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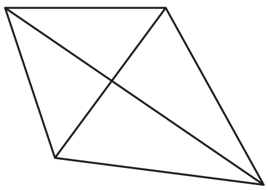
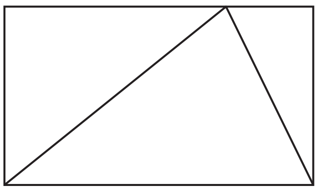
CHAPTER

7 The Pythagorean Theorem

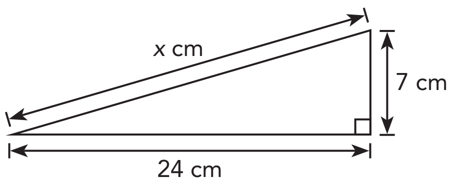
Lesson 7.1 Understanding the Pythagorean Theorem and Plane Figures

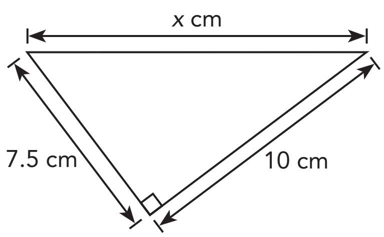
For each figure, shade two right triangles and label the hypotenuse of each triangle with an arrow.

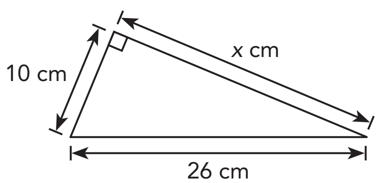
1. 

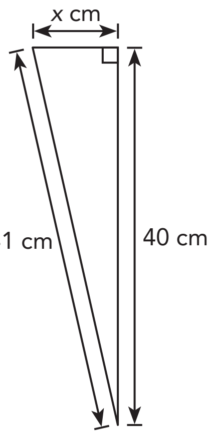
2. 

Find the value of *x*.

3. 

4. 

5. 

6. 

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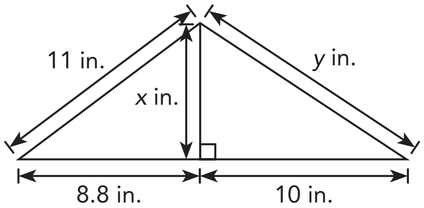
Calculate each unknown side length. Round your answer to the nearest tenth.

7.



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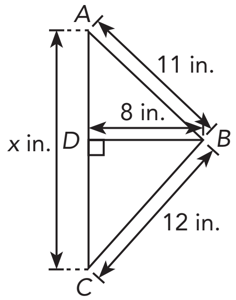

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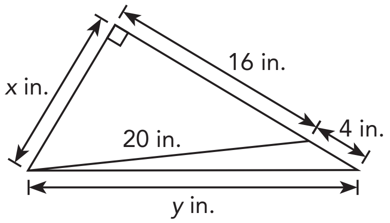

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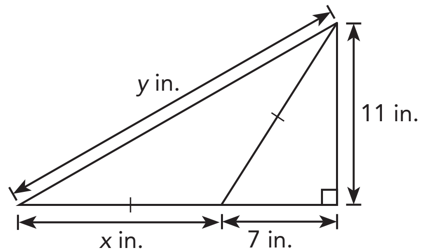
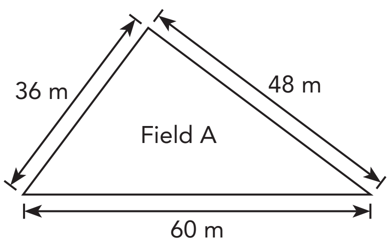
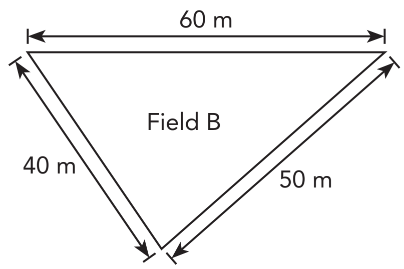


Solve. Show your work. Round your answer to the nearest tenth.

