Thevenin Equivalent Circuits

Introduction

In each of these problems, we are shown a circuit and its Thevenin or Norton equivalent circuit. The Thevenin and Norton equivalent circuits are described using three parameters: V_{oc}, the open circuit voltage of the circuit, I_{sc}, the short circuit of the circuit and R_{th}, the Thevenin resistance of the circuit. Each problem, asks us to determine the value of asked to determine the value of V_{oc}, I_{sc} or R_{th}.

Thevenin equivalent circuits are discussed in Section 5.5 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda. Norton equivalent circuits are discussed in Section 5.6.

Worked Examples

Example 1:
The circuit shown in Figure 1b is the Thevenin equivalent circuit of the circuit shown in Figure 1a. Find the value of the open circuit voltage, V_{oc} and Thevenin resistance, R_{th}.

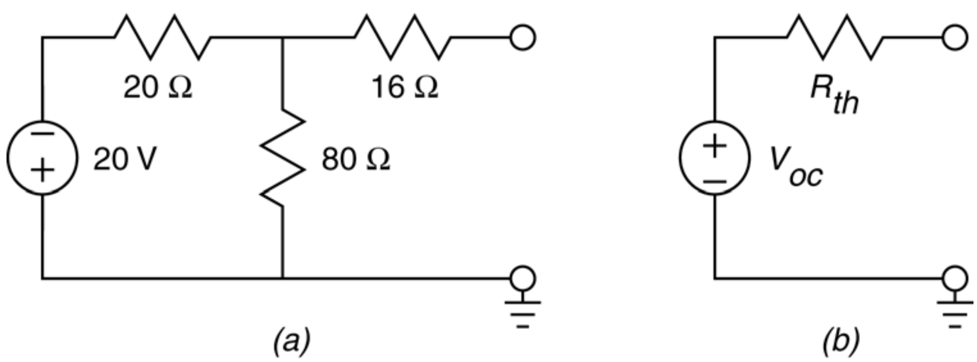

Figure 1 The circuit considered in Example 1.

Solution: The circuit from Figure 1a can be reduced to its Thevenin equivalent circuit in four steps shown in Figure 2a, b, c and d.

A source transformation transforms the series voltage source and 20 Ω resistor in Figure 1a into the parallel current source and 20 Ω resistor in Figure 2a. The current source current is calculated from the voltage source voltage and resistance as $\frac{20\text{ V}}{20\ \Omega} = 1\text{ A}$. After the source transformation, the 20 Ω resistor is parallel to the 80 Ω resistor. Replacing these parallel resistors with the equivalent 16 Ω resistor produces the circuit shown in Figure 2b.

A second source transformation transforms the parallel current source and 16 Ω resistor in Figure 2b into the series voltage source and 16 Ω resistor in Figure 2c. The voltage source voltage is calculated from the current source current and resistance as $(1\ \text{A})(16\ \Omega) = 16\ \text{V}$. After the source transformation, the two16 Ω resistors are in series. Replacing these series resistors with the equivalent 32 Ω resistor produces the circuit shown in Figure 2d.

Comparing Figure 2d to Figure 1b shows that the Thevenin resistance is R_{th} = 32 Ω and the open circuit voltage, V_{oc} = -16 V.

