G, M)$

where p is the performance.
When we can separate F,G, and M, then the problem is greatly simplified since we can select the material to maximize performance irregardless of the details of the problem, i.e. when :

$$p = f_1(F)f_2(G)f_3(M) \qquad (5.2)$$

And the f_1, f_2 and f_3 are separate functions, which are simply multiplied together.

In summary, the design requirements of a component, which performs mechanical, thermal or electrical functions, can be formulated in terms of one or more functions such as those above. Often the equations will yield a group of materials problems (the material index that can be maximized or minimized in selecting a material. The charts of these properties then allow the selection of the material using the methods already outlined. Sometimes this is independent of shape, sometimes we need to consider shape, which will be the topic of Chapter 7. A large number of design problems have been considered using this approach and these are covered in Chapter 6. The selection charts appear in Appendix C of Ashby's book.

Chapter 7 Selection of Material and Shape

As indicated briefly already, shaped sections like tubes and I beams carry bending, torsional and axial compressive loads more efficiently than solid sections. Here by efficiency we mean that the section uses less material and mass to carry the same load. This efficiency can be extended even further with sandwich panels (thin load bearing skins bonded to a foam or honeycomb interior). We will look at some aspects of shape here. The scheme for design is now shown in Fig. 7-1. In the example mentioned before of bicycles, the forks are loaded in bending. What are the pros and cons of using beams of wood or steel in this case? It is interesting to note that early bicycles were made of wood. However as steel became available in tubular sections wood disappeared. It turns out that wood is better in solid rods but steel in tubes is better than wood in a rod shape. What about aluminium or titanium tubes? This chapter outlines a method for answering this question. We will not look at all the details in this chapter, but will only consider bending and compression, if you are interested in torsion, you should refer to the text.
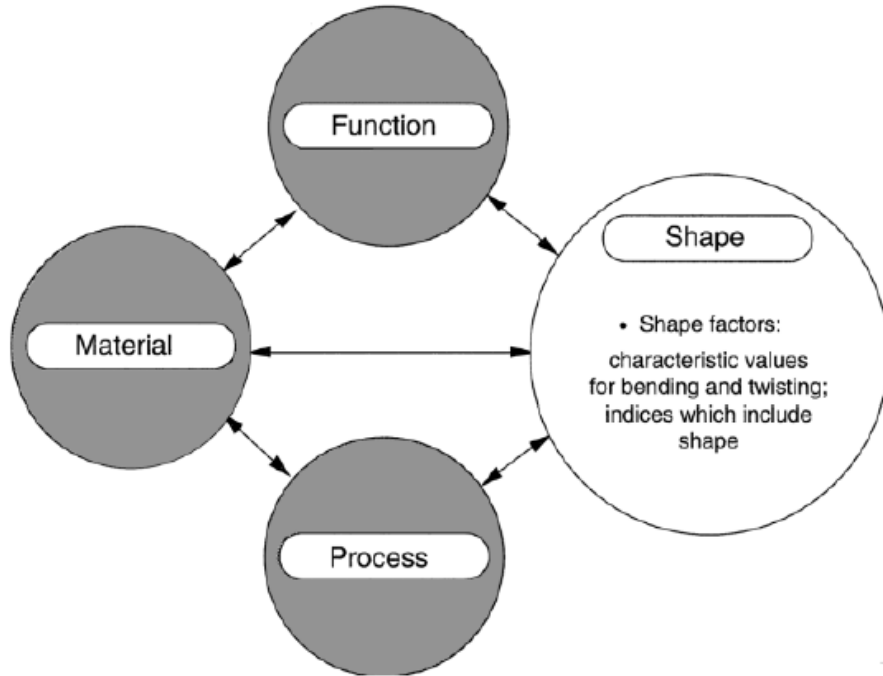
Fig. 7.1 Section shape is important for certain modes of loading. When shape is a variable a new term, the shape factor, appears in some of the material indices: they then allow optimum selection of material and shape.

Shape Factors

Fig. 7.2 shows the modes of loading and the shapes that resist them well. Only the last three are affected by shape and we will only deal with bending and a little about compression. To deal with these we need to define shape factors, which measure the efficiency of a shaped section for each mode of loading.

A material has properties but no shape, when we shape the material such as the I beam in Fig. 7.3 then we need to introduce the shape factor, φ, to describe the performance of the material in that application. All shape factors are described in comparison to a circular cross section rod which is taken as having φ of 1. We consider φ^e_B and φ^f_B for elastic bending of beams and failure of beams respectively.