11. Fritz mows two triangular fields. Determine which field is a right triangle.





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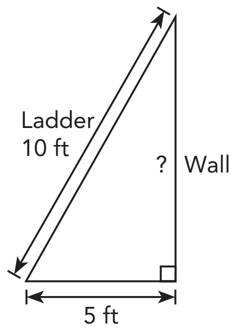
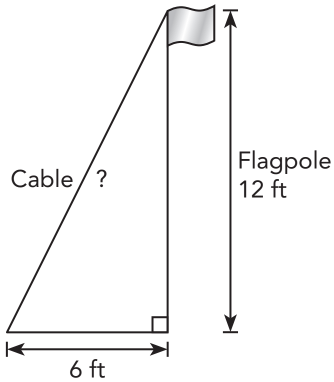
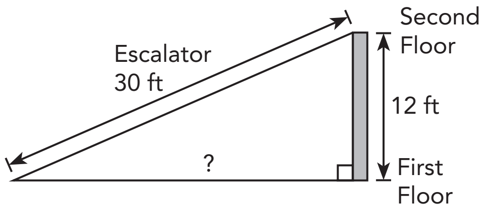
12. Alan placed a ladder against a wall. The bottom of the ladder was 5 feet away from the wall. Find the height of the wall.



13. One end of a cable is attached to the top of a flagpole and the other end is attached 6 feet away from the base of the pole. If the height of the flagpole is 12 feet, find the length of the cable.



14. An escalator runs from the first floor of a shopping mall to the second floor. The length of the escalator is 30 feet and the distance between the floors is 12 feet. Find the distance from the base of the escalator to the point on the first floor directly below the top of the escalator.



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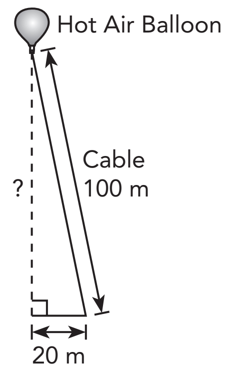
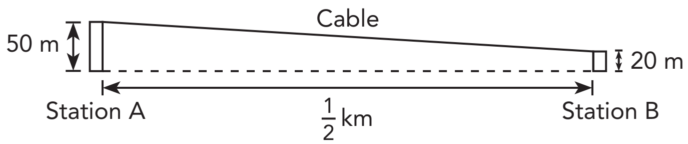
15. A hot air balloon is attached to the ground by a taut 100-meter cable, as shown in the diagram. Find the vertical height of the balloon above the ground.



16. A taut cable connects two cable car stations A and B which are positioned 50 meters and 20 meters above the ground, respectively. The horizontal distance between the stations is $\frac{1}{2}$ kilometer. Find the length of the cable.



17. A whiteboard is 6 feet long and 3 feet wide. Find the length of the longest straight line that can be drawn on the whiteboard.

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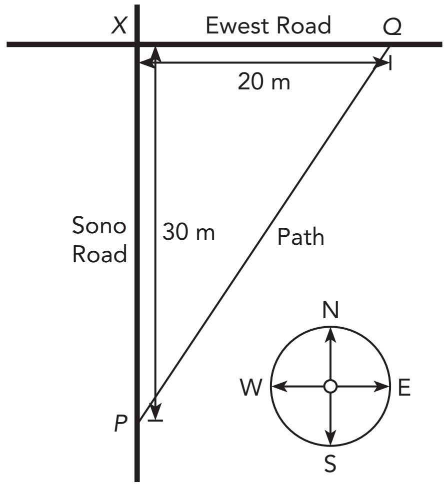
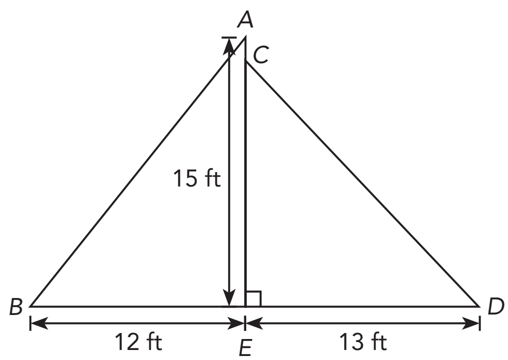
18. Sono Road runs from South to North and Ewest Road runs from East to West intersecting at point *X*. Jeb and Jill are at point *P* on Sono Road 30 meters from point *X*. Jeb walks along Sono Road to point *X* then turns east and walks 20 meters to point *Q* on Ewest Road. Jill walks on a path linking point *P* to point *Q*. Find the difference in distance between the two routes.



