(5.4) Multiple-Angle Identities

Objective: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Why: These identities will be used in calculus.

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Double Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos\ u - \sin\ u$

$= 2\cos\ u - 1$

$= 1 - 2\sin\ u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan\ u}$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

1. Use the sum and difference formulas to prove the identity.

$$\sin 2u = 2 \sin u \cos u$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

2. Use the sum and difference formulas to prove the identity.

$$\cos 2u = 2\cos^2 u - 1$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

3. Find all solutions in the interval $[0, 2\pi)$

$$2\cos x + \sin 2x = 0$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

4. $\cos 2x = \sin x$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Prove the identity.

$$\sin 3x = (\sin x)(3 - 4\sin^2 x)$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

* Proof involves taking the double angle identities and solving for $\sin^2 u$ or $\cos^2 u$.

ex.

$$\cos 2u = 1 - 2\sin^2 u$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Rewrite in terms of trigonometric functions with no power greater than 1.

$\cos^4 x$

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Obj: To learn the Double-Angle Identities, Power-Reducing
ngle Identies.

Half-Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\ \dfrac{1-\cos u}{\sin u} \\ \dfrac{\sin u}{1+\cos u} \end{cases}$$

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Using the Half-Angle Identites, give the exact value of sin 105°.

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

Find all solutions in the interval [0, 2π)

$$\sin^2 x = \cos^2 \frac{x}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\frac{2}{1}\left[\frac{1-\cos 2x}{2} = \frac{1+\cos x}{2}\right]$$

$$1-\cos 2x = 1+\cos x$$

$$1-(2\cos^2 x-1) = 1+\cos x$$

$$1-2\cos^2 x+1 = 1+\cos x$$

$$2-2\cos^2 x = 1+\cos x$$

$$-1-\cos x \qquad -1-\cos x$$

$$1-2\cos^2 x-\cos x = 0$$

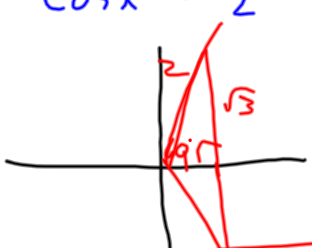
$$-2\cos^2 x-\cos x+1 = 0$$

$$2\cos^2 x+\cos x-1 = 0$$

$$(2\cos x-1)(\cos x+1) = 0$$

$$2\cos x-1=0 \quad \text{or} \quad \cos x+1=0$$

$$\cos x = \tfrac{1}{2} \qquad \cos x = -1$$



$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

OR

$$\frac{2}{1}\left[1-\cos^2 x = \frac{1+\cos x}{2}\right]$$

$$2-2\cos^2 x = 1+\cos x$$

HW:
(HR) (5.4) Pg.432: 5, 7, 9, 15, 19, 23, 39

Obj: To learn the Double-Angle Identities, Power-Reducing Identities, and Half-Angle Identies.

(9) $\sin 2x - \tan x = 0$

undef when $\cos x = 0$: $\frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x \left[2\sin x\cos x - \frac{\sin x}{\cos x} = 0 \right]$

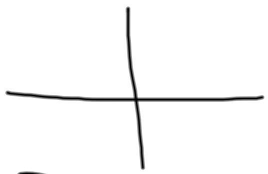
$2\sin x\cos^2 x - \sin x = 0$

$\sin x\,(2\cos^2 x - 1) = 0$

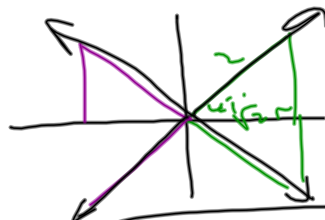
$\sin x = 0$ or $2\cos^2 x - 1 = 0$

$\sqrt{\cos^2 x} = \pm\sqrt{\frac{1}{2}}$

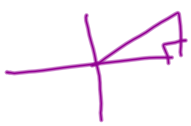
$\cos x = \pm\frac{\sqrt{2}}{2}$



$x = 0, \pi$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



$\sin 15 = \sin\frac{30}{2} = \sqrt{\frac{1-\cos 30}{2}} = \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$

$\sin(45-30)$

$= \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

(15) $\sin 4x = 2\sin 2x\cos 2x$ GOAL

$\sin(2x+2x) =$

$\sin 2x\cos 2x + \cos 2x\sin 2x =$

$2\sin 2x\cos 2x =$