

A SET OF SEQUENCES IN NUMBER THEORY

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Abstract: New sequences are introduced in number theory, and for each one a general question: how many primes each sequence has.

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Introduction.

74 new integer sequences are defined below, followed by references and some open questions.

1. Consecutive sequence:

1,12,123,1234,12345,123456,1234567,12345678,123456789,
12345678910,1234567891011,123456789101112,
12345678910111213,...

How many primes are there among these numbers?

In a general form, the Consecutive Sequence is considered in an arbitrary numeration base B .

Reference:

- a) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

3. Symmetric sequence:

1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321,
1234554321, 12345654321, 123456654321, 1234567654321,
12345677654321, 123456787654321, 1234567887654321,
12345678987654321, 123456789987654321, 12345678910987654321,
1234567891010987654321, 123456789101110987654321,
12345678910111110987654321, ...

How many primes are there among these numbers?

In a general form, the Symmetric Sequence is considered
in an arbitrary numeration base B.

References:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.
- b) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

4. Deconstructive sequence:

1,23,456,7891,23456,789123,4567891,23456789,123456789,1234567891, ...
| || || || | | | | ||
----- ----- ----- ----- ----- ----- ----- ...

How many primes are there among these numbers?

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

5. Mirror sequence:

1, 212, 32123, 4321234, 543212345, 65432123456, 7654321234567,
876543212345678, 98765432123456789, 109876543212345678910,
1110987654321234567891011, ...

Question: How many of them are primes?

6. Permutation sequence:

12,1342,135642,13578642,13579108642,135791112108642,
1357911131412108642,13579111315161412108642,
135791113151718161412108642,1357911131517192018161412108642,...

Question: Is there any perfect power among these numbers?

(Their last digit should be:

either 2 for exponents of the form $4k+1$,

either 8 for exponents of the form $4k+3$, where $k \geq 0$.)

Conjecture: no!

7. Generalized permutation sequence:

If $g(n)$, as a function, gives the number of digits of $a(n)$,

and F is a permutation of $g(n)$ elements, then:

$$a(n) = \overline{F(1)F(2)\dots F(g(n))} .$$

8. Simple numbers:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 25, 26, 27, 29, 31,
33, 34, 35, 37, 38, 39, 41, 43, 45, 46, 47, 49, 51, 53, 55, 57, 58, 61, 62, 65,
67, 69, 71, 73, 74, 77, 78, 79, 82, 83, 85, 86, 87, 89, 91, 93, 94, 95, 97, 101, 103, ...

(A number n is called <simple number> if the product of its proper divisors is less than or equal to n .)

Generally speaking, n has the form:

$n = p$, or p^2 , or p^3 , or pq , where p and q are distinct primes.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

Reference:

a) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

9. Digital sum:

0,1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,2,3,4,5,6,7,8,9,10,11,
 | _____ | | _____ | | _____ |

3,4,5,6,7,8,9,10,11,12,4,5,6,7,8,9,10,11,12,13,5,6,7,8,9,10,11,12,13,14,...
 | _____ | | _____ | | _____ |

(d (n) is the sum of digits.)
 s

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

10. Digital products:

0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5,6,7,8,9,0,2,4,6,8,19,12,14,16,18,
| _____ | | _____ | | _____ |

0,3,6,9,12,15,18,21,24,27,0,4,8,12,16,20,24,28,32,36,0,5,10,15,20,25,...
| _____ | | _____ | | _____ ...

(d (n) is the product of digits.)
p

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

11. Code puzzle:

151405,202315,2008180505,06152118,06092205,190924,1905220514,

0509070820,14091405,200514,051205220514,...

Using the following letter-to-number code:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

then $c(n) =$ the numerical code of the spelling of n in English
language; for exemple: 1 = ONE = 151405, etc.

Find a better codification (one sign only for each letter).

12. Pierced chain:

101,1010101,10101010101,101010101010101,10101010101010101,

101010101010101010101,10101010101010101010101,...

$$(c(n) = 101 * 1 \begin{array}{cccc} 0001 & 0001 & \dots & 0001 \end{array}, \text{ for } n \geq 1)$$
$$\begin{array}{cccc} | & | & | & | & \dots & | & | \\ \hline & & & & & & \\ 1 & & 2 & & & & n-1 \end{array}$$

How many $c(n)/101$ are primes ?

References:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.
- b) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

13. Divisor products:

1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19,
8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000, 31, 32768,
1089, 1156, 1225, 10077696, 37, 1444, 1521, 2560000, 41, ...

($P(n)$ is the product of all positive divisors of n .)
 d

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

14. Proper divisor products:

1, 1, 1, 2, 1, 6, 1, 8, 3, 10, 1, 144, 1, 14, 15, 64, 1, 324, 1, 400, 21, 22, 1,
13824, 5, 26, 27, 784, 1, 27000, 1, 1024, 33, 34, 35, 279936, 1, 38, 39,
64000, 1, ...

($p_d(n)$ is the product of all positive divisors of n but n .)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

15. Square complements:

1,2,3,1,5,6,7,2,1,10,11,3,14,15,1,17,2,19,5,21,22,23,6,1,26,
3,7,29,30,31,2,33,34,35,1,37,38,39,10,41,42,43,11,5,46,47,3,
1,2,51,13,53,6,55,14,57,58,59,15,61,62,7,1,65,66,67,17,69,70,71,2,...

Definition:

for each integer n to find the smallest integer k such that
 nk is a perfect square..

(All these numbers are square free.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

16. Cubic complements:

1, 4, 9, 2, 25, 36, 49, 1, 3, 100, 121, 18, 169, 196, 225, 4, 289, 12, 361, 50,
441, 484, 529, 9, 5, 676, 1, 841, 900, 961, 2, 1089, 1156, 1225, 6, 1369,
1444, 1521, 25, 1681, 1764, 1849, 242, 75, 2116, 2209, 36, 7, 20, ...

Definition:

for each integer n to find the smallest integer k such that
 nk is a perfect cub.

(All these numbers are cube free.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

17. m-power complements:

Definition:

for each integer n to find the smallest integer k such that
 nk is a perfect m -power ($m \geq 2$).

(All these numbers are m -power free.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

18. Cube free sieve:

2,3,4,5,6,7,9,10,11,12,13,14,15,17,18,19,20,21,22,23,25,26,
28,29,30,31,33,34,35,36,37,38,39,41,42,43,44,45,46,47,49,50,
51,52,53,55,57,58,59,60,61,62,63,65,66,67,68,69,70,71,73,...

Definition: from the set of natural numbers (except 0 and 1):

- take off all multiples of 2^3 (i.e. 8, 16, 24, 32, 40, ...)
- take off all multiples of 3^3
- take off all multiples of 5^3
- ... and so on (take off all multiples of all cubic primes).

(One obtains all cube free numbers.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

19. m-power free sieve:

Definition: from the set of natural numbers (except 0 and 1)

- take off all multiples of 2^m , afterwards all multiples of 3^m , ...
- and so on (take off all multiples of all m-power primes, $m \geq 2$).

(One obtains all m-power free numbers.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

20. Irrational root sieve:

2,3,5,6,7,10,11,12,13,14,15,17,18,19,20,21,22,23,24,26,28,
29,30,31,33,34,35,37,38,39,40,41,42,43,44,45,46,47,48,50,
51,52,53,54,55,56,57,58,59,60,61,62,63,65,66,67,68,69,70,71,72,73,...