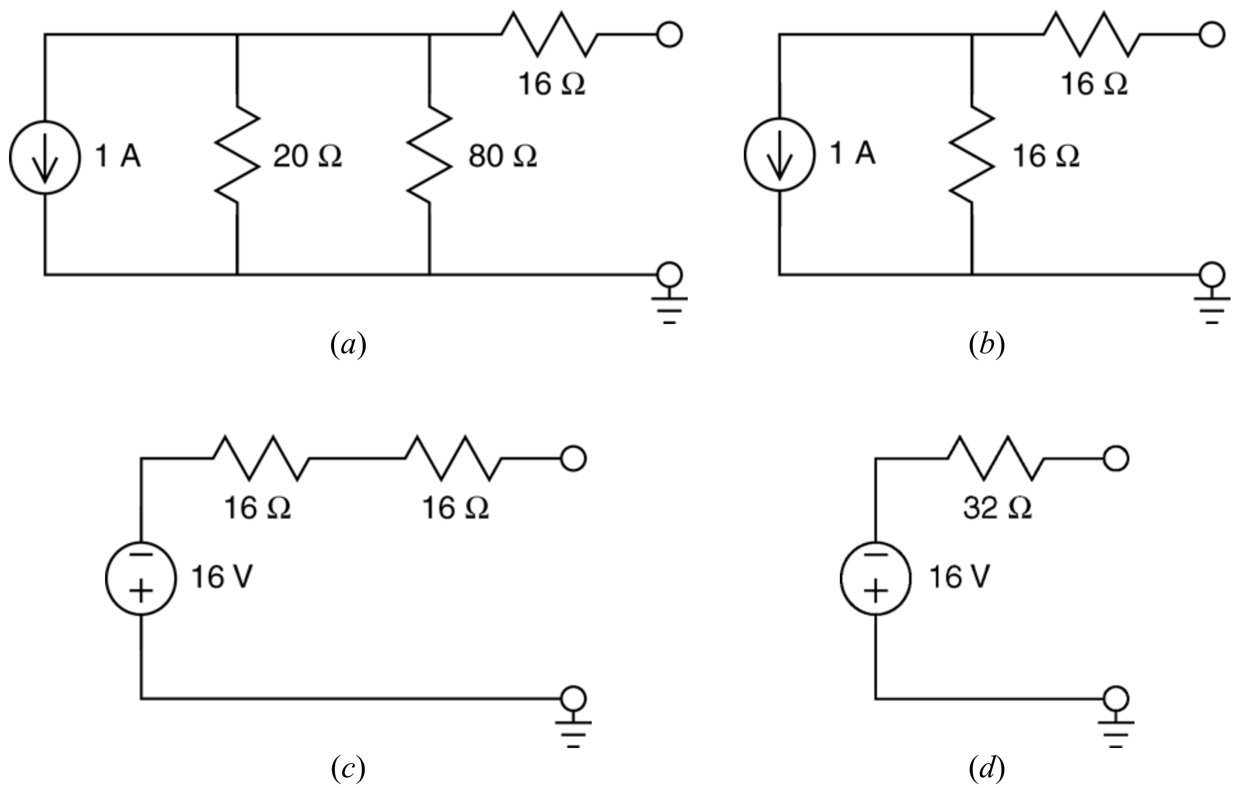

Figure 2 The circuit from Figure 1a can be reduced to its Thevenin equivalent circuit in four steps shown here as (*a*), (*b*), (*c*), and (*d*).

Example 2:

The circuit shown in Figure 3b is the Thevenin equivalent circuit of the circuit shown in Figure 1a. Find the value of the open circuit voltage, V_{oc} and Thevenin resistance, R_{th}.

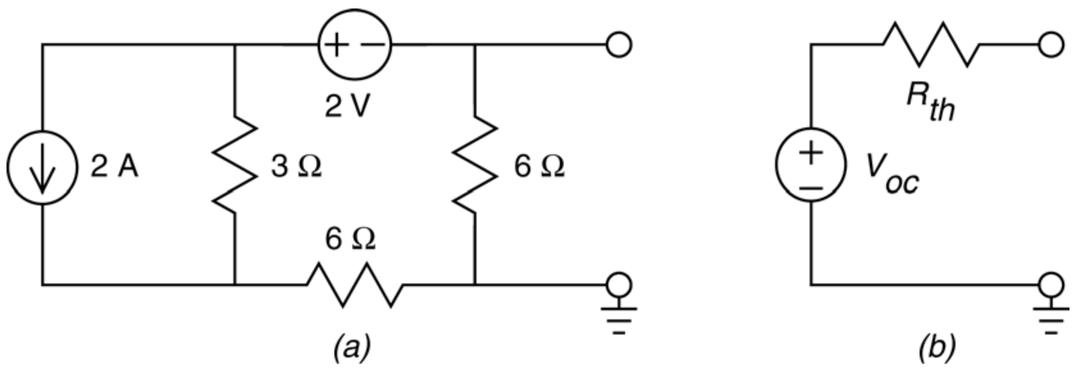

Figure 3 The circuit considered in Example 2.

Solution: The circuit from Figure 3a can be reduced to its Thevenin equivalent circuit in five steps shown in Figure 4a, b, c, d and e.

A source transformation transforms the parallel current source and 3 Ω resistor in Figure 3a into the series voltage source and 3 Ω resistor in Figure 4a. The voltage source voltage is calculated from the current source current and resistance as $(2\text{ A})(3\text{ }\Omega) = 6\text{ V}$. After the source transformation, the 3 Ω and 6 Ω resistors are in series. Also, the 6V and 3 V voltage sources are in series. Replacing the series resistors with the equivalent 9 Ω resistor and the series voltage sources with the equivalent 8 V source produces the circuit shown in Figure 4b.