19. A 15-foot vertical pole has two strings of equal length attached to it at different points. The other end of one string, represented by $\overline{AB}$ in the diagram is tethered to the ground 12 feet from the base of the pole. The other end of the other string, represented by $\overline{CD}$ in the diagram is tethered to the ground 13 feet from the base of the pole.

a) Find the length of the string.



b) Find the distance between the points *A* and *C*.

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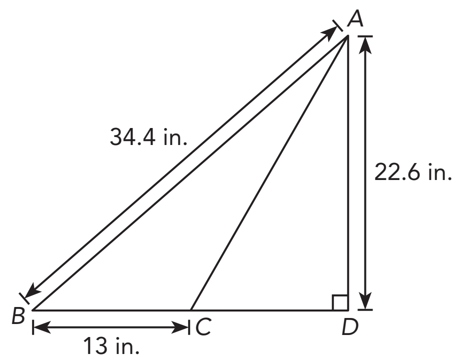
20. The diagonal of a square piece of cardboard is 28 inches.

a) Find the perimeter of the square.

b) Find the area of the square.

21. In the diagram, m$\angle ADB$ is 90°, AD is 22.6 inches, BC is 13 inches, and AB is 34.4 inches.

a) Find the length of $\overline{AC}$.



b) Find the area of triangle ACD.

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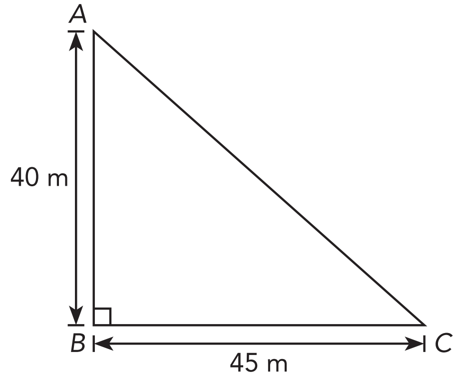
22. Points A, B, and C are corners of a triangular field where $m\angle ABC$ is 90°, AB is 40 meters and BC is 45 meters.

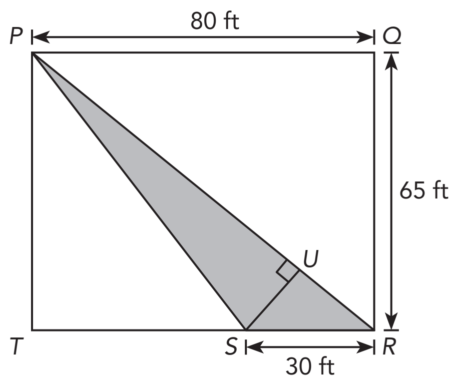
a) Find the length of $\overline{AC}$.



b) John walks along the edge of the field from point A to point C. If P is the point on $\overline{AC}$ when John is nearest to point B, find the length of $\overline{BP}$.

23. In rectangle $PQRT$, PQ is 80 feet, QR is 65 feet, RS is 30 feet, and $m\angle SUP$ is 90°.

a) Find the perimeter of the shaded triangle.



b) Find the area of the shaded triangle.

c) Find the length of $\overline{SU}$.

