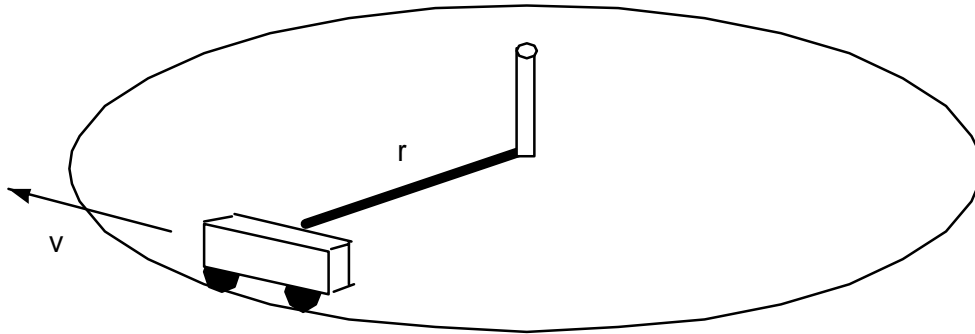


PID Theory and System Type

MEM 639 Real-Time Microcomputer Control 1

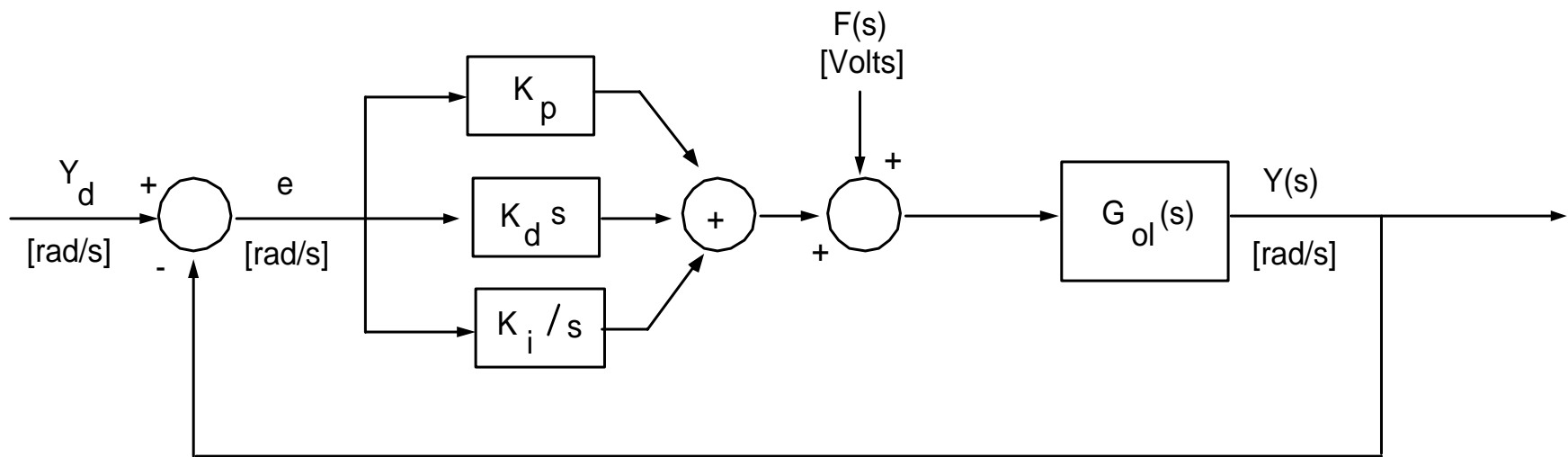
PID Control (Closed-Loop) of Motorized Tethered Cart

