

ME 3600 Control Systems

Performance Indices for Closed-Loop Control

Ref: Dorf and Bishop, *Modern Control Systems*, Pearson/Prentice-Hall, 2005.

To optimize the performance of a closed-loop control system, we can try to adjust the control system parameters to **maximize or minimize some performance index** of our choosing. Some common performance indices are

$$\begin{aligned} ISE &= \int_0^T e^2(t) dt, & IAE &= \int_0^T |e(t)| dt, \\ ITSE &= \int_0^T t e^2(t) dt, & ITAE &= \int_0^T t |e(t)| dt. \end{aligned}$$

Each of the indices are calculated over some interval of time $0 \leq t \leq T$. The time T is chosen to span much of the transient response of the system, so a reasonable choice for second-order systems is T_s the settling time. The first two indices weight the error equally over the entire interval, while the last two give higher weight to the error at later times. Hence, the system is not penalized for having large initial error.

ITAE Optimal Response for a Step Input

The text (Dorf & Bishop, 2005) indicates for systems with transfer functions of the form

$$T(s) = \frac{b_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1s + b_0}$$

that the form of the denominator required to provide ITAE optimal response to a *step* input is as presented in the following table (Table 5.6). This form of transfer function has **zero steady-state error for a step input**.

Order	Form of Denominator
2	$s^2 + 1.4\omega_n s + \omega_n^2$
3	$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$
4	$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$
5	$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$
6	$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.6\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$

ITAE Optimal Response for a Ramp Input

The text (Dorf & Bishop, 2005) indicates for systems with transfer functions of the form

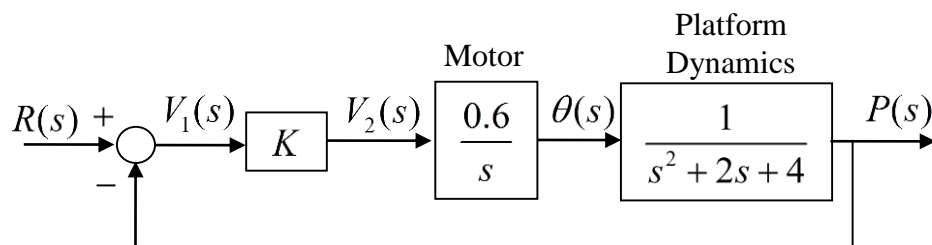
$$T(s) = \frac{b_1 s + b_0}{s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

that the form of the denominator required to provide ITAE optimal response to a *ramp* input is as presented in the following table (Table 5.7). This form of transfer function has **zero steady-state error for a ramp input**.

Order	Form of Denominator
2	$s^2 + 3.2\omega_n s + \omega_n^2$
3	$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3$
4	$s^4 + 2.41\omega_n s^3 + 4.93\omega_n^2 s^2 + 5.14\omega_n^3 s + \omega_n^4$
5	$s^5 + 2.19\omega_n s^4 + 6.5\omega_n^2 s^3 + 6.3\omega_n^3 s^2 + 5.24\omega_n^4 s + \omega_n^5$

Example

- As presented in previous notes, proportional position control of a certain space platform can be expressed by the block diagram below. The open loop platform dynamics is **second order and under-damped**.



- Question: Can we choose a value for K to give the closed loop system ITAE optimal response for a step input?

- Closed Loop Transfer Function:
$$\frac{P}{R}(s) = \frac{0.6K}{s^3 + 2s^2 + 4s + 0.6K}$$

- The transfer function for the system is of the proper form for **ITAE optimal step response** so long as the characteristic equation can be put into **optimal form**. For a third order system, we require

$$s^3 + 2s^2 + 4s + 0.6K = s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$

- Unfortunately, comparing the s^2 and s coefficients gives **two different values** for ω_n . So, it is not possible to adjust K to force the characteristic equation to have the proper form.
- Suppose, however, that we consider changes to the damping coefficient in the platform dynamics transfer function. In this case, we have

$$s^3 + cs^2 + 4s + 0.6K = s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$

- Now, equating the coefficients gives $\omega_n = 1.364$ (r/s), $c = 2.39$, and $K = 4.23$. Note that the original damping coefficient was $c = 2$, so this result requires us to **increase the platform damping**.
- One could also consider changing the **platform stiffness**.