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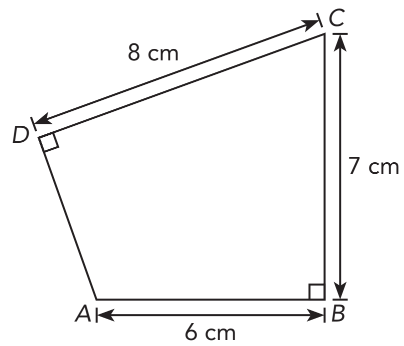
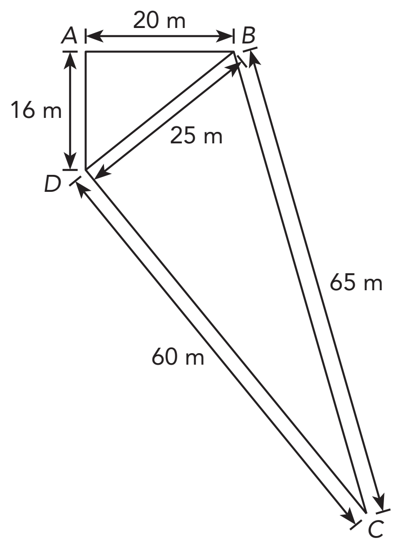
24. A map with a scale of 1 : 50,000 shows the locations of four towns A, B, C, and D. The distance between town A and town B is 6 centimeters, the distance between town B and town C is 7 centimeters, and the distance between town C and town D is 8 centimeters. Given that m∠*ABC* = m∠*ADC* = 90°, find the actual distance between town A and town D.



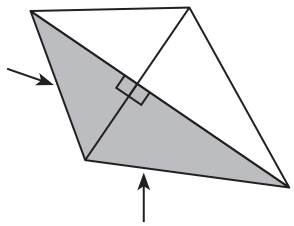
25. In the diagram, *AB* is 20 meters, *BC* is 65 meters, *CD* is 60 meters, *AD* is 16 meters, and *BD* is 25 meters. Determine if triangle *ABD* and triangle *BDC* are right triangles. Explain.

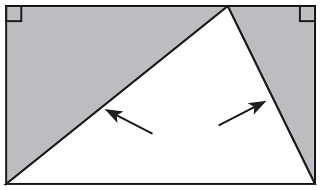


Answers

Chapter 7

Lesson 7.1

1. 

2. 

3. $x^2 = 24^2 + 7^2$
$x^2 = 576 + 49$
$x^2 = 625$
$x = \sqrt{625}$
$x = 25$

4. $x^2 = 10^2 + 7.5^2$
$x^2 = 100 + 56.25$
$x^2 = 156.25$
$x = \sqrt{156.25}$
$x = 12.5$

5. $26^2 = 10^2 + x^2$
$676 = 100 + x^2$
$676 - 100 = 100 + x^2 - 100$
$576 = x^2$
$x = \sqrt{576}$
$x = 24$

6. $41^2 = 40^2 + x^2$
$1{,}681 = 1{,}600 + x^2$
$1{,}681 - 1{,}600 = 1{,}600 + x^2 - 1{,}600$
$81 = x^2$
$x = \sqrt{81}$
$x = 9$

7. $11^2 = 8.8^2 + x^2$
$121 = 77.44 + x^2$
$121 - 77.44 = 77.44 + x^2 - 77.44$
$43.56 = x^2$
$x = \sqrt{43.56}$
$x = 6.6$
The value of x is 6.6.
$y^2 = x^2 + 10^2$
$y^2 = 43.56 + 100$
$y^2 = 143.56$
$y = \sqrt{143.56}$
$y \approx 12.0$
The value of y is approximately 12.0.