Goal: Want cart velocity to always be 3 rad/s

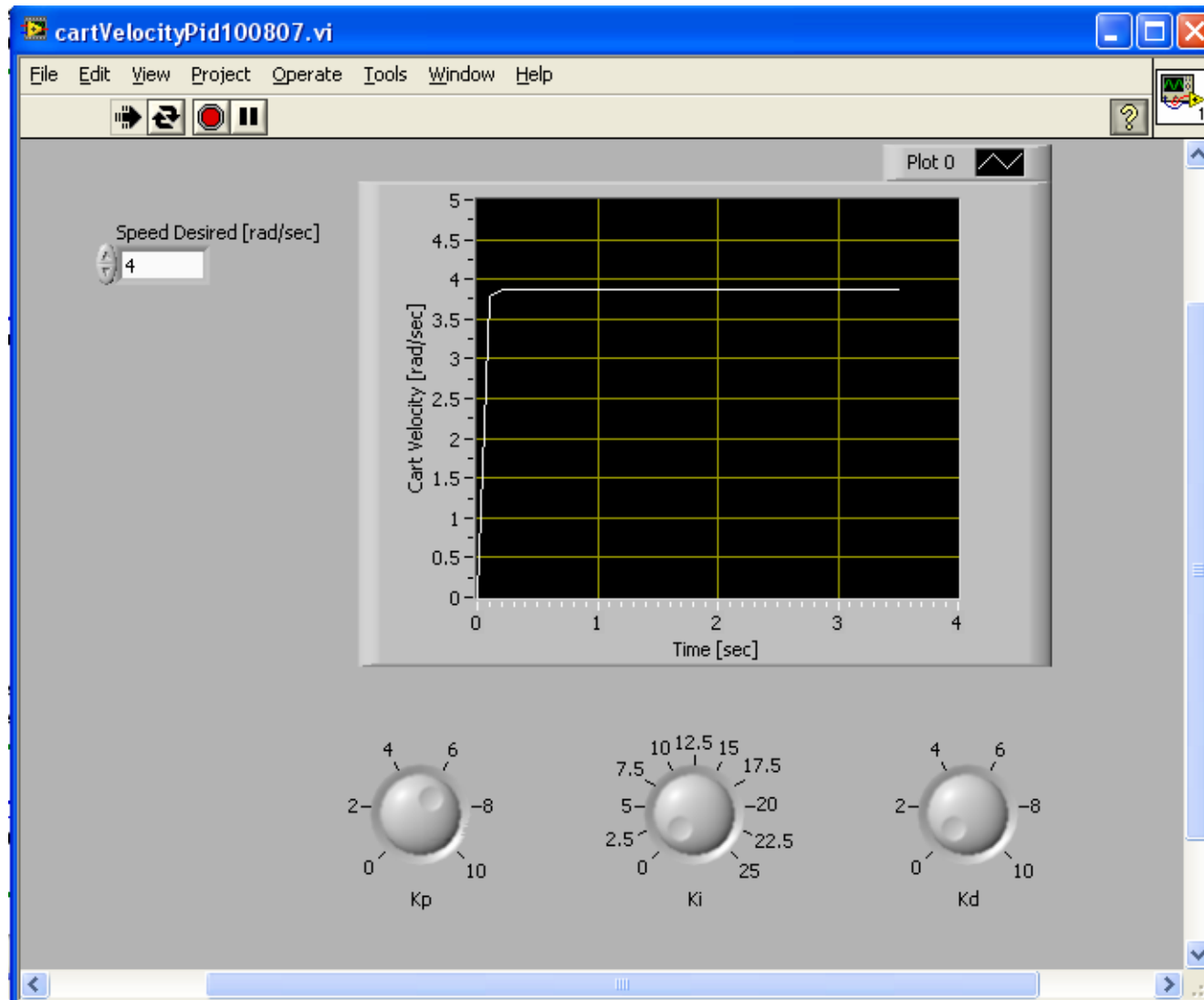


- Even if terrain has hills and valleys
- Even if cart weight changes

PID is the most common form of closed-loop control:

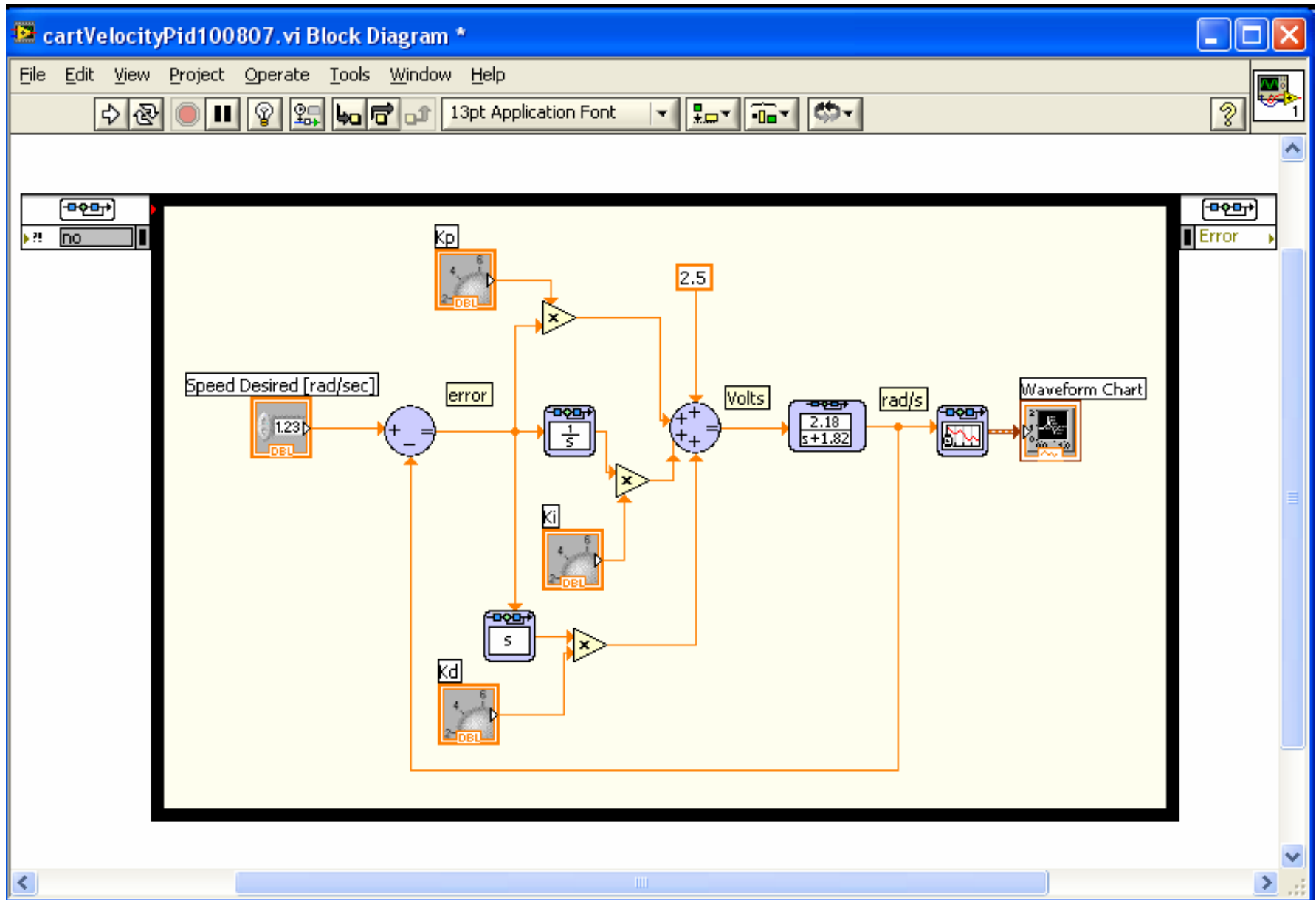


Why?