Definition: from the set of natural numbers (except 0 and 1):

- take off all powers of 2^k , $k \geq 2$, (i.e. 4, 8, 16, 32, 64, ...)
- take off all powers of 3^k , $k \geq 2$;
- take off all powers of 5^k , $k \geq 2$;
- take off all powers of 6^k , $k \geq 2$;
- take off all powers of 7^k , $k \geq 2$;
- take off all powers of 10^k , $k \geq 2$;

... and so on (take off all k -powers, $k \geq 2$, of all square free numbers).

One gets all square free numbers by the following method (sieve):

from the set of natural numbers (except 0 and 1):

- take off all multiples of 2^2 (i.e. 4, 8, 12, 16, 20, ...)
- take off all multiples of 3^2
- take off all multiples of 5^2

... and so on (take off all multiples of all square primes);

one obtains, therefore:

2,3,5,6,7,10,11,13,14,15,17,19,21,22,23,26,29,30,31,33,34,
35,37,38,39,41,42,43,46,47,51,53,55,57,58,59,61,62,65,66,
67,69,70,71,...

which are used for irrational root sieve.

(One obtains all natural numbers whose m -th roots, for any $m \geq 2$, are irrational.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

References:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.
- b) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

21. Odd sieve:

7,13,19,23,25,31,33,37,43,47,49,53,55,61,63,67,73,75,79,83,

85,91,93,97,...

(All odd numbers that are not equal to the difference of two primes.)

A sieve is used to get this sequence:

- subtract 2 from all prime numbers and obtain a temporary sequence;
- choose all odd numbers that do not belong to the temporary one.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

22. Binary sieve:

1, 3, 5, 9, 11, 13, 17, 21, 25, 27, 29, 33, 35, 37, 43, 49, 51, 53, 57, 59, 65,
67, 69, 73, 75, 77, 81, 85, 89, 91, 97, 101, 107, 109, 113, 115, 117, 121,
123, 129, 131, 133, 137, 139, 145, 149, ...

(Starting to count on the natural numbers set at any step from 1:

- delete every 2-nd numbers
- delete, from the remaining ones, every 4-th numbers
- ... and so on: delete, from the remaining ones, every (2^k) -th numbers, $k = 1, 2, 3, \dots$.)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which are not prime.

23. Trinary sieve:

1, 2, 4, 5, 7, 8, 10, 11, 14, 16, 17, 19, 20, 22, 23, 25, 28, 29, 31, 32, 34, 35,
37, 38, 41, 43, 46, 47, 49, 50, 52, 55, 56, 58, 59, 61, 62, 64, 65, 68, 70, 71,
73, 74, 76, 77, 79, 82, 83, 85, 86, 88, 91, 92, 95, 97, 98, 100, 101, 103, 104,
106, 109, 110, 112, 113, 115, 116, 118, 119, 122, 124, 125, 127, 128, 130,
131, 133, 137, 139, 142, 143, 145, 146, 149, ...

(Starting to count on the natural numbers set at any step from 1:

- delete every 3-rd numbers
- delete, from the remaining ones, every 9-th numbers
- ... and so on: delete, from the remaining ones, every (3^k) -th numbers, $k = 1, 2, 3, \dots$.)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which are not prime.

24. n-ary sieve (generalization, $n \geq 2$):

(Starting to count on the natural numbers set at any step from 1:

- delete every n -th numbers;
- delete, from the remaining ones, every (n^2) -th numbers;
- ... and so on: delete, from the remaining ones, every (n^k) -th numbers, $k = 1, 2, 3, \dots$.)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which

are not prime.

25. Consecutive sieve:

1, 3, 5, 9, 11, 17, 21, 29, 33, 41, 47, 57, 59, 77, 81, 101, 107, 117, 131, 149.
153, 173, 191, 209, 213, 239, 257, 273, 281, 321, 329, 359, 371, 401, 417,
441, 435, 491, ...

(From the natural numbers set:

- keep the first number,
delete one number out of 2 from all remaining numbers;
- keep the first remaining number,
delete one number out of 3 from the next remaining numbers;
- keep the first remaining number,
delete one number out of 4 from the next remaining numbers;
- ... and so on, for step k ($k \geq 2$):
- keep the first remaining number,
delete one number out of k from the next remaining numbers;
-)

This sequence is much less dense than the prime number sequence,
and their ratio tends to $\frac{p}{n}$ as n tends to infinity.

For this sequence we chosen to keep the first remaining
number at all steps,

but in a more general case:

the kept number may be any among the remaining k -plet
(even at random).

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

26. General-sequence sieve:

Let $u_i > 1$, for $i = 1, 2, 3, \dots$, a strictly increasing positive integer sequence. Then:

From the natural numbers set:

- keep one number among $1, 2, 3, \dots, u_1 - 1$,
and delete every u_1 -th numbers;
- keep one number among the next $u_2 - 1$ remaining numbers,
and delete every u_2 -th numbers;
- ... and so on, for step k ($k \geq 1$):
- keep one number among the next $u_k - 1$ remaining numbers,
and delete every u_k -th numbers;
- ...

Problem: study the relationship between sequence u_i , $i = 1, 2, 3, \dots$, and the remaining sequence resulted from the general sieve.

u_i , previously defined, is called sieve generator.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

27. Digital sequences:

(This a particular case of sequences of sequences.)

General definition:

in any numeration base B , for any given infinite integer or rational sequence S_1, S_2, S_3, \dots , and any digit D from 0 to $B-1$,

it's built up a new integer sequence which

associates to S_1 the number of digits D of S_1 in base B ,
to S_2 the number of digits D of S_2 in base B , and so on...

For example, considering the prime number sequence in base 10, then the number of digits 1 (for example) of each prime number following their order is: 0,0,0,0,2,1,1,1,0,0,1,0,...

(the digit-1 prime sequence).

Second example if we consider the factorial sequence $n!$ in base 10, then the number of digits 0 of each factorial number following their order is: 0,0,0,0,0,1,1,2,2,1,3,...

(the digit-0 factorial sequence).

Third example if we consider the sequence n^n in base 10, $n=1,2,\dots$, then the number of digits 5 of each term $1^1, 2^2, 3^3,\dots$, following their order is: 0,0,0,1,1,1,1,0,0,0,...

(The digit-5 n^n sequence)

References:

- a) E. Grosswald, University of Pennsylvania, Philadelphia,
Letter to the Author, August 3, 1985;
- b) R. K. Guy, University of Calgary, Alberta, Canada,

Letter to the Author, November 15, 1985;

c) Arizona State University, Hayden Library, "The Florentin
Smarandache papers" special collection, Tempe, AZ 85287-
1006, USA.

28. Construction sequences:

(This a particular case of sequences of sequences.)

General definition:

in any numeration base B , for any given infinite integer or rational sequence S_1, S_2, S_3, \dots , and any digits D_1, D_2, \dots, D_k ($k < B$), it's built up a new integer sequence such that each of its terms $Q_1 < Q_2 < Q_3 < \dots$ is formed by these digits D_1, D_2, \dots, D_k only (all these digits are used), and matches a term S_i of the previous sequence.

For exemple, considering in base 10 the prime number sequence, and the digits 1 and 7 (for exemple), we construct a written-only-with-these-digits (all these digits are used) prime number new sequence: 17,71,... (the digit-1-7-only prime sequence).