8. $11^2 = AD^2 + 8^2$
$121 = AD^2 + 64$
$121 - 64 = AD^2 + 64 - 64$
$57 = AD^2$
$AD = \sqrt{57}$
$AD \approx 7.55$
$12^2 = DC^2 + 8^2$
$144 = DC^2 + 64$
$144 - 64 = DC^2 + 64 - 64$
$80 = DC^2$
$DC = \sqrt{80}$
$DC \approx 8.94$
$x = 7.55 + 8.94$
$x = 16.49$
$x \approx 16.5$
The value of x is approximately 16.5.

9. $20^2 = 16^2 + x^2$
$400 = 256 + x^2$
$400 - 256 = 256 + x^2 - 256$
$144 = x^2$
$x = \sqrt{144}$
$x = 12$
The value of x is 12.
$y^2 = (16 + 4)^2 + 12^2$
$y^2 = 20^2 + 12^2$
$y^2 = 400 + 144$
$y^2 = 544$
$y = \sqrt{544}$
$y \approx 23.3$
The value of y is approximately 23.3.

10. $x^2 = 7^2 + 11^2$
$x^2 = 49 + 121$
$x^2 = 170$
$x = \sqrt{170}$
$x \approx 13.04$
The value of x is approximately 13.0.
$y^2 \approx 11^2 + (13.04 + 7)^2$
$y^2 = 11^2 + 20.04^2$
$y^2 \approx 121 + 401.60$
$y^2 = 522.60$
$y = \sqrt{522.60}$
$y \approx 22.9$
The value of y is approximately 22.9.

11. Field A:

$$36^2 + 48^2 \stackrel{?}{=} 60^2$$
$$1{,}296 + 2{,}304 \stackrel{?}{=} 3{,}600$$
$$3{,}600 = 3{,}600$$

So, field A has a right angle.

Field B:

$$40^2 + 50^2 \stackrel{?}{=} 60^2$$
$$1{,}600 + 2{,}500 \stackrel{?}{=} 3{,}600$$
$$4{,}100 \neq 3{,}600$$

So, field B does not have a right angle.

12. Let the height of the wall be x feet.

$$10^2 = x^2 + 5^2$$
$$100 = x^2 + 25$$
$$100 - 25 = x^2 + 25 - 25$$
$$75 = x^2$$
$$x = \sqrt{75}$$
$$x \approx 8.7$$

The height of the wall is approximately 8.7 feet.

13. Let the length of the cable be x feet.

$$x^2 = 6^2 + 12^2$$
$$x^2 = 36 + 144$$
$$x^2 = 180$$
$$x = \sqrt{180}$$
$$x \approx 13.4$$

The length of the cable is approximately 13.4 feet.

14. Let the distance from the base of the escalator to the point on the first floor directly below the top of the escalator be x feet.

$$30^2 = 12^2 + x^2$$
$$900 = 144 + x^2$$
$$900 - 144 = 144 + x^2 - 144$$
$$756 = x^2$$
$$x = \sqrt{756}$$
$$x \approx 27.5$$

The distance from the base of the escalator to the point on the first floor directly below the top of the escalator is approximately 27.5 feet.

15. Let the vertical height of the balloon above the ground be x meters.

$$100^2 = 20^2 + x^2$$
$$10{,}000 = 400 + x^2$$
$$10{,}000 - 400 = 400 + x^2 - 400$$
$$9{,}600 = x^2$$
$$x = \sqrt{9{,}600}$$
$$x \approx 98.0$$

The vertical height of the balloon is approximately 98 meters above the ground.

16. Difference in height between stations A and B

$= 50 - 20$

$= 30$ m

$\frac{1}{2}$ km $= 500$ m

Let the length of the cable be x meters.

$$x^2 = 500^2 + 30^2$$
$$x^2 = 250{,}000 + 900$$
$$x^2 = 250{,}900$$
$$x = \sqrt{250{,}900}$$
$$x \approx 500.9$$

The length of the cable is approximately 500.9 meters.