A second source transformation transforms the series 8 V voltage source and 9 Ω resistor in Figure 4b into the parallel current source and 9 Ω resistor in Figure 4c. The current source current is calculated from the voltage source voltage and resistance as $\frac{8\text{ V}}{9\text{ }\Omega} = 0.89\text{ A}$. After the source transformation, the 9 Ω resistor is parallel to the 6 Ω resistor. Replacing these parallel resistors with the equivalent 3.6 Ω resistor produces the circuit shown in Figure 4d.

A third source transformation transforms the parallel 0.89 A current source and 3.6 Ω resistor in Figure 4d into the series voltage source and 3.6 Ω resistor in Figure 4e. The voltage source voltage is calculated from the current source current and resistance as $(0.89\text{ A})(3.6\text{ }\Omega) = 3.2\text{ V}$.

Comparing Figure 4e to Figure 3b shows that Thevenin resistance is R_{th} = 3.6 Ω and that the open circuit voltage, V_{oc} = -3.2 V.

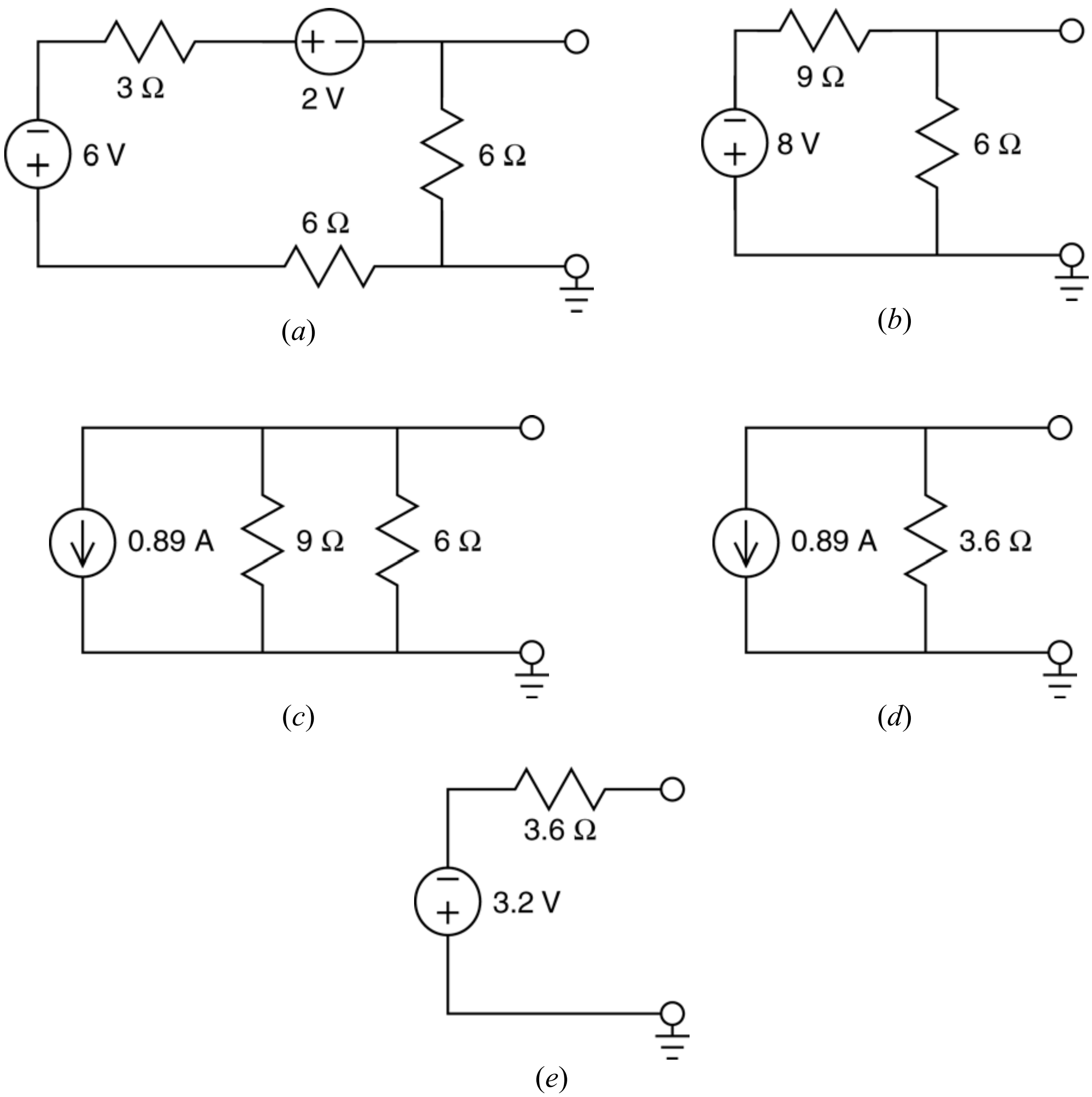

Figure 4 The circuit from Figure 3a can be reduced to its Thevenin equivalent circuit in five steps shown here as (*a*), (*b*), (*c*), (*d*) and (*e*).

Example 3:
The circuit shown in Figure 5b is the Thevenin equivalent circuit of the circuit shown in Figure 5a. Find the value of the open circuit voltage, V_{oc} and Thevenin resistance, R_{th}.

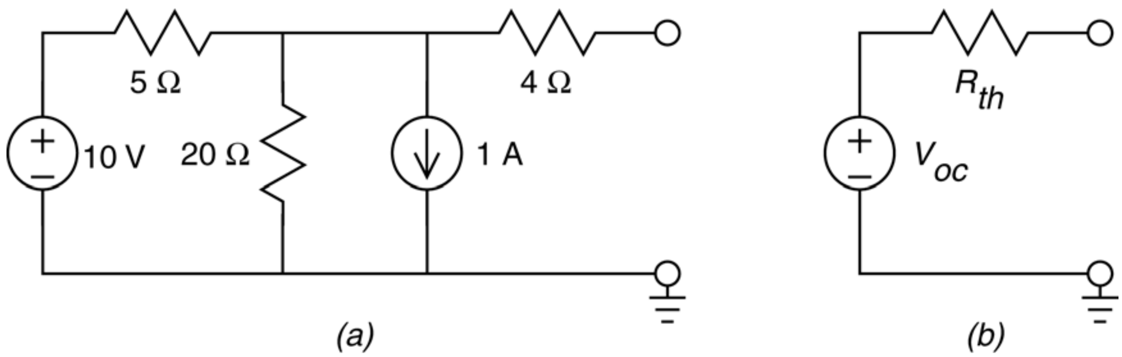