Elastic bending

The bending stiffness is given by S_B, the force per unit displacement as in the next equation:

$$S_B = \frac{C_1 EI}{l^3}$$, where C_1 is a constant that depends on end constraint (see Appendix A

p. 381), I is the second moment of area about the axis of bending, l is the span of the beam and E is Young's modulus. The I's can be found in Table 7.1 on p. 164. By

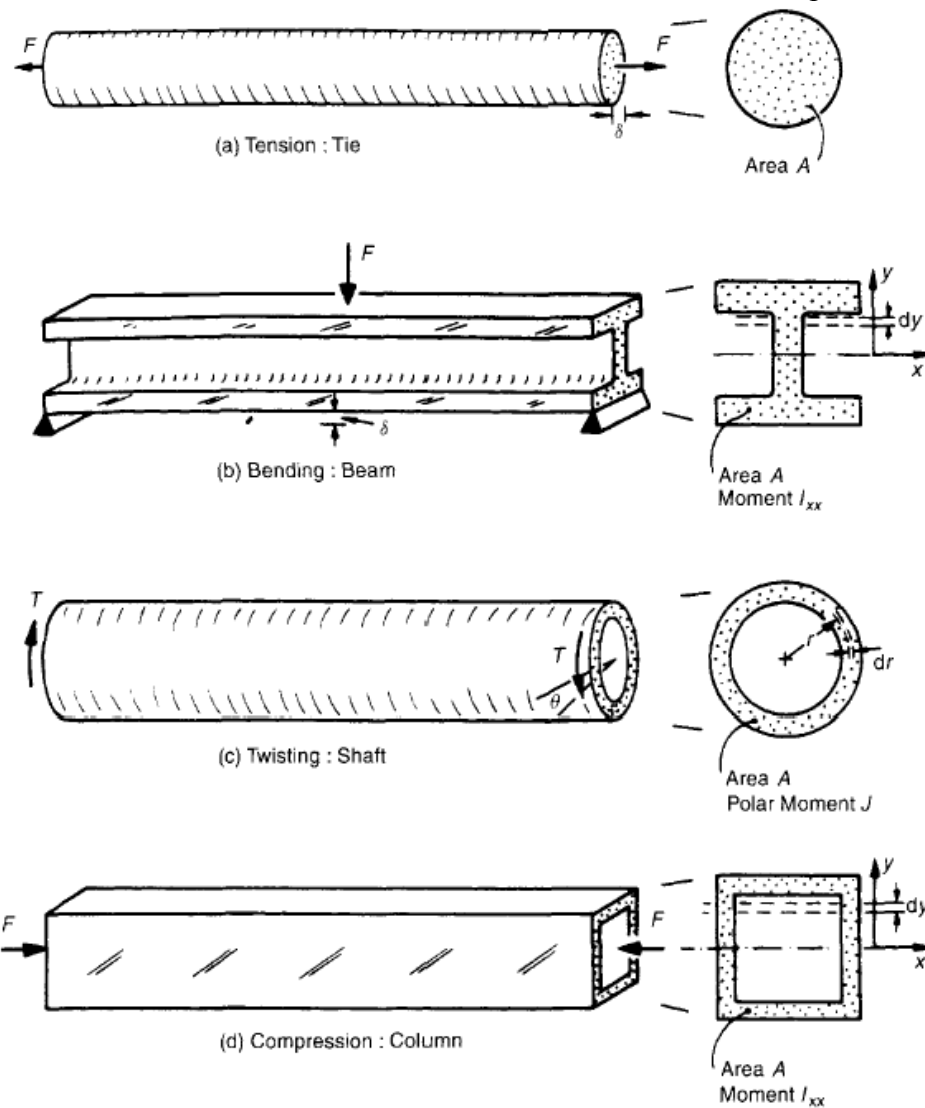comparison to a round beam of solid section we can then define the shape factor as S_B for

Fig. 7.2 Common modes of loading: (a) axial tension; (b) bending; (c) torsion; and (d) axial compression, which can lead to buckling.