Second exemple, considering in base 10 the multiple of 3 sequence, and the digits 0 and 1, we construct a written-only-with-these-digits (all these digits are used) multiple of 3 new sequence: 1011,1101,1110,10011,10101,10110,11001,11010,11100,... (the digit-0-1-only multiple of 3 sequence).

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

References:

a) E. Grosswald, University of Pennsylvania, Philadelphia,

Letter to F.

Smarandache, August 3, 1985;

b) R. K. Guy, University of Calgary, Alberta, Canada, Letter
to F. Smarandache, November 15, 1985;

c) Arizona State University, Hayden Library, "The Florentin
Smarandache papers" special collection, Tempe, AZ 85287-
1006, USA.

29. General residual sequence:

$(x + C_1) \dots (x + C_{F(m)}), m = 2, 3, 4, \dots,$

where $C_i, 1 \leq i \leq F(m)$, forms a reduced set of residues mod m ,

x is an integer, and F is Euler's totient.

The General Residual Sequence is induced from the

The Residual Function (see <Libertas Mathematica>):

Let $L : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be a function defined by

$$L(x,m) = (x + C_1) \dots (x + C_{F(m)}),$$

where $C_i, 1 \leq i \leq F(m)$, forms a reduced set of residues mod m ,

$m \geq 2, x$ is an integer, and F is Euler's totient.

The Residual Function is important because it generalizes the classical theorems by Wilson, Fermat, Euler, Wilson, Gauss, Lagrange, Leibnitz, Moser, and Sierpinski all together.

For $x=0$ it's obtained the following sequence:

$$L(m) = C_1 \dots C_{F(m)}, \text{ where } m = 2, 3, 4, \dots$$

(the product of all residues of a reduced set mod m):

1, 2, 3, 24, 5, 720, 105, 2240, 189, 3628800, 385, 479001600, 19305, 896896, 2027025, 20922789888000, 85085, 6402373705728000, 8729721, 47297536000, 1249937325, ...

which is found in "The Handbook of Integer Sequences", by N.

J. A. Sloane, Academic Press, USA, 1973.

The Residual Function extends it.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

References:

- a) F. Smarandache, "A numerical function in the congruence theory",
in <Libertah Mathematica>, Texas State University, Arlington,
12, pp. 181-185, 1992;
see <Mathematical Reviews> 93i:11005 (11A07), p.4727,
and <Zentralblatt fur Mathematik>, Band 773(1993/23),
11004 (11A);
- b) F. Smarandache, "Collected Papers" (Vol. 1), Ed. Tempus,
Bucharest, 1994 (to appear);
- c) Arizona State University, Hayden Library, "The Florentin
Smarandache papers" special collection, Tempe, AZ 85287-
1006, USA.
- d) Student Conference, University of Craiova, Department of
Mathematics, April 1979, "Some problems in number
theory" by Florentin Smarandache.

30. (Inferior) prime part:

2, 3, 3, 5, 5, 7, 7, 7, 7, 11, 11, 13, 13, 13, 13, 17, 17, 19, 19, 19, 19, 23, 23,
23, 23, 23, 23, 29, 29, 31, 31, 31, 31, 31, 31, 37, 37, 37, 37, 41, 41, 43, 43,
43, 43, 47, 47, 47, 47, 47, 47, 53, 53, 53, 53, 53, 53, 59, ...

(For any positive real number n one defines $p(n)$ as the
largest prime number less than or equal to n .)

31. (Superior) prime part:

2, 2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 11, 13, 13, 17, 17, 17, 17, 19, 19, 23, 23,
23, 23, 29, 29, 29, 29, 29, 29, 31, 31, 37, 37, 37, 37, 37, 37, 41, 41, 41,
41, 43, 43, 47, 47, 47, 47, 53, 53, 53, 53, 53, 53, 59, 59, 59, 59, 59, 59, 61, ...

(For any positive real number n one defines $P(n)$ as the
smallest prime number greater than or equal to n .)

Study these sequences.

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

32. (Inferior) square part:

0, 1, 1, 1, 4, 4, 4, 4, 4, 9, 9, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 16, 16,
25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 36, 36, 36, 36, 36, 36, 36, 36, 36,
36, 36, 36, 36, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 49, 64, 64, ...

(The largest square less than or equal to n .)

33. (Superior) square part:

0, 1, 4, 4, 4, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 25, 25, 25, 25, 25, 25,
25, 25, 25, 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, 49, 49, 49, 49, 49, 49,
49, 49, 49, 49, 49, 49, 49, 49, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64,
64, 64, 81, 81, ...

(The smallest square greater than or equal to n .)

Study these sequences.

34. (Inferior) cube part:

0,1,1,1,1,1,1,1,8,27,27,
27,
27,64,64,64,...

(The largest cube less than or equal to n.)

35. (Superior) cube part:

0,1,8,8,8,8,8,8,8,8,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,
27,27,27,27,27,27,64,64,64,64,64,64,64,64,64,64,64,64,64,64,
64,
64,64,64,64,64,125,125,125,...

(The smallest cube greater than or equal to n.)

Study these sequences.

36. (Inferior) factorial part:

1, 2, 2, 2, 2, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 24, 24, 24, 24,
24, ...

(F (n) is the largest factorial less than or equal to n.)
p

37. (Superior) factorial part:

1, 2, 6, 6, 6, 6, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
24, 24, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, ...

(f (n) is the smallest factorial greater than or equal to n.)
p

Study these sequences.

38. Double factorial complements:

1,1,1,2,3,8,15,1,105,192,945,4,10395,46080,1,3,2027025,2560,
34459425,192,5,3715891200,13749310575,2,81081,1961990553600,
35,23040,213458046676875,128,6190283353629375,12,...

(For each n to find the smallest k such that nk is a double
factorial, i.e. $nk =$ either $1*3*5*7*9*...*n$ if n is odd,
either $2*4*6*8*...*n$ if n is even.)

Study this sequence in interrelation with Smarandache
function { $S(n)$ is the smallest integer such that $S(n)!$ is
divisible by n }.

39. Prime additive complements:

1,0,0,1,0,1,0,3,2,1,0,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,
1,0,5,4,3,2,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,...

(For each n to find the smallest k such that $n+k$ is prime.)

Remark: is it possible to get as large as we want
but finite decreasing $k, k-1, k-2, \dots, 2, 1, 0$ (odd k)
sequence included in the previous sequence -- i.e. for any
even integer are there two primes whose difference is equal
to it? I conjecture the answer is negative.

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

40. Prime base:

0,1,10,100,101,1000,1001,10000,10001,10010,10100,100000,
 100001,1000000,1000001,1000010,1000100,10000000,10000001,
 100000000,100000001,100000010,100000100,1000000000,1000000001,
 1000000010,1000000100,1000000101,...

(Each number n written in the prime base.)

(I define over the set of natural numbers the following infinite

base: $p_0 = 1$, and for $k \geq 1$ p_k is the k-th prime number.)

Every positive integer A may be uniquely written in

the prime base as:

$$A = \underbrace{(a_n \dots a_1 a_0)}_{10 \text{ (SP)}} \stackrel{\text{def}}{=} \prod_{i=0}^n a_i p_i, \text{ with all } a_i = 0 \text{ or } 1, \text{ (of course } a_n = 1),$$

in the following way:

- if $p_n \leq A < p_{n+1}$ then $A = p_n + r$;
 - if $p_m \leq r < p_{m+1}$ then $r = p_m + r_2$, $m < n$;
- and so on until one obtains a rest $r = 0$.