Figure 5 The circuit considered in Example 3.

Solution: The circuit from Figure 5a can be reduced to its Thevenin equivalent circuit in four steps shown in Figure 6a, b, c and d.

A source transformation transforms the series 10 V voltage source and 5 Ω resistor in Figure 5a into the parallel current source and 5 Ω resistor in Figure 6a. The current source current is calculated from the voltage source voltage and resistance as $\frac{10\text{ V}}{5\ \Omega} = 2\text{ A}$. After the source transformation, the 5 Ω resistor is parallel to the 20 Ω resistor. Also, the 2 A current source is parallel to the 1 A current source. Replacing these parallel resistors with the equivalent 4 Ω resistor and replacing the parallel current sources with the equivalent 1 A current source produces the circuit shown in Figure 6b.

A second source transformation transforms the parallel 1 A current source and 4 Ω resistor in Figure 6b into the series voltage source and 4 Ω resistor in Figure 6c. The voltage source voltage is calculated from the current source current and resistance as $(1\text{ A})(4\ \Omega) = 4\text{ V}$. After the source transformation, the two 4 Ω resistors are in series. Replacing the series resistors with the equivalent 8 Ω produces the circuit shown in Figure 6d.

Comparing Figure 6d to Figure 5b shows that Thevenin resistance is $R_{th} = 8\ \Omega$ and that the open circuit voltage, $V_{oc} = 4$ V.