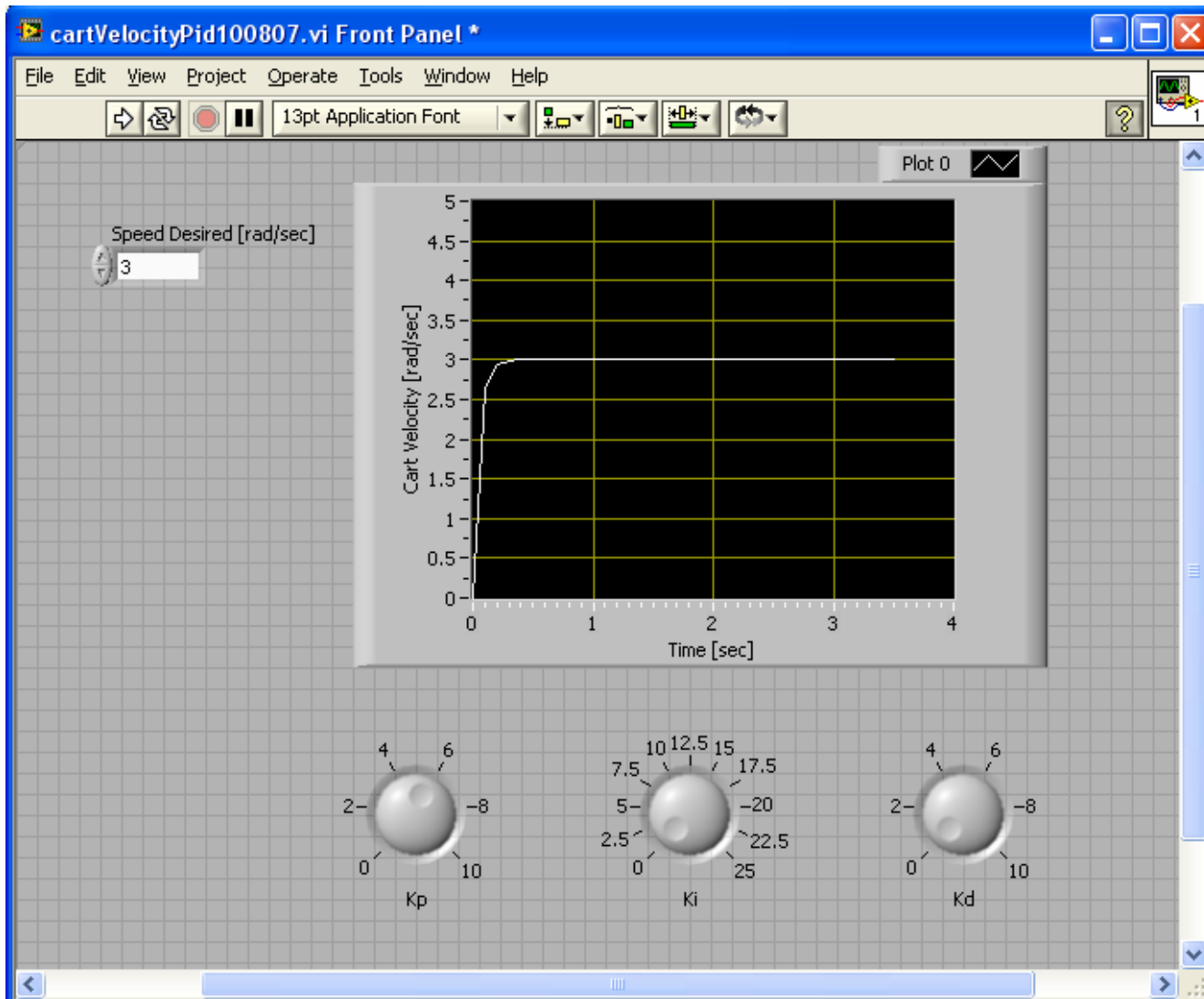


Answer: One can tune for desired performance without full knowledge of dynamics

