

- **Summation formulas identities:**

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4} = \left(\frac{n(n+1)}{2}\right)^2.$$

- 

$$1 - \sin^2 x = \cos^2 x, \quad 1 + \tan^2 x = \sec^2 x, \quad \sec^2 x - 1 = \tan^2 x$$

- **Trigonometric Half-angle formulas:**

$$\cos^2 x = \frac{1 + \cos(2x)}{2}, \quad \sin^2 x = \frac{1 - \cos(2x)}{2}, \quad \sin(2x) = 2 \sin x \cos x$$

- **Indefinite integrals:**

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C, \quad \int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

- **Approximation's Rules and their error bound:** Let  $y = f(x)$  be a continuous function defined on the interval  $[a, b]$  such that its derivatives  $f''$  and  $f^{(4)}$  are continuous. The midpoint rule approximation  $M(n)$ , the Trapezoid Rule approximation  $T(n)$  and the Simpson's Rule approximation  $S(n)$  to  $\int_a^b f(x) dx$  using  $n$  equally spaced subintervals on  $[a, b]$  are given by the following formulas:

$$\begin{aligned} M(n) &= \sum_{k=1}^n f\left(a + \left(k - \frac{1}{2}\right)\Delta x\right) \Delta x, \\ T(n) &= \left(\frac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(a + k\Delta x) + \frac{1}{2}f(x_n)\right) \Delta x, \\ S(n) &= \left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)\right) \frac{\Delta x}{3}, \end{aligned}$$

where  $\Delta x = \frac{b-a}{n}$ . The absolute errors in approximating the integral  $\int_a^b f(x) dx$  by the Midpoint rule, Trapezoid Rule satisfy the inequalities

$$E_M \leq \frac{k(b-a)}{24} (\Delta x)^2 \quad \text{and} \quad E_T \leq \frac{k(b-a)}{12} (\Delta x)^2,$$

where  $k$  is a real number such that  $|f''(x)| \leq k$  for all  $x$  in  $[a, b]$ .

The absolute errors in approximating the integral  $\int_a^b f(x) dx$  by the Simpson's Rule satisfies the inequalities

$$E_S \leq \frac{K(b-a)}{180} (\Delta x)^4$$

where  $K$  is a real number such that  $|f^{(4)}(x)| \leq K$  for all  $x$  in  $[a, b]$ .

Equivalently, the error bounds can also be written as:

$$\begin{aligned} E_M &\leq \frac{k(b-a)^3}{24n^2}, \\ E_T &\leq \frac{k(b-a)^3}{12n^2}, \\ E_S &\leq \frac{K(b-a)^5}{180n^4}. \end{aligned}$$

(You can use any of these two versions for error bounds, as they give the same result.)