ENGINEERING OPTIMIZATION

Theory and Practice
Third Edition

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PREFACE

The ever-increasing demand on engineers to lower production costs to withstand competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products both economically and efficiently. Optimization techniques, having reached a degree of maturity over the past several years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems being solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a simple manner. Essential proofs and explanations of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using nonlinear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. This book presents the theory and applications of all optimization techniques in a comprehensive manner. Some of the recently developed methods of optimization, such as genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy optimization, are also discussed in the book.

A large number of solved examples, review questions, problems, figures, and references are included to enhance the presentation of the material. Although emphasis is placed on engineering design problems, examples and problems are taken from several fields of engineering to make the subject appealing to all branches of engineering.

This book can be used either at the junior/senior or first-year-graduate-level optimum design or engineering optimization courses. At Purdue University, I cover Chapters 1, 2, 3, 5, 6, and 7 and parts of Chapters 8, 10, 12, and 13 in a dual-level course entitled *Optimal Design: Theory with Practice*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his or her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. The book can also be used, with some supplementary material, for a second course on engineering optimization or optimum design or structural optimization. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

The book consists of thirteen chapters and two appendices. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn–Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and postoptimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5 through 7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and in-direct methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems with mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, geometric, and dynamic programming. The theory and applications of calculus of variations, optimal control theory, multiple objective optimization, optimality criteria methods, genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy system optimization are discussed briefly in Chapter 12. The various approximation techniques used to speed up the convergence of practical mechanical and structural optimization problems are outlined in Chapter 13. Appendix A presents the definitions and properties of convex and concave functions. Finally, a brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B.

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