PID Control Block Diagram Simulation



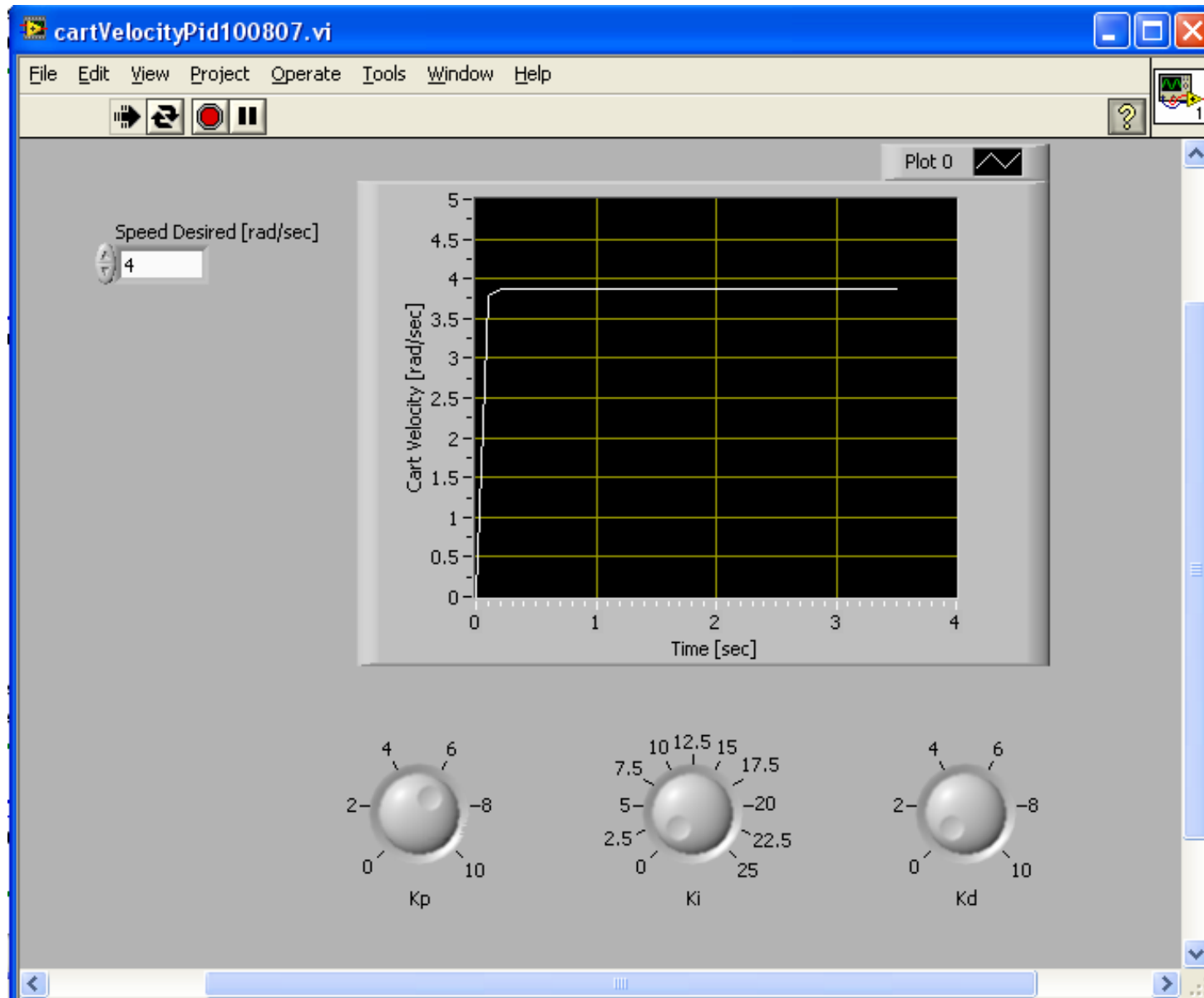
Lab: PID Simulation for Motorized Tethered Cart – Closed-loop Velocity Control



