The hyperbolic functions

Introduction

In a number of applications, the exponential functions e^x and e^{-x} occur in particular combinations and these combinations are referred to as the **hyperbolic functions**. This leaflet defines these functions and show their graphs.

1. The hyperbolic functions

The **hyperbolic cosine** is defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The **hyperbolic sine** is defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

These are often referred to as the 'cosh' function and the 'shine' function. They are nothing more than combinations of the exponential functions e^x and e^{-x}.

Your scientific calculator can be used to evaluate these functions. Usually the 'hyp cos' and 'hyp sin' buttons are used. You may need to refer to your calculator manual. Check that you can use your calculator by verifying that

$$\sinh 3 = 10.018 \qquad \text{and} \qquad \cosh 4.2 = 33.351$$

You may like to verify that the same values can be obtained by using the exponential functions, that is

$$\sinh 3 = \frac{e^3 - e^{-3}}{2} \qquad \text{and} \qquad \cosh 4.2 = \frac{e^{4.2} + e^{-4.2}}{2}$$

The **hyperbolic tangent** is defined as

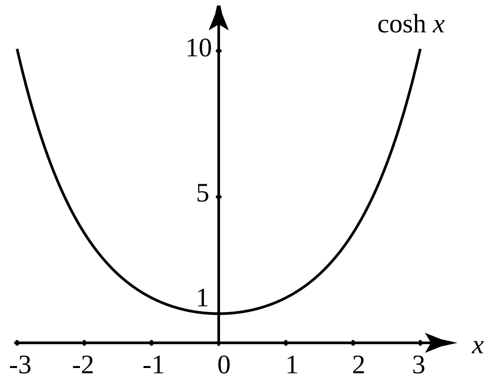
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Other hyperbolic functions are

$$\operatorname{sech} x = \frac{1}{\cosh x}, \qquad \operatorname{cosech} x = \frac{1}{\sinh x}, \qquad \coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$$

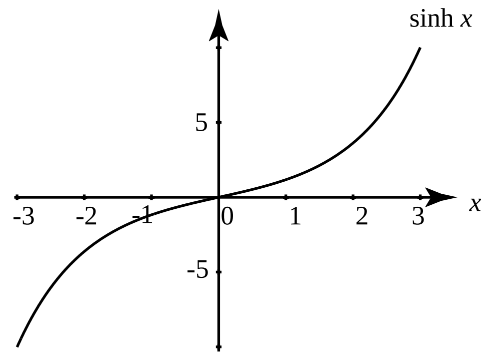
By drawing up tables of values, or indeed by using the properties of the exponential functions, graphs can be plotted. The graphs of $\cosh x$, $\sinh x$ and $\tanh x$ are shown below.

2. Graphs of the hyperbolic functions



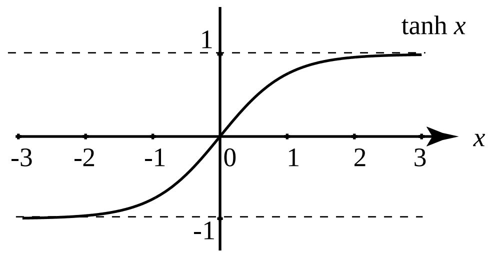
Some properties of $\cosh x$

- $\cosh 0 = 1$ and $\cosh x$ is greater than 1 for all other values of x
- the graph is symmetrical about the y axis. Mathematically this means $\cosh(-x) = \cosh x$. Cosh x is said to be an **even function**,
- $\cosh x \to +\infty$ as $x \to \pm\infty$



Some properties of $\sinh x$

- $\sinh 0 = 0$, the graph passes through the origin
- $\sinh(-x) = -\sinh x$. Sinh x is said to be an **odd function** - it has rotational symmetry about the origin.
- $\sinh x \to +\infty$ as $x \to +\infty$, $\quad \sinh x \to -\infty$ as $x \to -\infty$



Some properties of $\tanh x$

- $\tanh 0 = 0$ and $-1 < \tanh x < 1$ for all x
- $\tanh(-x) = -\tanh x$. Tanh x is said to be an **odd function** - it has rotational symmetry about the origin.
- $\tanh x \to +1$ as $x \to +\infty$, $\qquad \tanh x \to -1$ as $x \to -\infty$