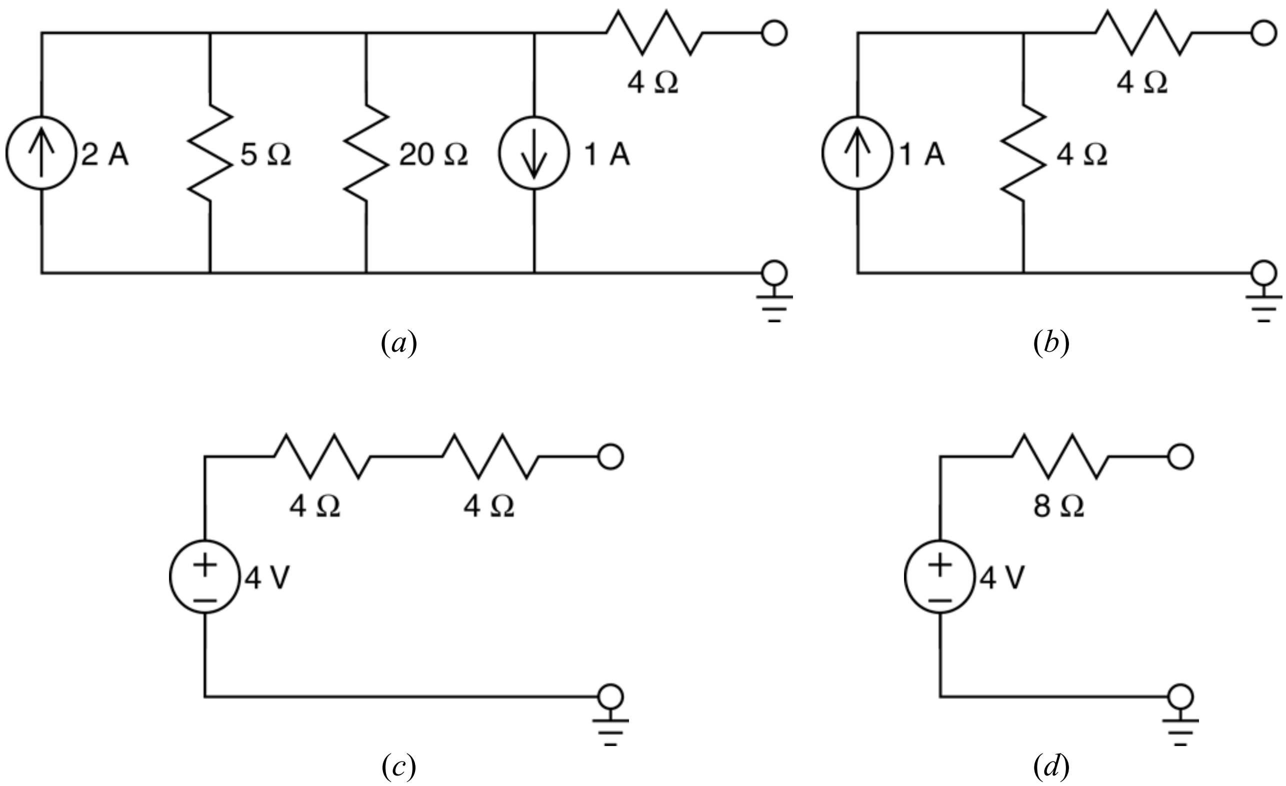

Figure 6 The circuit from Figure 5a can be reduced to its Thevenin equivalent circuit in four steps shown here as (*a*), (*b*), (*c*), and (*d*).

Example 4:

The circuit shown in Figure 7b is the Thevenin equivalent circuit of the circuit shown in Figure 7a. Find the value of the open circuit voltage, V_{oc} and Thevenin resistance, R_{th}. Also, determine the value of the short circuit current, I_{sc}.

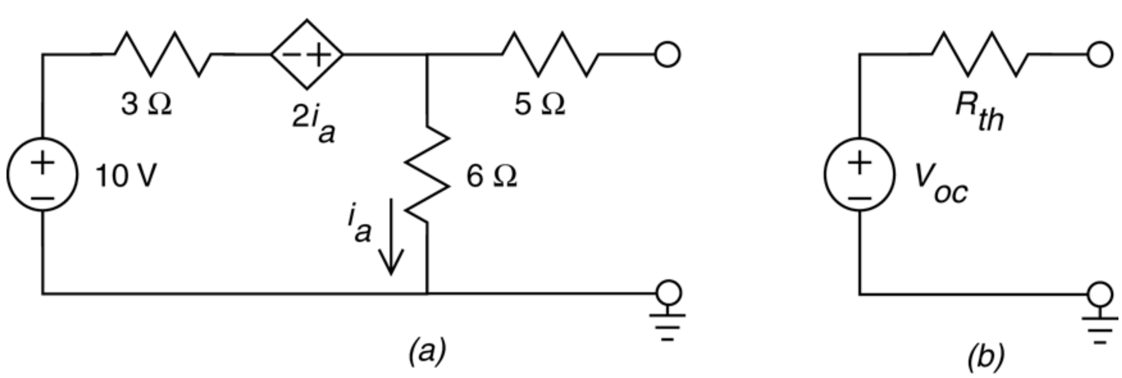

Figure 7 The circuit considered in Example 4.