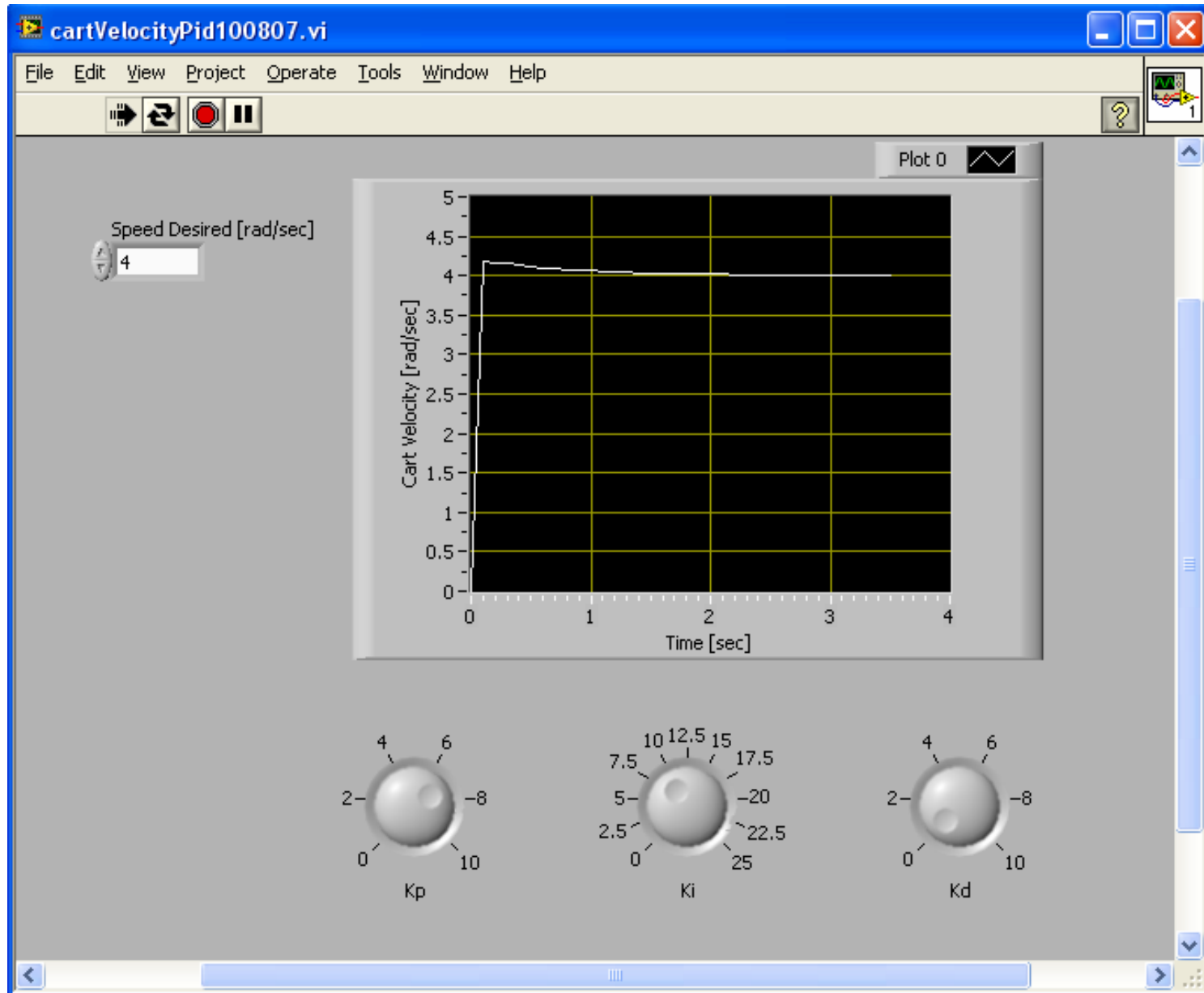
Implement simulation

Observe:

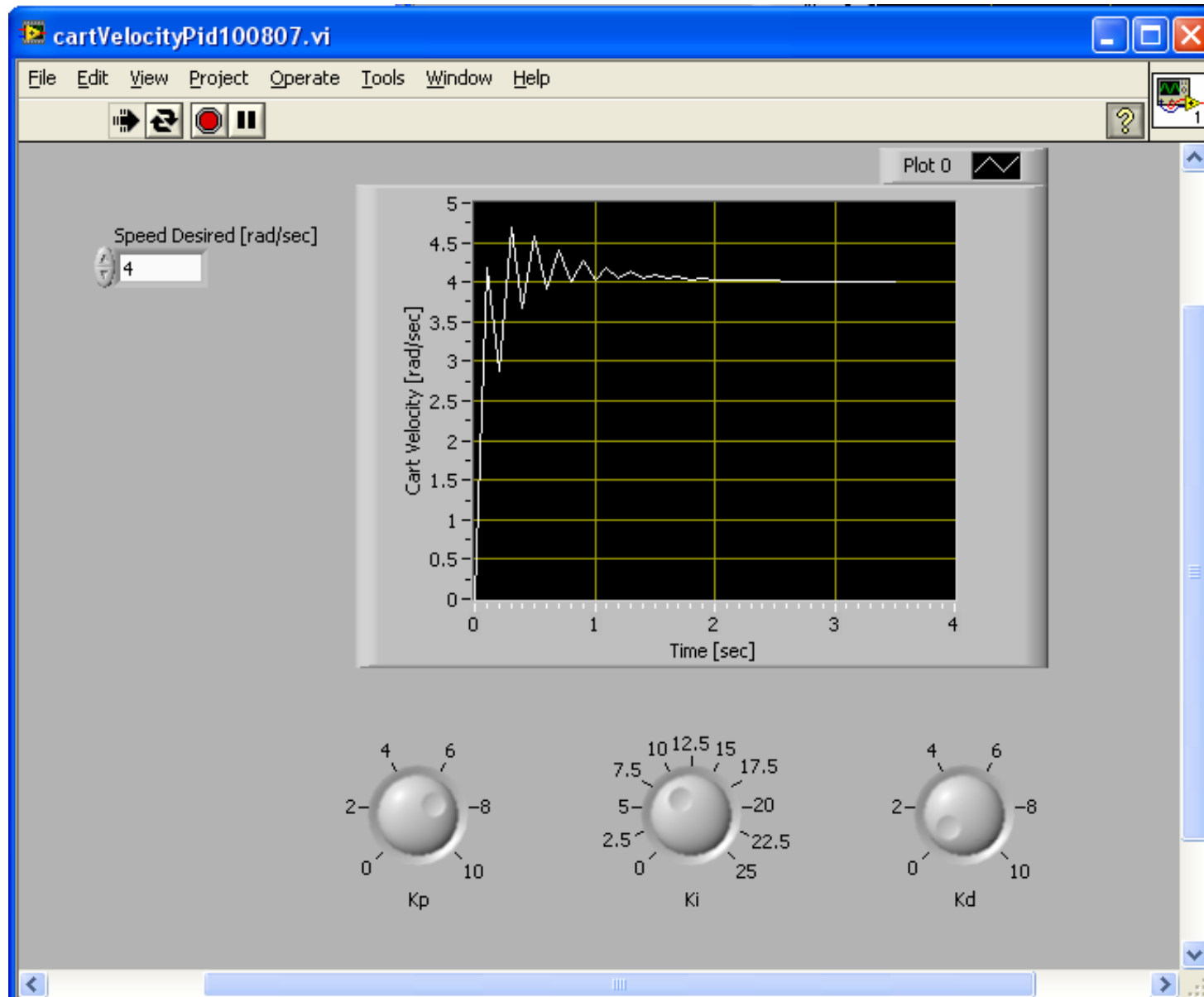
- Faster time constant
- Response vs. stability
- Steady-state error
- When goes unstable



Proportional only: faster time response, but always steady-state error!



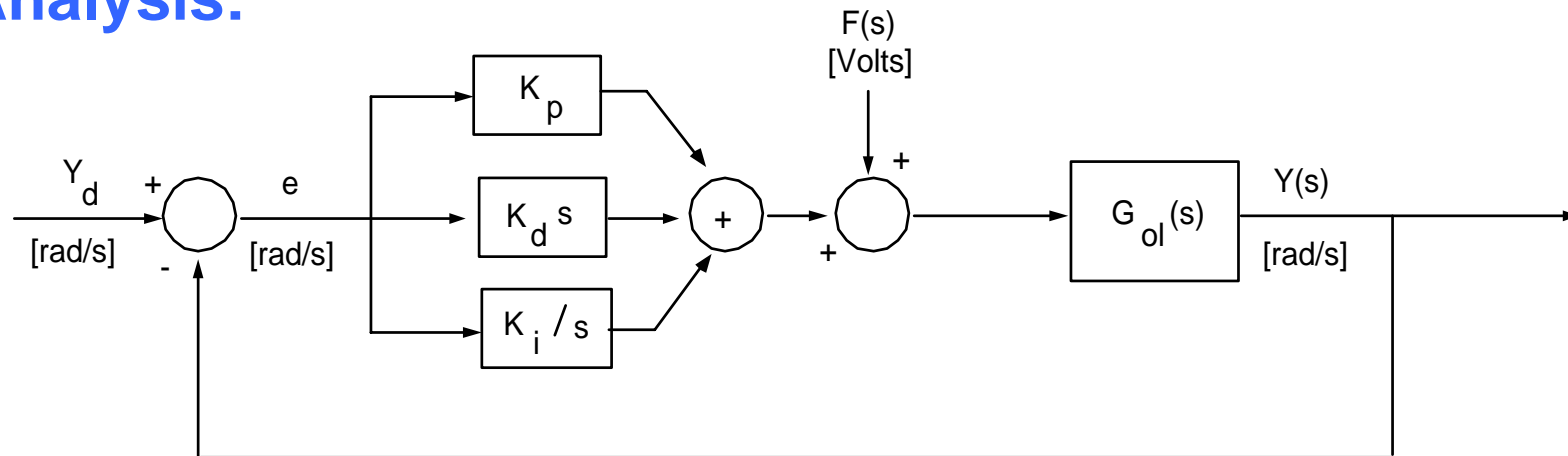
Adding integral control eliminates steady-state error (but overshoot)



Adding derivative yields more overshoot – and possibly unstable

However, tuning can be very tedious. Some knowledge of system type aids in tuning, make performance expectations realistic and avoid instability.