Therefore, any number may be written as a sum of prime numbers + e, where $e = 0$ or 1.

If we note by $p(A)$ the superior part of A (i.e. the largest prime less than or equal to A), then

A is written in the prime base as:

$$A = p(A) + p(A-p(A)) + p(A-p(A)-p(A-p(A))) + \dots$$

This base is important for partitions with primes.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

41. Square base:

0,1,2,3,10,11,12,13,20,100,101,102,103,110,111,112,1000,1001,
 1002,1003,1010,1011,1012,1013,1020,10000,10001,10002,10003,
 10010,10011,10012,10013,10020,10100,10101,100000,100001,100002,
 100003,100010,100011,100012,100013,100020,100100,100101,100102,
 100103,100110,100111,100112,101000,101001,101002,101003,101010,
 101011,101012,101013,101020,101100,101101,101102,1000000,...

(Each number n written in the square base.)

(I define over the set of natural numbers the following infinite

base: for $k \geq 0$ $s_k = k^2$.)

Every positive integer A may be uniquely written in

the square base as:

$$A = \overbrace{(a_n \dots a_1 a_0)}_{(S_2)} \stackrel{\text{def}}{=} \sum_{i=0}^n a_i s_i, \text{ with } a_i = 0 \text{ or } 1 \text{ for } i \geq 2,$$

$0 \leq a_0 \leq 3$, $0 \leq a_1 \leq 2$, and of course $a_n = 1$,

in the following way:

- if $s_n \leq A < s_{n+1}$ then $A = s_n + r$;
 - if $s_m \leq r < s_{m+1}$ then $r = s_m + r_1$, $m < n$;
- and so on until one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of squares

(1 not counted as a square -- being obvious) + e, where

$e = 0, 1, \text{ or } 3$.

If we note by $s(A)$ the superior square part of A (i.e. the largest square less than or equal to A), then A is written

in the square base as:

$$A = s(A) + s(A-s(A)) + s(A-s(A)-s(A-s(A))) + \dots .$$

This base is important for partitions with squares.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

42. m-power base (generalization):

(Each number n written in the m -power base,
where m is an integer ≥ 2 .)

(I define over the set of natural numbers the following infinite

m -power base: for $k \geq 0$ $t_k = k^m$.)

Every positive integer A may be uniquely written in
the m -power base as:

$$A = \overbrace{(a_n \dots a_1 a_0)}_{(SM)} \stackrel{\text{def}}{=} \sum_{i=0}^n a_i t_i, \text{ with } a_i = 0 \text{ or } 1 \text{ for } i \geq m,$$

$0 \leq a_i \leq \lfloor ((i+2)^m - 1) / (i+1)^m \rfloor$ (integer part)

for $i = 0, 1, \dots, m-1$, $a_i = 0$ or 1 for $i \geq m$, and of course $a_n = 1$,

in the following way:

- if $t_n \leq A < t_{n+1}$ then $A = t_n + r$;
 - if $t_m \leq r < t_{m+1}$ then $r = t_m + r_1$, $m < n$;
- and so on until one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of m -powers

(1 not counted as an m -power -- being obvious) + e , where

$e = 0, 1, 2, \dots$, or 2^{m-1} .

If we note by $t(A)$ the superior m -power part of A (i.e. the

largest m -power less than or equal to A), then A is written in the

m -power base as:

$$A = t(A) + t(A-t(A)) + t(A-t(A)-t(A-t(A))) + \dots$$

This base is important for partitions with m -powers.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

43. Generalized base:

(Each number n written in the generalized base.)

(I define over the set of natural numbers the following infinite

generalized base: $1 = g_0 < g_1 < \dots < g_k < \dots$.)

Every positive integer A may be uniquely written in

the generalized base as:

$$A = \underbrace{(a_n \dots a_1 a_0)}_{(SG)} \stackrel{\text{def}}{=} \prod_{i=0}^n a_i g_i, \text{ with } 0 \leq a_i \leq \lfloor (g_{i+1} - 1) / g_i \rfloor$$

(integer part) for $i = 0, 1, \dots, n$, and of course $a_n \geq 1$,

in the following way:

- if $g_n \leq A < g_{n+1}$ then $A = g_n + r_1$;
 - if $g_m \leq r_1 < g_{m+1}$ then $r_1 = g_m + r_2$, $m < n$;
- and so on until one obtains a rest $r_j = 0$.

If we note by $g(A)$ the superior generalized part of A (i.e. the largest g_i less than or equal to A), then A is written in the m -power base as:

$$A = g(A) + g(A - g(A)) + g(A - g(A) - g(A - g(A))) + \dots$$

This base is important for partitions: the generalized base may be any infinite integer set (primes, squares, cubes, any m -powers, Fibonacci/Lucas numbers, Bernoulli numbers, etc.) those partitions are studied.

A particular case is when the base verifies: $2g_i \geq g_{i+1}$ for any i , and $g_0 = 1$, because all coefficients of a written number in this base will be 0 or 1.

Remark: another particular case: if one takes $g_i = p^{i-1}$, $i = 1, 2, 3, \dots$, p an integer ≥ 2 , one gets the representation of a number in the numerical base p (p may be 10 (decimal), 2 (binar), 16 (hexadecimal),

etc.}.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

44. Factorial quotients:

1,1,2,6,24,1,720,3,80,12,3628800,2,479001600,360,8,45,
20922789888000,40,6402373705728000,6,240,1814400,
1124000727777607680000,1,145152,239500800,13440,180,
304888344611713860501504000000,...

(For each n to find the smallest k such that nk is a factorial number.)

Study this sequence in interrelation with Smarandache function.

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

45. Double factorial numbers:

1,2,3,4,5,6,7,4,9,10,11,6,13,14,5,6,17,12,19,10,7,22,23,6,
15,26,9,14,29,10,31,8,11,34,7,12,37,38,13,10,41,14,43,22,9,
46,47,6,21,10,...

($d_f(n)$ is the smallest integer such that $d_f(n)!!$ is a
multiple of n .)

Study this sequence in interrelation with Smarandache function.

Reference:

a) Arizona State University, Hayden Library, "The Florentin
Smarandache papers" special collection, Tempe, AZ 85287-
1006, USA.

46. Primitive numbers (of power 2):

2, 4, 4, 6, 8, 8, 8, 10, 12, 12, 14, 16, 16, 16, 16, 18, 20, 20, 22, 24, 24, 24,
26, 28, 28, 30, 32, 32, 32, 32, 32, 34, 36, 36, 38, 40, 40, 40, 42, 44, 44, 46,
48, 48, 48, 48, 50, 52, 52, 54, 56, 56, 56, 58, 60, 60, 62, 64, 64, 64, 64, 64, 66, ...

($S(n)$ is the smallest integer such that $S(n)!$ is divisible by 2^n .)

Curious property: this is the sequence of even numbers,
each number being repeated as many times as its exponent
(of power 2) is.

This is one of irreducible functions, noted $S(k)$, which
helps to calculate the Smarandache function.

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

47. Primitive numbers (of power 3):

3,6,9,9,12,15,18,18,21,24,27,27,27,30,33,36,36,39,42,45,45,
48,51,54,54,54,57,60,63,63,66,69,72,72,75,78,81,81,81,81,84,
87,90,90,93,96,99,99,102,105,108,108,108,111,...

($S(n)$ is the smallest integer such that $S(n)!$ is divisible by 3^n .)