Solution: To determine the value of the open circuit voltage, V_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure 8 shows the circuit from Figure 7a after adding the open circuit and labeling the open circuit voltage. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure 8, mesh current i_2 is equal to the current in the open circuit. Consequently, $i_2 = 0$ A. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - 0 = i_1$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 3\,i_1 - 2\left(i_1 - 0\right) + 6\left(i_1 - 0\right) - 10 = 0$$

$$\Rightarrow \quad i_1 = \frac{10}{7} = 1.43 \text{ A}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + V_{oc} - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad V_{oc} = 6\left(i_1\right) = 6\left(1.43\right) = 8.58 \text{ V}$$

Next, to determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure 9 shows the circuit from Figure 7a after adding the short circuit and labeling the short circuit current. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure 9, mesh current i_2 is equal to the current in the short circuit. Consequently, $i_2 = I_{sc}$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - I_{sc}$$

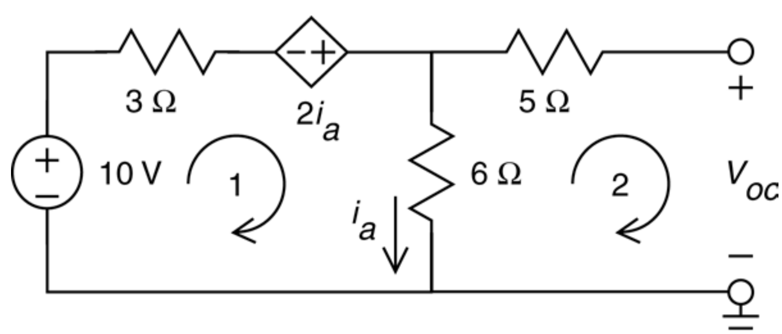

Figure 8 Calculating the open circuit voltage, V_{oc}, using mesh equations.

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 10 \tag{1}$$

Apply KVL to mesh 2 to get

$$5\,i_2 - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{11}{6}\,i_2$$

Substituting into equation 1 gives

$$7\left(\frac{11}{6}\,i_2\right) - 4\,i_2 = 10 \quad \Rightarrow \quad i_2 = 1.13\text{ A} \quad \Rightarrow \quad I_{sc} = 1.13\text{ A}$$

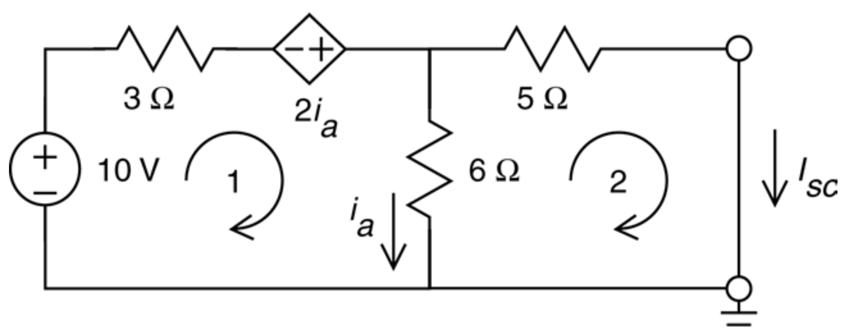

Figure 9 Calculating the short circuit current, I_{sc}, using mesh equations.