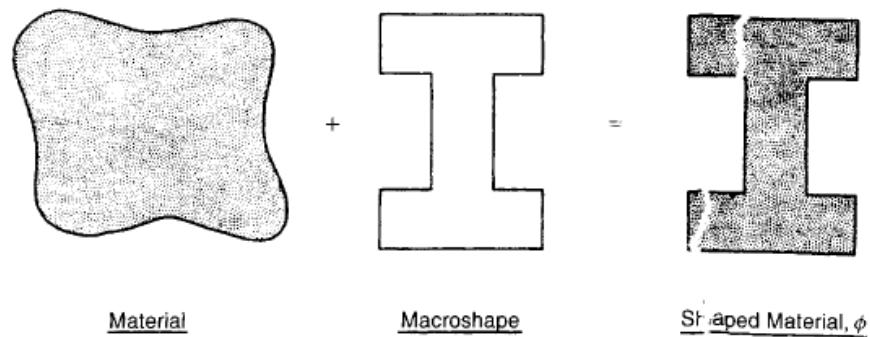

Material + Macroshape = Shaped Material, ϕ

Fig. 7.3 Mechanical efficiency is obtained by combining material with macroscopic shape. The shape is characterized by a dimensionless shape factor, ϕ. The schematic is suggested by Parkhouse (1987).

the shape divided by S_B for the cylindrical rod. By putting in the actual I's for the shapes

we determine that: $\phi_B^e = \dfrac{4\pi I}{A^2}$, where A is the cross sectional area which is the same

for the rod and the shape. To find the value we need to express A in the same terms as I for the shape considered. Some shapes with ϕ^e_B of value 2 and 10 are shown in Fig. 7.4. Note that it actual dimension of the shape is not important, only how the material is concentrated in its displacement from the neutral axis for bending. In bending the more material that is far from the neutral axis the higher the ϕ^e_B. The shape factors for common shapes are given in Table 7.2 p. 165. For a tube the shape factor is simply r/t,
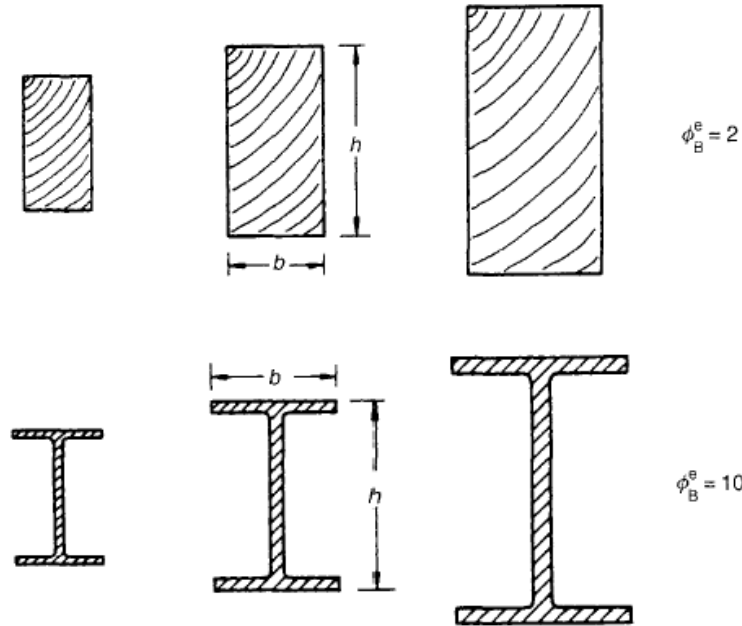

Fig. 7.4 A set of rectangular sections with $\phi_B^e = 2$, and a set of I-sections with $\phi_B^e = 10$. Members of a set differ in size but not in shape.

where, r is the tube radius and t the wall thickness. Thus as r is increased and t is decreased the shape factor increases. There are limits to this process as we shall see.

Failure in Bending and Column Buckling.
Failure can be considered when plasticity occurs based on σ_y or when fracture occurs based on σ_{fr} or based on fatigue failure based on σ_e the fatigue endurance limit. The symbol σ_f is the stress at which failure occurs be it by yielding, fracture or fatigue. One shape factor, Z, will cover all three.