17. Let the longest line be x feet.

$$x^2 = 6^2 + 3^2$$
$$x^2 = 36 + 9$$
$$x^2 = 45$$
$$x = \sqrt{45}$$
$$x \approx 6.7$$

The longest line that can be drawn across the whiteboard is approximately 6.7 feet.

18. Let the length of the path be p meters.

$$p^2 = 30^2 + 20^2$$
$$p^2 = 900 + 400$$
$$p^2 = 1{,}300$$
$$p = \sqrt{1{,}300}$$
$$p \approx 36.1$$

Jill walked approximately 36.1 meters.

Difference between the two routes

$\approx 50 - 36.1$

$= 13.9$ m

The difference in distance between the two routes is about 13.9 meters.

19. a) In $\triangle ABC$,

$$AB^2 = BE^2 + AE^2$$
$$AB^2 = 12^2 + 15^2$$
$$AB^2 = 144 + 225$$
$$AB^2 = 369$$
$$AB = \sqrt{369}$$

$AB \approx 19.2$ feet

$CD = AB$

$CD \approx 19.2$ ft

The length of the string is approximately 19.2 feet.

b) In $\triangle CED$,

$$\begin{aligned} CD^2 &= CE^2 + DE^2 \\ AB^2 &= CE^2 + 13^2 \\ 369 &= CE^2 + 169 \\ 369 - 169 &= CE^2 + 169 - 169 \\ 200 &= CE^2 \\ CE &= \sqrt{200} \\ CE &\approx 14.1 \text{ ft} \\ AC &= AE - CE \\ AC &\approx 15 - 14.1 \\ AC &= 0.9 \text{ ft} \end{aligned}$$

The difference between the points A and C is about 0.9 feet.

20. a) Let the side of the square be x inches.

$$\begin{aligned} 28^2 &= x^2 + x^2 \\ 784 &= 2x^2 \\ \frac{784}{2} &= \frac{2x^2}{2} \\ 392 &= x^2 \\ x &= \sqrt{392} \\ x &\approx 19.8 \end{aligned}$$

$$\begin{aligned} \text{Perimeter of square} &\approx 4 \cdot 19.8 \\ &= 79.2 \text{ in.} \end{aligned}$$

The perimeter of the square is about 79.2 inches.

b)
$$\begin{aligned} \text{Area of the square} &= x \cdot x \\ &= x^2 \\ &= 392 \text{ in}^2 \end{aligned}$$

The area of the square is 392 square inches.

21. a) In $\triangle ADB$,

$$\begin{aligned} 34.4^2 &= BD^2 + 22.6^2 \\ 1{,}183.36 &= BD^2 + 510.76 \\ 1{,}183.36 - 510.76 &= BD^2 + 510.76 - 510.76 \\ 672.6 &= BD^2 \\ BD &= \sqrt{672.6} \\ BD &\approx 25.93 \end{aligned}$$

$$\begin{aligned} CD &\approx 25.93 - 13 \\ &= 12.93 \text{ in.} \end{aligned}$$

In $\triangle ADC$,

$$\begin{aligned} AC^2 &= 12.93^2 + 22.6^2 \\ AC^2 &\approx 677.94 \\ AC &= \sqrt{677.94} \\ AC &\approx 26.0 \text{ in.} \end{aligned}$$

The length of $\overline{AC}$ is approximately 26 inches.

b)
$$\begin{aligned} \text{Area of } \triangle ACD &\approx \frac{1}{2} \cdot 12.93 \cdot 22.6 \\ &\approx 146.1 \text{ in}^2 \end{aligned}$$

The area of triangle ACD is approximately 146.1 square inches.

22. a)
$$\begin{aligned} AC^2 &= 40^2 + 45^2 \\ AC^2 &= 3{,}625 \\ AC &= \sqrt{3{,}625} \\ AC &\approx 60.2 \text{ m} \end{aligned}$$

The length of $\overline{AC}$ is approximately 60.2 meters.

b) The length of $\overline{BP}$ is the perpendicular distance between B and $\overline{AC}$.

