Exercise: Electromagnetism

(a) A core with three legs is shown in Figure 1. Its depth is 5 cm, and there are 200 turns on the leftmost leg. The relative permeability of the core is 1500 and constant. Assume a 4% increase in the effective area of the air gap due to fringing effects.

i) Calculate the total reluctance, R_{TOT}?

ii) Calculate the flux, Φ in each legs of the core.

iii) Calculate the flux density, B in each of the legs.

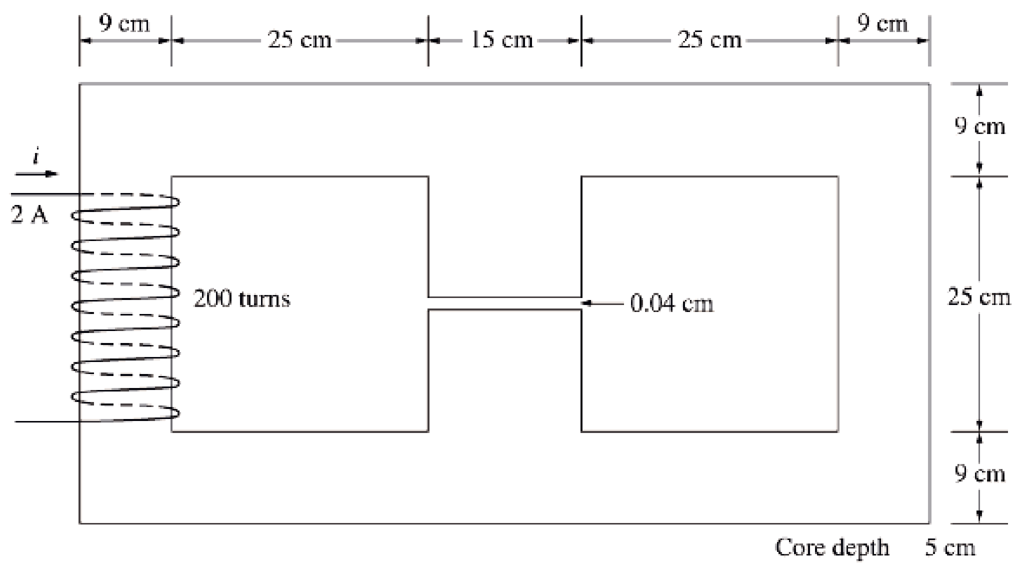

Figure 1

(a)

SOLUTION This core can be divided up into four regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the center leg of the core, $\mathcal{R}_3$ be the reluctance of the center air gap, and $\mathcal{R}_4$ be the reluctance of the right-hand portion of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.08 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.09 \text{ m})(0.05 \text{ m})} = 127.3 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_r \mu_0 A_2} = \frac{0.34 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.05 \text{ m})} = 24.0 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_0 A_3} = \frac{0.0004 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.05 \text{ m})(1.04)} = 40.8 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_r \mu_0 A_4} = \frac{1.08 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.09 \text{ m})(0.05 \text{ m})} = 127.3 \text{ kA} \cdot \text{t/Wb}$$

i) The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 127.3 + \frac{(24.0 + 40.8)127.3}{24.0 + 40.8 + 127.3} = 170.2 \text{ kA} \cdot \text{t/Wb}$$

ii) The total flux in the core is equal to the flux in the left leg:

$$\phi_{\text{left}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(200 \text{ t})(2.0 \text{ A})}{170.2 \text{ kA} \cdot \text{t/Wb}} = 0.00235 \text{ Wb}$$

The fluxes in the center and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{center}} = \frac{\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{127.3}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00156 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{\mathcal{R}_2 + \mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{24.0 + 40.8}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00079 \text{ Wb}$$

iii)

The flux density in the legs can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00235 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.522 \text{ T}$$

$$B_{\text{center}} = \frac{\phi_{\text{center}}}{A} = \frac{0.00156 \text{ Wb}}{(0.15 \text{ cm})(0.05 \text{ cm})} = 0.208 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00079 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.176 \text{ T}$$

b) A magnetic circuit containing a core winded by a 400 turns coil, 4A current and an air gap on the opposite side as shown in the Figure 2 below.

i) Determine the air gap reluctance, R_g and core reluctance, R_c

ii) Determine the energy stored in the core and in the air-gap.

iii) Determine the ***excitation current*** and ***induced emf*** in the coil to produce a flux of 0.4sin314t mWb in the air gap.

iv) Determine the inductance of the coil.

v) If the coil is connected with a source 200Vrms and 60Hz, determine the maximum flux density

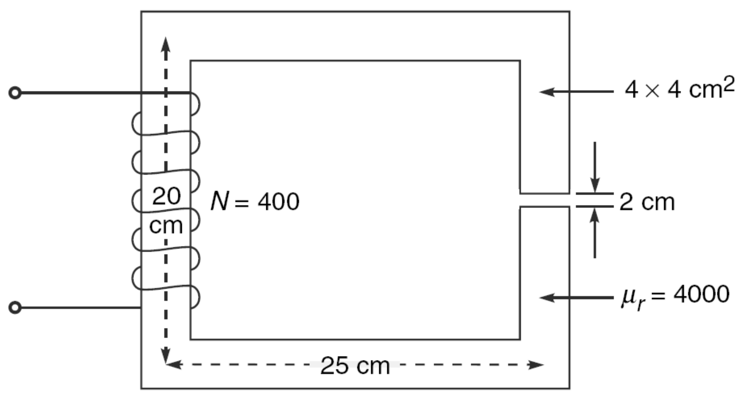

Figure 2

(b) i) $\Re = \dfrac{l}{\mu A}$

$$\Re_g = \frac{l}{\mu_0 A} = \frac{2x10^{-3}}{(4\pi x10^{-7})(4x4x10^{-4})} = 9.95x10^5$$

$$\Re_c = \frac{l}{\mu_0 \mu_r A} = \frac{[2(25+20)-0.2]x10^{-3}}{(4\pi x10^{-7})(4000)(4x4x10^{-4})} = 1.11x10^5$$

ii) $\Re_g + \Re_c = (9.95+1.11)x10^5 = 11.06x10^5$

$$\phi = \frac{400x4}{11.06x10^5} = 1.45 \quad mWb$$

$$W_{f(air-gap)} = \frac{1}{2} R_g \phi^2 = \frac{1}{2} x9.95x10^5 x(1.45)^2 x10^{-6} = 1.046J$$

$$W_{f(core)} = \frac{1}{2} R_c \phi^2 = \frac{1}{2} x1.11x10^5 x(1.45)^2 x10^{-6} = 0.036J$$

iii)

$$i = \frac{\phi R_{total}}{N} = \frac{(0.4\sin 314t)x10^{-5} x11.06x10^5}{400} = 1.11\sin 314t \quad A$$

$$e = \omega N \phi_{\max} \cos \omega t = 314x400x0.4x10^{-3} \cos 314t = 50.24 \cos 314t \quad V$$

iv) $$L = \frac{N^2}{R_{total}} = \frac{(400)^2}{11.06x10^5} = 144.7mH$$

v)

$$V = E = 4.44 f \phi_{\max} N \qquad (rms)$$

$$200 = (4.44)(60)\phi_{\max}(400)$$

$$\phi_{\max} = 1.876 \ mWb$$