In bending, the stress is largest at the point on the surface that lies farthest from the

neutral axis, i.e.: $\sigma = \dfrac{My_m}{I} = \dfrac{M}{Z}$, where M is the bending moment and y_m is the

distance to the farthest point from the neutral axis. If this stress exceeds the failure stress then the beam will fail and this defines the failure moment: $M_f = Z\sigma_f$. As before the shape factor for failure is defined as the ratio of the failure moment, M_f, of the

18

equivalent sectional area to that of a circular cross section. So: $\phi_B^f = \dfrac{4\sqrt{\pi}Z}{A^{3/2}}$. For a

tube the expression for Z is about πr^2 so that φ_B^f is $\sqrt{(2r/t)}$. The failure criterion is the square root of twice the stiffness criterion. A detailed compilation of the failure criteria is given in Table 7.2 p. 165-166.

A column, loaded in compression, buckles elastically when the load exceeds the Euler

load: $F_c = \dfrac{n^2\pi^2 EI_{min}}{l^2}$, where n is a constant that depends on the end constraints.

The resistance to buckling then depends on the minimum second moment of area and the appropriate shape factor is the same as the shape factor for elastic bending, but considering the minimum second moment of area. Since beams can buckle in any direction in general, square of circular columns are preferred to I beams that are only optimized for bending in one direction.

The Efficiency of Standard Sections

There are of course limits to the process of making sections into thin walled tubes or I beams because of kinking and buckling of the tube if the wall is too thin. The details of this are explained in the text, for those who are interested. In practical terms as shown in Fig. 7.6 the upper limit for steel sections is about 65 for φ_B^e. Upper limits for other shape factors are given in Table 7.3. We see that wood is much more limited than steels, with other materials lying in between. The book explores some of the reasons for these limits like manufacturing limitations and local buckling. If you want to understand this you are referred to section 7.4.

Table 7.3 Upper limits for the shape factors ϕ_B^e, ϕ_T^e, ϕ_B^f and ϕ_T^f

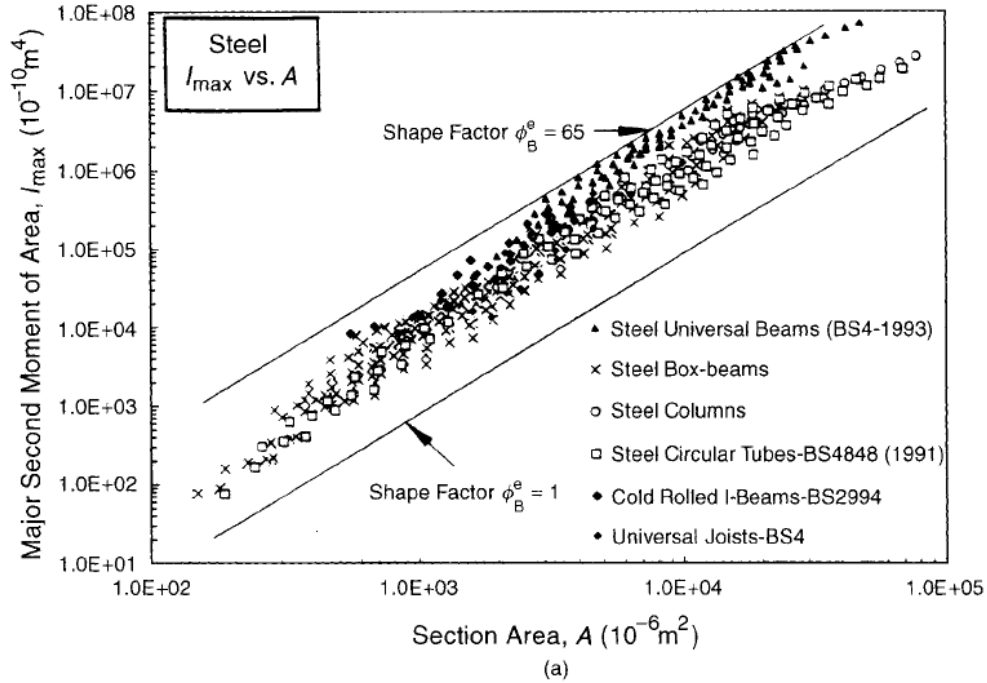| Material	$(\phi_B^e)_{max}$	$(\phi_T^e)_{max}$	$(\phi_B^f)_{max}$	$(\phi_T^f)_{max}$
Structural steels	65	25	13	7
Aluminium alloys	44	31	10	8
GFRP and CFRP	39	26	9	7
Polymers (e.g. nylons)	12	8	5	4
Woods (solid sections)	5	1	3	1
Elastomers	<6	3	---	—

Fig. 7.6 Empirical upper limits for shape factors for steel sections: (a) log(I) plotted against log(A); (b) log(Z) plotted against log(A); (c) log(K) plotted against log(A); (d) log(Q) plotted against log(A).