Analysis:



Input-Output Relationship given by:

$$Y(s) = G_{ol} \left\{ F(s) + \left(K_p + \frac{K_i}{s} + K_d s \right) (Y_d - Y) \right\} \quad (1)$$

Can reduce to show that (1) becomes:

$$Y(s) = \frac{G_{ol}s}{s + G_{ol}(K_p s + K_i + K_d s^2)} F + \frac{G_{ol}(K_p s + K_i + K_d s^2)}{s + G_{ol}(K_p s + K_i + K_d s^2)} Y_d \quad (2)$$

Case Study 1: Proportional only control $Y_d = \frac{A}{s}$ So (2) becomes:

$$Y(s) = \frac{G_{ol}s}{s + G_{ol}(K_p s)} F + \frac{G_{ol}(K_p s)}{s + G_{ol}(K_p s)} \cdot \frac{A}{s} \quad (3)$$

Final Value Theorem states that:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot Y(s)$$

Thus steady-state part of (3) becomes:

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot Y(s) = \lim_{s \rightarrow 0} \frac{G_{ol}(K_p s)}{s + G_{ol}(K_p s)} A \quad (4)$$

Given that cart velocity has an open-loop transfer function of the following form:

$$G_{ol}(s) = \frac{b}{s+a} \quad (5)$$

Substitution of (5) into (4) yields

$$y_{ss} = \lim_{s \rightarrow 0} \frac{\frac{b}{s+a}(K_p s)}{s + \frac{b}{s+a}(K_p s)} A = \lim_{s \rightarrow 0} \frac{bK_p s}{s(s+a) + bK_p s} A$$

Applying L'Hopital to calculate the limit yields:

$$y_{ss} = \lim_{s \rightarrow 0} \frac{bK_p}{2s + a + bK_p} A = \frac{bK_p}{a + bK_p} A$$

Note: If K_p is very large, then $y_{ss} \approx A$

Will always have
steady-state error

Case Study 2: Proportional + Integral control $Y_d = \frac{A}{s}$

So (2) becomes:

$$Y(s) = \frac{G_{ol}s}{s + G_{ol}(K_p s + K_i)} F + \frac{G_{ol}(K_p s + K_i)}{s + G_{ol}(K_p s + K_i)} \cdot \frac{A}{s}$$

Apply Final Value Theorem:

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot Y(s) = \frac{G_{ol}(K_p s + K_i)}{s + G_{ol}(K_p s + K_i)} A \quad (6)$$

Substituting the OLTF (2) into (6) yields:

$$y_{ss} = \lim_{s \rightarrow 0} \frac{\frac{b}{s+a}(K_p s + K_i)}{s + \frac{b}{s+a}(K_p s + K_i)} A = \lim_{s \rightarrow 0} \frac{b(K_p s + K_i)}{s(s+a) + b(K_p s + K_i)} A = \frac{bK_i}{bK_i} A = A$$

Hence integral action ensures zero steady-state error

Systems like the motorized tethered cart are called Type 0 systems:

$$G_{ol}(s) = \frac{b}{s + a}$$

All transfer functions can be factored into the general form:

$$G_{ol}(s) = \frac{(\tau_a s + 1) \cdots (\tau_m s + 1)}{s^i (\tau_1 s + 1) \cdots (\tau_n s + 1)}$$

$i = 0$	Type 0 System
$i = 1$	Type 1 System
$i = 2$	Type 2 System

General form any TF

A Type i system is the number of “free” integrators, i . The motorized tethered cart, for velocity control, is a Type 0 system

For Type 0 Systems:

- Will always have steady-state error (with proportional only control): see Case 1
- Integral action will eliminate steady-state error (see Case 2)
- Derivative action may increase transient response but cause instability

Homework:

- Show steady-state response for derivative action on Type 0 system
- Response to ramp input