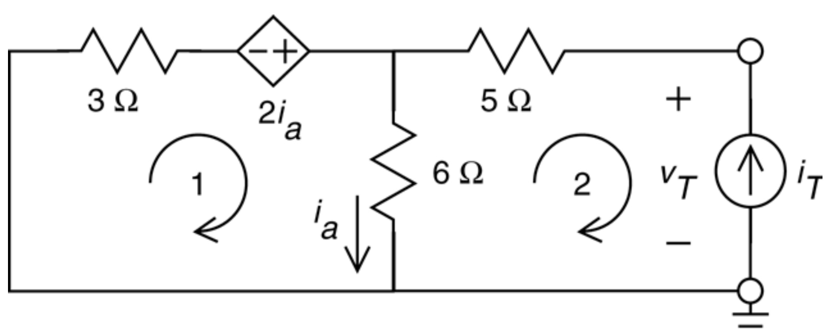

Figure 10 Calculating the Thevenin resistance, $R_{th} = \dfrac{v_T}{i_T}$, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 10 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source across the terminals of the circuit and then label the voltage across that current source as shown in Figure 10. The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

In Figure 10, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure 10, mesh current i_2 is equal to the negative of the current source current. Consequently, $i_2 = -i_T$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 + i_T$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{4}{7}\,i_2 \tag{2}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + v_T - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = -v_T$$

Substituting for i_1 using equation 2 gives

$$-6\left(\frac{4}{7}\,i_2\right) + 11\,i_2 = -v_T \quad \Rightarrow \quad 7.57\,i_2 = -v_T$$

Finally,

$$R_{th} = \frac{v_T}{i_T} = \frac{-v_T}{-i_T} = \frac{-v_T}{i_2} = 7.57\ \Omega$$

As a check, notice that

$$R_{th}\,I_{sc} = \left(7.57\right)\left(1.13\right) = 8.55 \approx V_{oc}$$

Example 5:

The circuit shown in Figure 11b is the Thevenin equivalent circuit of the circuit shown in Figure 11a. Find the value of the open circuit voltage, V_{oc} and Thevenin resistance, R_{th}. Also, determine the value of the short circuit current, I_{sc}.

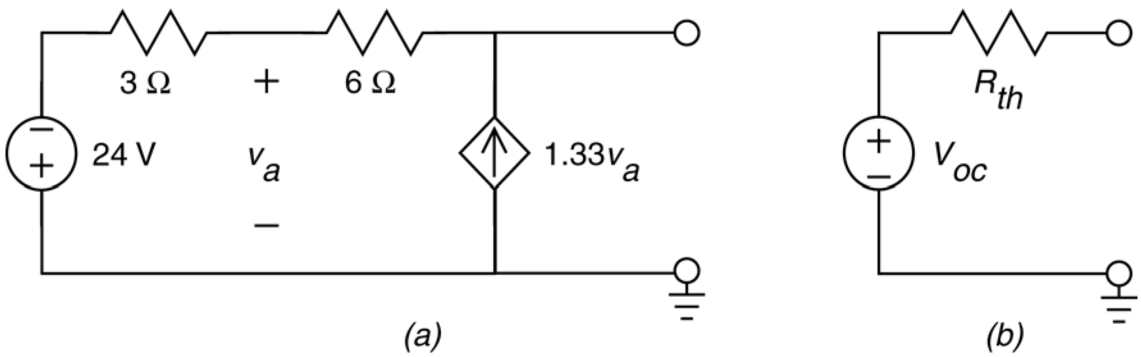

Figure 11 The circuit considered in Example 5.

Solution: To determine the value of the open circuit voltage, V_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure 12 shows the circuit from Figure 11a after adding the open circuit and labeling the open circuit voltage. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure 12, node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the open circuit voltage, i.e. $v_3 = V_{oc}$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 + V_{oc} = 3\,v_a$$

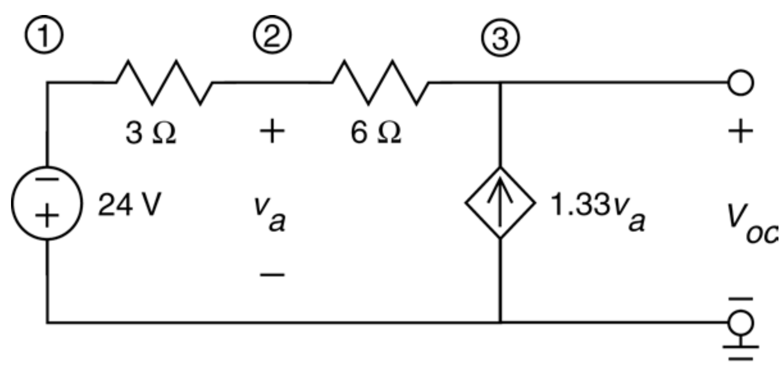

Figure 12 Calculating the open circuit voltage, V_{oc}, using node equations.