Curious property: this is the sequence of multiples of 3,
each number being repeated as many times as its exponent
(of power 3) is.

This is one of irreducible functions, noted $S(k)$, which helps
to calculate the Smarandache function.

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

**48. Primitive numbers (of power p , p prime)
{generalization}:**

($S_p(n)$ is the smallest integer such that $S_p(n)!$ is divisible by p^n .)

Curious property: this is the sequence of multiples of p , each number being repeated as many times as its exponent (of power p) is.

These are the irreducible functions, noted $S_p(k)$, for any prime number p , which helps to calculate the Smarandache function.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

49. Square residues:

1, 2, 3, 2, 5, 6, 7, 2, 3, 10, 11, 6, 13, 14, 15, 2, 17, 6, 19, 10, 21, 22, 23, 6,
5, 26, 3, 14, 29, 30, 31, 2, 33, 34, 35, 6, 37, 38, 39, 10, 41, 42, 43, 22, 15,
46, 47, 6, 7, 10, 51, 26, 53, 6, 14, 57, 58, 59, 30, 61, 62, 21, ...

($s_r(n)$ is the largest square free number which divides n .)

Or, $s_r(n)$ is the number n released of its squares:

if $n = (p_1^{a_1}) \cdot \dots \cdot (p_r^{a_r})$, with all p_i primes and all $a_i \geq 1$,
then $s_r(n) = p_1 \cdot \dots \cdot p_r$.

Remark: at least the $(2^2) \cdot k$ -th numbers ($k = 1, 2, 3, \dots$)
are released of their squares;

and more general: all $(p^2) \cdot k$ -th numbers (for all p prime,
and $k = 1, 2, 3, \dots$) are released of their squares.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

50. Cubical residues:

1, 2, 3, 4, 5, 6, 7, 4, 9, 10, 11, 12, 13, 14, 15, 4, 17, 18, 19, 20, 21, 22, 23,
 12, 25, 26, 9, 28, 29, 30, 31, 4, 33, 34, 35, 36, 37, 38, 39, 20, 41, 42, 43,
 44, 45, 46, 47, 12, 49, 50, 51, 52, 53, 18, 55, 28, ...

($c_r(n)$ is the largest cube free number which divides n .)

Or, $c_r(n)$ is the number n released of its cubicals:

if $n = (p_1^{a_1}) \cdot \dots \cdot (p_r^{a_r})$, with all p_i primes and all $a_i \geq 1$,
 then $c_r(n) = (p_1^{b_1}) \cdot \dots \cdot (p_r^{b_r})$, with all $b_i = \min\{2, a_i\}$.

Remark: at least the $(2^3) \cdot k$ -th numbers ($k = 1, 2, 3, \dots$)
 are released of their cubicals;
 and more general: all $(p^3) \cdot k$ -th numbers (for all p prime,
 and $k = 1, 2, 3, \dots$) are released of their cubicals.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

51. m-power residues (generalization):

$m_r(n)$ is the largest m-power free number which divides n.)

Or, $m_r(n)$ is the number n released of its m-powers:

if $n = (p_1^{a_1}) * \dots * (p_r^{a_r})$, with all p_i primes and all $a_i \geq 1$,
then $m_r(n) = (p_1^{b_1}) * \dots * (p_r^{b_r})$, with all $b_i = \min \{ m-1, a_i \}$.

Remark: at least the $(2^m)*k$ -th numbers ($k = 1, 2, 3, \dots$)

are released of their m-powers;

and more general: all $(p^m)*k$ -th numers (for all p prime,

and $k = 1, 2, 3, \dots$) are released of their m-powers.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

52. Exponents (of power 2):

0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,4,0,1,0,2,0,1,0,2,0,1,0,2,0,
1,0,5,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,3,0,1,
0,2,0,1,0,6,0,1,...

($e_2(n)$ is the largest exponent (of power 2) which divides n .)

Or, $e_2(n) = k$ if 2^k divides n but $2^{(k+1)}$ does not.

53. Exponents (of power 3):

0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,3,0,0,
1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,
0,1,0,0,2,0,0,1,0,...

($e_3(n)$ is the largest exponent (of power 3) which divides n .)

Or, $e_3(n) = k$ if 3^k divides n but $3^{(k+1)}$ does not.

54. Exponents (of power p) {generalization}:

($e_p(n)$ is the largest exponent (of power p) which divides n ,
where p is an integer ≥ 2 .)

Or, $e_p(n) = k$ if p^k divides n but $p^{(k+1)}$ does not.

Study these sequences.

Reference:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

55. Pseudo-primes:

2,3,5,7,11,13,14,16,17,19,20,23,29,30,31,32,34,35,37,38,41,
43,47,50,53,59,61,67,70,71,73,74,76,79,83,89,91,92,95,97,98,
101,103,104,106,107,109,110,112,113,115,118,119,121,124,125,
127,128,130,131,133,134,136,137,139,140,142,143,145,146, ...

(A number is pseudo-prime if some permutation of the digits
is a prime number, including the identity permutation.)

(Of course, all primes are pseudo-primes,
but not the reverse!)

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

56. Pseudo-squares:

1, 4, 9, 10, 16, 18, 25, 36, 40, 46, 49, 52, 61, 63, 64, 81, 90, 94, 100, 106,
108, 112, 121, 136, 144, 148, 160, 163, 169, 180, 184, 196, 205, 211, 225,
234, 243, 250, 252, 256, 259, 265, 279, 289, 295, 297, 298, 306, 316, 324,
342, 360, 361, 400, 406, 409, 414, 418, 423, 432, 441, 448, 460, 478, 481,
484, 487, 490, 502, 520, 522, 526, 529, 562, 567, 576, 592, 601, 603, 604,
610, 613, 619, 625, 630, 631, 640, 652, 657, 667, 675, 676, 691, 729, 748,
756, 765, 766, 784, 792, 801, 810, 814, 829, 841, 844, 847, 874, 892, 900,
904, 916, 925, 927, 928, 940, 952, 961, 972, 982, 1000, ...

(A number is a pseudo-square if some permutation of the digits is a perfect square, including the identity permutation.)

(Of course, all perfect squares are pseudo-squares, but not the reverse!)

One listed all pseudo-squares up to 1000.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

57. Pseudo-cubes:

1, 8, 10, 27, 46, 64, 72, 80, 100, 125, 126, 152, 162, 207, 215, 216, 251,
261, 270, 279, 297, 334, 343, 406, 433, 460, 512, 521, 604, 612, 621,
640, 702, 720, 729, 792, 800, 927, 972, 1000, ...

(A number is a pseudo-cube if some permutation of the digits is a cube, including the identity permutation.)

(Of course, all perfect cubes are pseudo-cubes, but not the reverse!)

One listed all pseudo-cubes up to 1000.

58. Pseudo-m-powers:

(A number is a pseudo-m-power if some permutation of the digits is an m-power, including the identity permutation; $m \geq 2$.)

Study these sequences.

Reference:

a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

59. Pseudo-factorials:

1,2,6,10,20,24,42,60,100,102,120,200,201,204,207,210,240,270,
402,420,600,702,720,1000,1002,1020,1200,2000,2001,2004,2007,
2010,2040,2070,2100,2400,2700,4002,4005,4020,4050,4200,4500,
5004,5040,5400,6000,7002,7020,7200,...

(A number is a pseudo-factorial if some permutation of the digits
is a factorial number, including the identity permutation.)