Material indices that include shape.

If we again consider the problem of a beam of minimum weight to support a given load as was done in chapter 5. The material index to be maximized, M_1, is just $\dfrac{E^{0.5}}{\rho}$. If materials can have equivalent shape factors then this material index will suffice. However, if we wish to select a material shape combination for the beam then we must include the material factor φ^e_B, and the factor M1 becomes $\dfrac{E^{0.5}\left(\phi^e_B\right)^{0.5}}{\rho}$. We can use the factors in Table 7.3 for this purpose. An example of this calculation for four materials is given in Table 7.4. We see that without the shape factor (column 5) wood is the best material for a beam (we use it to build houses). However, when maximum shape factors are included Al becomes the best material (hence its use in aircraft). Steel is not as good but is more than 50% better than wood and when cost is also considered steel is found to be quite competitive. Other factors such as durability in the environment and construction costs can then become decisive. A graphical method is illustrated in Fig. 7.12. The shape factor is introduced as moving the point along a line of slope 1 by the value of the shape factor hence increasing its performance. For more details see section 7.7 p. 186.

Table 7.4 The selection of material and shape for a light, stiff, beam

Material	ρ Mg/m^3	E GPa	ϕ^*_{max}	$\dfrac{E^{1/2}}{\rho}$	$\dfrac{(\phi_{\mathrm{max}}E)^{1/2}}{\rho}$
1020 Steel	7.85	205	65	1.8	14.7
6061-T4 Al	2.7	70	44	3.1	**20.5**
GFRP (isotropic)	1.75	28	39	2.9	19.0
Wood (oak)	0.9	13.5	5	**4.1**	9.1

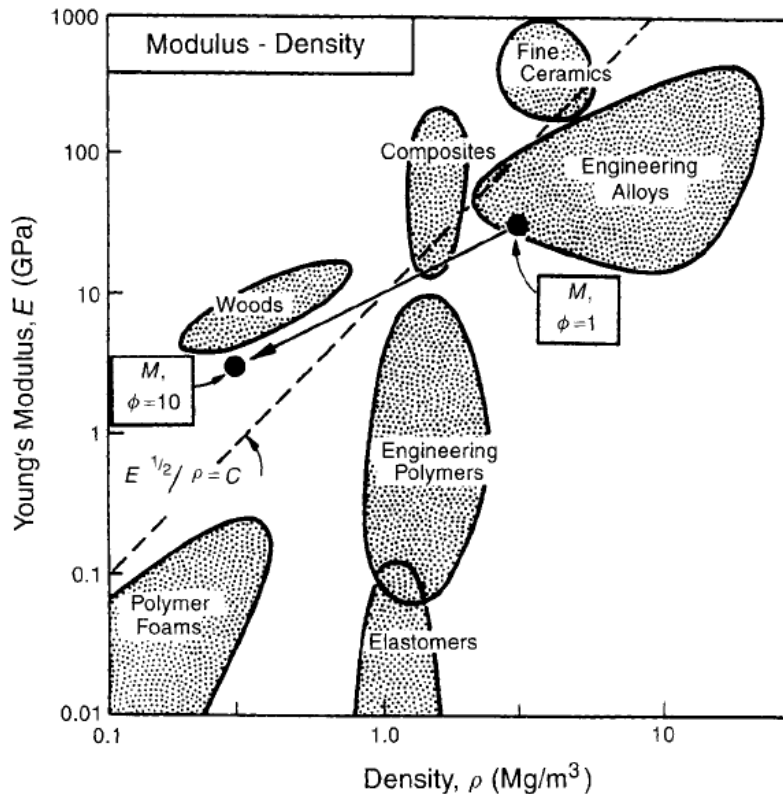$^*\phi_{\mathrm{max}}$ means the maximum permitted value of ϕ from Table 7.3.

Fig. 7.12 Schematic of Materials Selection Chart 1: Young's modulus plotted against density. The best material-and-shape for a light, stiff beam is that with the greatest value of $E^{1/2}/\rho$. The structured material behaves in bending like a new material with modulus $E^* = E/\phi$ and density $\rho^* = \rho/\phi$ (where ϕ means ϕ^e_B) and can be plotted onto the charts. All the material-selection criteria still apply. A similar procedure is used for torsion.