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = 0 \quad \Rightarrow \quad 9\, v_2 - v_3 = 0 \quad \Rightarrow \quad 9\, v_a = V_{oc}$$

Combining these equations gives

$$3\left(-48 + V_{oc}\right) = 9\, v_a = V_{oc} \quad \Rightarrow \quad V_{oc} = 72 \text{ V}$$

Next, to determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure 13 shows the circuit from Figure 7a after adding the short circuit and labeling the short circuit current. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure 13, node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The voltage at node 3 is equal to the voltage across a short, $v_3 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across a short, i.e. $v_3 = 0$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\, v_1 + v_3 = 3\, v_2 \quad \Rightarrow \quad -48 = 3\, v_a \quad \Rightarrow \quad v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = I_{sc} \quad \Rightarrow \quad \frac{9}{6} v_a = I_{sc} \quad \Rightarrow \quad I_{sc} = \frac{9}{6}\left(-16\right) = -24 \text{ A}$$

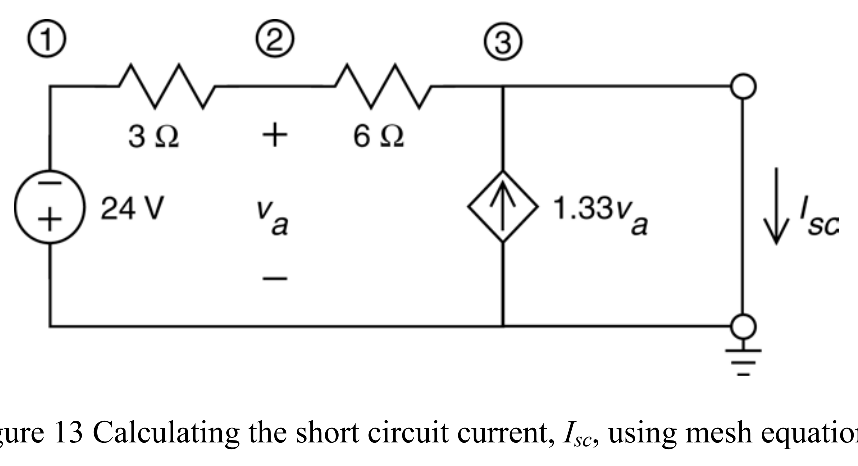

Figure 13 Calculating the short circuit current, I_{sc}, using mesh equations.

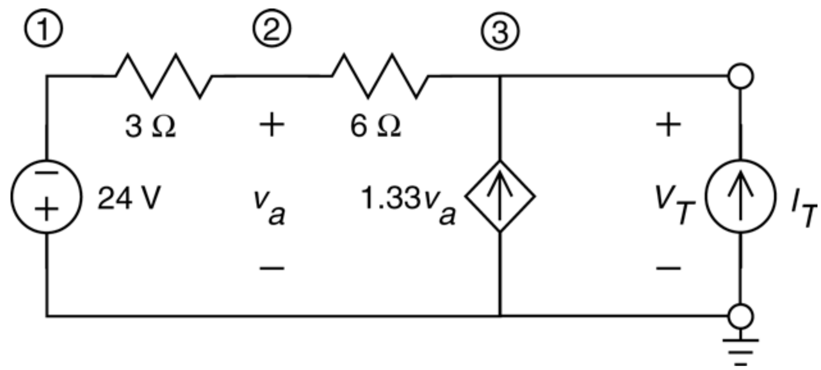

Figure 14 Calculating the Thevenin resistance, $R_{th} = \dfrac{v_T}{i_T}$, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 24 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source circuit across the terminals of the circuit and then label the voltage across that current source as shown in Figure 14. The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure 14, node voltage v_1 is equal to the across a short circuit, i.e. $v_1 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across the current source, i.e. $v_3 = v_T$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

$$\begin{aligned} \frac{v_2 - v_3}{6} + \frac{4}{3}v_2 + i_T = 0 \quad &\Rightarrow \quad 9\,v_2 - v_3 + 6\,i_T = 0 \\ &\Rightarrow \quad 9\,v_a - v_T + 6\,i_T = 0 \\ &\Rightarrow \quad 3\,v_T - v_T + 6\,i_T = 0 \quad \Rightarrow \quad 2\,v_T = -6\,i_T \end{aligned}$$

Finally,

$$R_{th} = \frac{v_T}{i_T} = -3\,\Omega$$

As a check, notice that