(Of course, all factorials are pseudo-factorials,
but not the reverse!)

One listed all pseudo-factorials up to 10000.

Procedure to obtain this sequence:

- calculate all factorials with one digit only ($1!=1$, $2!=2$,
and $3!=6$), this is line_1 (of one digit pseudo-factorials):
1,2,6;
 - add 0 (zero) at the end of each element of line_1,
calculate all factorials with two digits ($4!=24$ only)
and all permutations of their digits:
this is line_2 (of two digits pseudo-factorials):
10,20,60; 24, 42;
 - add 0 (zero) at the end of each element of line_2 as well
as anywhere in between their digits,
calculate all factorials with three digits ($5!=120$,
and $6!=720$) and all permutations of their digits:
this is line_3 (of three digits pseudo-factorials):
100,200,600,240,420,204,402; 120,720, 102,210,201,702,270,720;
and so on ...
- to get from line_k to line_(k+1) do:

- add 0 (zero) at the end of each element of line_k as well
as anywhere in between their digits,
calculate all factorials with (k+1) digits
and all permutations of their digits;
The set will be formed by all line_1 to the last line elements
in an increasing order.

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

60. Pseudo-divisors:

1,10,100,1,2,10,20,100,200,1,3,10,30,100,300,1,2,4,10,20,40,
100,200,400,1,5,10,50,100,500,1,2,3,6,10,20,30,60,100,200,
300,600,1,7,10,70,100,700,1,2,4,8,10,20,40,80,100,200,400,
800,1,3,9,10,30,90,100,300,900,1,2,5,10,20,50,100,200,500,1000,...

(The pseudo-divisors of n .)

(A number is a pseudo-divisor of n if some permutation of the digits is a divisor of n , including the identity permutation.)

(Of course, all divisors are pseudo-divisors,
but not the reverse!)

A strange property: any integer has an infinity of
pseudo-divisors !!

because $10\dots0$ becomes $0\dots01 = 1$, by a circular permutation
of its digits, and 1 divides any integer !

One listed all pseudo-divisors up to 1000 for the numbers 1, 2, 3,
..., 10.

Procedure to obtain this sequence:

- calculate all divisors with one digit only,
this is line_1 (of one digit pseudo-divisors);
- add 0 (zero) at the end of each element of line_1,
calculate all divisors with two digits
and all permutations of their digits:
this is line_2 (of two digits pseudo-divisors);
- add 0 (zero) at the end of each element of line_2 as well
as anywhere in between their digits,
calculate all divisors with three digits

and all permutations of their digits:
this is line_3 (of three digits pseudo-divisors);
and so on ...
to get from line_k to line_(k+1) do:
- add 0 (zero) at the end of each element of line_k as well
as anywhere in between their digits,
calculate all divisors with (k+1) digits
and all permutations of their digits;
The set will be formed by all line_1 to the last line elements
in an increasing order.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

61. Pseudo-odd numbers:

1, 3, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 27, 29, 30,
31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 45, 47, 49, 50, 51, 52, 53, 54,
55, 56, 57, 58, 59, 61, 63, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, ...

(Some permutation of digits is an odd number.)

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

62. Pseudo-triangular numbers:

1, 3, 6, 10, 12, 15, 19, 21, 28, 30, 36, 45, 54, 55, 60, 61, 63, 66, 78, 82,
87, 91, ...

(Some permutation of digits is a triangular number.)

A triangular number has the general form: $n(n+1)/2$.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

63. Pseudo-even numbers:

0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30,
32, 34, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 58,
60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 80, 81, 82, 83,
84, 85, 86, 87, 88, 89, 90, 92, 94, 96, 98, 100, ...

(The pseudo-even numbers.)

(A number is a pseudo-even number if some permutation of
the digits is a even number, including the identity permutation.)

(Of course, all even numbers are pseudo-even numbers,
but not the reverse!)

A strange property: an odd number can be a pseudo-even
number!

One listed all pseudo-even numbers up to 100.

How many primes, squares, and cubes are in this sequence? What
interesting properties has it?

64. Pseudo-multiples (of 5):

0,5,10,15,20,25,30,35,40,45,50,51,52,53,54,55,56,57,58,59,
60,65,70,75,80,85,90,95,100,101,102,103,104,105,106,107,
108,109,110,115,120,125,130,135,140,145,150,151,152,153,
154,155,156,157,158,159,160,165,...

(The pseudo-multiples of 5.)

(A number is a pseudo-multiple of 5 if some permutation of the digits is a multiple of 5, including the identity permutation.)

(Of course, all multiples of 5 are pseudo-multiples, but not the reverse!)

65. Pseudo-multiples of p (p is an integer ≥ 2)

{generalizations}:

(The pseudo-multiples of p .)

(A number is a pseudo-multiple of p if some permutation of the digits is a multiple of p , including the identity permutation.)

(Of course, all multiples of p are pseudo-multiples, but not the reverse!)

Procedure to obtain this sequence:

- calculate all multiples of p with one digit only (if any),
this is line_1 (of one digit pseudo-multiples of p);
- add 0 (zero) at the end of each element of line_1,
calculate all multiples of p with two digits (if any)
and all permutations of their digits:

this is line_2 (of two digits pseudo-multiples of p);

- add 0 (zero) at the end of each element of line_2 as well as anywhere in between their digits,

calculate all multiples with three digits (if any)

and all permutations of their digits:

this is line_3 (of three digits pseudo-multiples of p);

and so on ...

to get from line_k to line_(k+1) do:

- add 0 (zero) at the end of each element of line_k as well as anywhere in between their digits,

calculate all multiples with (k+1) digits (if any)

and all permutations of their digits;

The set will be formed by all line_1 to the last line elements in an increasing order.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

Reference:

a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

66. The Generalized Palindrome:

has one of the forms:

$a(1)a(2)\dots a(n-1)a(n)a(n-1)\dots a(2)a(1)$ or

$a(1)a(2)\dots a(n-1)a(n)a(n)a(n-1)\dots a(2)a(1),$

where all $a(k)$ are positive integers of one or more digits, and all above $a(k)$ integers are concatenated.

(When all $a(k)$ integers have a digit only, one gets the classical definition of the palindrome.)

Obviously, when $n=1$ in the first case, one can consider any positive integer as a SGP because, say $1743902 = (1743902)$, i.e. $a(1) = 1743902$. But let's take in the first case $n > 1$.

Examples of GP:

1457567145 because $1457567145 = (145)(7)(56)(7)(145),$

also 145756567145 because $145756567145 = (145)(7)(56)(56)(7)(145).$

Question: how many terms from the following sequences (known as Smarandache symmetric sequences) are primes?

a) $121, 12321, \dots, 12\dots 898\dots 21, 12\dots 9109\dots 21,$
 $12\dots 91011109\dots 21, \text{ etc.}$

b) $1221, 123321, 123\dots 8998\dots 21, 123\dots 89101098\dots 21,$
 $123\dots 89101111098\dots 21, \text{ etc.}$

Reference:

a) F.Smarandache, Properties of numbers, University of Craiova, 1972.

67. Constructive set (of digits 1,2):

1, 2, 11, 12, 21, 22, 111, 112, 121, 122, 211, 212, 221, 222, 1111, 1112,
1121, 1122, 1211, 1212, 1221, 1222, 21112112, 2121, 2122, 2211, 2212,
2221, 2222, ...

(Numbers formed by digits 1 and 2 only.)

Definition:

a1) 1, 2 belong to S;

a2) if a, b belong to S, then \overline{ab} belongs to S too;

a3) only elements obtained by rules a1) and a2) applied a
finite number of times belong to S.

Remark:

- there are 2^k numbers of k digits in the sequence, for
k = 1, 2, 3, ... ;
- to obtain from the k-digits number group the (k+1)-digits
number group, just put first the digit 1 and second the
digit 2 in the front of all k-digits numbers.

Constructive set (of digits 1,2,3):

1,2,3,11,12,13,21,22,23,31,32,33,111,112,113,121,122,123,
131,132,133,211,212,213,221,222,223,231,232,233,311,312,
313,321,322,323,331,332,333,...

(Numbers formed by digits 1, 2, and 3 only.)

Definition:

- a1) 1, 2, 3 belong to S;
- a2) if a, b belong to S, then \overline{ab} belongs to S too;
- a3) only elements obtained by rules a1) and a2) applied
a finite number of times belong to S.

Remark:

- there are 3^k numbers of k digits in the sequence, for
 $k = 1, 2, 3, \dots$;
- to obtain from the k-digits number group the (k+1)-digits
number group, just put first the digit 1, second the digit 2,
and third the digit 3 in the front of all k-digits numbers.

68. Generalized constructive set:

(Numbers formed by digits d_1, d_2, \dots, d_m only,
all d_i being different each other, $1 \leq m \leq 9$.)

Definition:

- a1) d_1, d_2, \dots, d_m belong to S;
- a2) if a, b belong to S, then \overline{ab} belongs to S too;
- a3) only elements obtained by rules a1) and a2) applied
a finite number of times belong to S.

Remark:

- there are m^k numbers of k digits in the sequence, for
 $k = 1, 2, 3, \dots$;
- to obtain from the k -digits number group the $(k+1)$ -digits
number group, just put first the digit d_1 , second the digit d_2 ,
 \dots , and the m -th time digit d_m in the front of all k -digits
numbers.

More general: all digits d_i can be replaced by numbers as
large as we want (therefore of many digits each), and also
 m can be as large as we want.

Study these sequences.

69. Square roots:

0,1,1,1,2,2,2,2,2,3,3,3,3,3,3,3,4,4,4,4,4,4,4,4,4,4,5,5,5,5,
5,5,5,5,5,5,5,6,6,6,6,6,6,6,6,6,6,6,6,6,6,7,7,7,7,7,7,7,7,7,
7,7,7,7,7,7,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,9,9,9,9,9,9,
9,9,9,9,9,9,9,9,9,9,9,9,9,9,10,10,10,10,10,10,10,10,10,10,
10,10,10,10,10,10,10,10,10,10,10,...

($s(n)$ is the superior integer part of square root of n .)
q

Remark: this sequence is the natural sequence, where each number is repeated $2n+1$ times, because between n^2 (included) and $(n+1)^2$ (excluded) there are $(n+1)^2 - n^2$ different numbers.

How many primes, squares, and cubes are in this sequence? What interesting properties has it?

72. Numerical carpet:

has the general form

```
      .
      .
      .
      1
     1a1
    1aba1
   1abcba1
  1abcdcba1
 1abcdedcba1
1abcdefedcba1
...1abcdefgfedcba1...
 1abcdefedcba1
 1abcdedcba1
 1abcdcba1
 1abcba1
 1aba1
 1a1
 1
 .
 .
 .
```

On the border of level 0, the elements are equal to "1";

they form a rhomb.

Next, on the border of level 1, the elements are equal to "a",

where "a" is the sum of all elements of the previous border;

the "a"s form a rhomb too inside the previous one.

Next again, on the border of level 2, the elements are equal to "b",

where "b" is the sum of all elements of the previous border;

the "b"s form a rhomb too inside the previous one.

And so on...

The numerical carpet is symmetric and esthetic, in its middle g is the sum of all carpet numbers (the core).

Look at a few terms of the Numerical Carpet:

```

1
1
141
1
1
1 8 1
1 8 40 8 1
1 8 1
1
1
1 12 1
1 12 108 12 1
1 12 108 540 108 12 1
1 12 108 12 1
1 12 1
1
1
1 16 1
1 16 208 16 1
1 16 208 1872 208 16 1
1 16 208 1872 9360 1872 208 16 1
1 16 208 1872 208 16 1
1 16 208 16 1
1 16 1
1
1
1 20 1
1 20 340 20 1
1 20 340 4420 340 20 1
1 20 340 4420 39780 4420 340 20 1
1 20 340 4420 39780 198900 39780 4420 340 20 1
1 20 340 4420 39780 4420 340 20 1
1 20 340 4420 340 20 1
1 20 340 20 1
1 20 1
1
.
.
.
```


Or, under other form:

```

1
1 4
1 8 40
1 12 108 504
1 16 208 1872 9360
1 20 340 4420 39780 198900
1 24 504 8568 111384 1002456 5012280
1 28 700 14700 249900 3248700 29238300 146191500
1 32 928 23200 487200 8282400 107671200 969040800 4845204000
.....
.
.
.

```

General Formula:

$$C(n,k) = 4n \prod_{i=1}^k (4n-4i+1) \text{ for } 1 \leq k \leq n,$$

and $C(n,0) = 1$.

Study this multi-sequence.

References:

- a) Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.
- b) Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.
- c) F. Smarandache, "Collected Papers" (Vol. 1), Ed. Tempus, Bucharest, 1994 (to appear);

73. First Table:

6,10,14,18,26,30,38,42,42,54,62,74,74,90,...

($t(n)$ is the largest even number such that any other even number not exceeding it is the sum of two of the first n odd primes.)

It helps to better understand Goldbach's conjecture:

- if $t(n)$ is unlimited, then the conjecture is true;
- if $t(n)$ is constant after a certain rank, then the conjecture is false.

Also, the table gives how many times an even number is written as a sum of two odd primes, and in what combinations.

Of course, $t(n) \leq 2p_n$, where p_n is the n -th odd prime,
 $n = 1, 2, 3, \dots$

Here is the table:

+	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	6	8	10	14	16	20	22	26	32	34	40	44	46	50	.
5	10	12	16	18	22	24	28	34	36	42	46	48	52		.
7		14	18	20	24	26	30	36	38	44	48	50	54		.
11			22	24	28	30	34	40	42	48	52	54	58		.
13				26	30	32	36	42	44	50	54	56	60		.
17					34	36	40	46	48	54	58	60	64		.
19						38	42	48	50	56	60	62	66		.
23							46	52	54	60	64	66	70		.
29								58	60	66	70	72	76		.
31									62	68	72	74	78		.
37										74	78	80	84		.
41											82	84	88		.
43												86	90		.
47													94		.
.....															
															.
															.
															.

Study this table and table sequence.

74. Second table:

9,15,21,29,39,47,57,65,71,93,99,115,129,137,...

($v(n)$ is the largest odd number such that any odd number ≥ 9 not exceeding it is the sum of three of the first n odd primes.)

It helps to better understand Goldbach's conjecture for three primes:

- if $v(n)$ is unlimited, then the conjecture is true;
- if $v(n)$ is constant after a certain rank, then the conjecture is false.

(Vinogradov proved in 1937 that any odd number greater than $3^{(3^{15})}$ satisfies this conjecture.

But what about values less than $3^{(3^{15})}$?)

Also, the table gives you in how many different combinations an odd number is written as a sum of three odd primes, and in what combinations.

Of course, $v(n) \leq 3p_n$, where p_n is the n -th odd prime, $n = 1, 2, 3, \dots$. It is also generalized for the sum of m primes, and how many times a number is written as a sum of m primes ($m > 2$).

7																		
+																		
		3	5	7	11	13	17	19	23	29	31	37	41	43	47			
3		13	15	17	21	23	27	29	33	39	41	47	51	53	57	.	.	.
5			17	19	23	25	29	31	35	41	43	49	53	55	59	.	.	.
7				21	25	27	31	33	37	43	45	51	55	57	61	.	.	.
11					29	31	35	37	41	47	49	55	59	61	65	.	.	.
13						33	37	39	43	49	51	57	61	63	67	.	.	.
17							41	43	47	53	55	61	65	67	71	.	.	.
19								45	49	55	57	63	67	69	73	.	.	.
23									53	59	61	67	71	73	77	.	.	.
29										65	67	73	77	79	83	.	.	.
31											69	75	79	81	85	.	.	.
37												81	85	87	91	.	.	.
41													89	91	95	.	.	.
43														93	97	.	.	.
47															101	.	.	.

11																		
+																		
		3	5	7	11	13	17	19	23	29	31	37	41	43	47			
3		17	19	21	25	27	31	33	37	43	45	51	55	57	61	.	.	.
5			21	23	27	29	33	35	39	45	47	53	57	59	63	.	.	.
7				25	29	31	35	37	41	47	49	55	59	61	65	.	.	.
11					33	35	39	41	45	51	53	59	63	65	69	.	.	.
13						37	41	43	47	53	55	61	65	67	71	.	.	.
17							45	47	51	57	59	65	69	71	75	.	.	.
19								49	53	59	61	67	71	73	77	.	.	.
23									57	63	65	71	75	77	81	.	.	.
29										69	71	77	81	83	87	.	.	.
31											73	79	83	85	89	.	.	.
37												85	89	91	95	.	.	.
41													93	95	99	.	.	.
43														97	101	.	.	.
47															105	.	.	.

13															
+															
	3	5	7	11	13	17	19	23	29	31	37	41	43	47	

3	19	21	23	27	29	33	35	39	45	47	53	57	59	63	.
5		23	25	29	31	35	37	41	47	49	55	59	61	65	.
7			27	31	33	37	39	43	49	51	57	61	63	67	.
11				35	37	41	43	47	53	55	61	65	67	71	.
13					39	43	45	49	55	57	63	67	69	73	.
17						47	49	53	59	61	67	71	73	77	.
19							51	55	61	63	69	73	75	79	.
23								59	65	67	73	77	79	83	.
29									71	73	79	83	85	89	.
31										75	81	85	87	91	.
37											87	91	93	97	.
41												95	97	101	.
43													99	103	.
47														107	.
.....															
.															
.															
.															

17															
+															
	3	5	7	11	13	17	19	23	29	31	37	41	43	47	

3	23	25	27	31	33	37	39	43	49	51	57	61	63	67	.
5		27	29	33	35	39	41	45	51	53	59	63	65	69	.
7			31	35	37	41	43	47	53	55	61	65	67	71	.
11				39	41	45	47	51	57	59	65	69	71	75	.
13					43	47	49	53	59	61	67	71	73	77	.
17						51	53	57	63	65	71	75	77	81	.
19							55	59	65	67	73	77	79	83	.
23								63	69	71	77	81	83	87	.
29									75	77	83	87	89	93	.
31										79	85	89	91	95	.
37											91	95	97	101	.
41												99	101	105	.
43													103	107	.
47														111	.
.....															
.															
.															
.															

19																	
+																	
		3	5	7	11	13	17	19	23	29	31	37	41	43	47		
3	25	27	29	33	35	39	41	45	51	53	59	63	65	69	.	.	
5	29	31	35	37	41	43	47	53	55	61	65	67	71	.	.	.	
7		33	37	39	43	45	49	55	57	63	67	69	73	.	.	.	
11			41	43	47	49	53	59	61	67	71	73	77	.	.	.	
13				45	49	51	55	61	63	69	73	75	79	.	.	.	
17					53	55	59	65	67	73	77	79	83	.	.	.	
19						57	61	67	69	75	79	81	85	.	.	.	
23								65	71	73	79	83	85	89	.	.	
29									77	79	85	89	91	95	.	.	
31										81	87	91	93	97	.	.	
37											93	97	99	103	.	.	
41												101	103	107	.	.	
43													105	109	.	.	
47														113	.	.	

23																	
+																	
		3	5	7	11	13	17	19	23	29	31	37	41	43	47		
3	29	31	33	37	39	43	45	49	55	57	63	67	69	73	.	.	
5		33	35	39	41	45	47	51	57	59	65	69	71	75	.	.	
7			37	41	43	47	49	53	59	61	67	71	73	77	.	.	
11				45	47	51	53	57	63	65	71	75	77	81	.	.	
13					49	53	55	59	65	67	73	77	79	83	.	.	
17						57	59	63	69	71	77	81	83	87	.	.	
19							61	65	71	73	79	83	85	89	.	.	
23								69	75	77	83	87	89	93	.	.	
29									81	83	89	93	95	99	.	.	
31										85	91	95	97	101	.	.	
37											97	101	103	107	.	.	
41												105	107	111	.	.	
43													109	113	.	.	
47														117	.	.	

29															
+															
	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	35	37	39	43	45	49	51	55	61	63	69	73	75	79	.
5		39	41	45	47	51	53	57	63	65	71	75	77	81	.
7			43	47	49	53	55	59	65	67	73	77	79	83	.
11				51	53	57	59	63	69	71	77	81	83	87	.
13					55	59	61	65	71	73	79	83	85	89	.
17						63	65	69	75	77	83	87	89	93	.
19							67	71	77	79	85	89	91	95	.
23								75	81	83	89	93	95	99	.
29									87	89	95	99	101	105	.
31										91	97	101	103	107	.
37											103	107	109	113	.
41												111	113	117	.
43													115	119	.
47														123	.

.....

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.

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31															
+															
	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	37	39	41	45	47	51	53	57	63	65	71	75	77	81	.
5		41	43	47	49	53	55	59	65	67	73	77	79	83	.
7			45	49	51	55	57	61	67	69	75	79	81	85	.
11				53	55	59	61	65	71	73	79	83	85	89	.
13					57	61	63	67	73	75	81	85	87	91	.
17						65	67	71	77	79	85	89	91	95	.
19							69	73	79	81	87	91	93	97	.
23								77	83	85	91	95	97	101	.
29									89	91	97	101	103	107	.
31										93	99	103	105	109	.
37											105	109	111	115	.
41												113	115	119	.
43													117	121	.
47														125	.

.....

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.

Second table sequence:

0,0,0,0,1,2,4,4,6,7,9,10,11,15,17,16,19,19,23,25,26,26,28,
33,32,35,43,39,40,43,43,...

($a(2k+1)$ represents the number of different combinations
such that $2k+1$ is written as a sum of three odd primes.)

This sequence is deduced from the second table.

Study the second table and the second table sequence.

References:

- a) Florentin Smarandache, "Problems with and without ... problems!",
Ed. Somipress, Fes, Morocco, 1983;
- b) Arizona State University, Hayden Library, "The Florentin
Smarandache papers" special collection, Tempe, AZ 85287-
1006, USA.