Area of $\triangle ABC$:

$$\begin{aligned} \frac{1}{2} \cdot AC \cdot BP &= \frac{1}{2} \cdot BC \cdot AB \\ \frac{1}{2} \cdot 60.2 \cdot BP &\approx \frac{1}{2} \cdot 45 \cdot 40 \\ 30.1 \cdot BP &= 900 \\ \frac{30.1 \cdot BP}{30.1} &= \frac{900}{30.1} \\ BP &\approx 29.9 \text{ m} \end{aligned}$$

The length of $\overline{BP}$ is approximately 29.9 meters.

23. a) In $\triangle PQR$,

$$\begin{aligned} PR^2 &= 80^2 + 65^2 \\ PR^2 &= 6{,}400 + 4{,}225 \\ PR^2 &= 10{,}625 \\ PR &= \sqrt{10{,}625} \\ PR &\approx 103.1 \text{ ft} \end{aligned}$$

In $\triangle PTS$,

$$\begin{aligned} TS &= 80 - 30 \\ &= 50 \text{ ft} \\ PT &= QR \\ &= 65 \text{ ft} \\ PS^2 &= 50^2 + 65^2 \\ PS^2 &= 2{,}500 + 4{,}225 \\ PS^2 &= 6{,}725 \\ PS &= \sqrt{6{,}725} \\ &\approx 82.0 \text{ ft} \end{aligned}$$

Perimeter of shaded triangle
$\approx 103.1 + 82 + 30$
$= 215.1$ ft

The perimeter of the shaded triangle is approximately 215.1 feet.

b) Area of shaded triangle

$$\begin{aligned} &= \frac{1}{2} \cdot SR \cdot PT \\ &= \frac{1}{2} \cdot 30 \cdot 65 \\ &= 975 \text{ ft}^2 \end{aligned}$$

The area of the shaded triangle is 975 square feet.

c) Area of shaded triangle:

$$\frac{1}{2} \cdot PR \cdot SU = 975$$
$$\frac{1}{2} \cdot 103.1 \cdot SU \approx 975$$
$$\frac{51.55 \cdot SU}{51.55} = \frac{975}{51.55}$$
$$SU \approx 18.9 \text{ ft}$$

The length of $\overline{SU}$ is approximately 18.9 feet.

24. a) Let the distance between town *A* and town *C* on the map be *x* centimeters.

$x^2 = 6^2 + 7^2$
$x^2 = 36 + 49$
$x^2 = 85$

Let the distance between town *A* and town *D* on the map be *y* centimeters.

$$x^2 = 8^2 + y^2$$
$$85 = 64 + y^2$$
$$85 - 64 = 64 + y^2 - 64$$
$$21 = y^2$$
$$y = \sqrt{21}$$
$$y \approx 4.58 \text{ cm}$$

Let the actual distance between town *A* and town *D* be *d* kilometers.

Map scale:
1 : 50,000
50,000 cm = 0.5 km

$$\frac{1}{0.5} = \frac{4.58}{d}$$
$$0.5 \cdot \frac{1}{0.5} = \frac{4.58}{d} \cdot 0.5$$
$$1 = \frac{2.29}{d}$$
$$d \cdot 1 = \frac{2.29}{d} \cdot d$$
$$d \approx 2.3 \text{ km}$$

The actual distance between town *A* and town *D* is approximately 2.3 kilometers.

25. In $\triangle ABD$,

$$AB^2 + AD^2 \stackrel{?}{=} BD^2$$
$$20^2 + 16^2 \stackrel{?}{=} 25^2$$
$$656 \neq 625$$

So, triangle *ABD* is not a right triangle.

In $\triangle BCD$,

$$BD^2 + CD^2 \stackrel{?}{=} BC^2$$
$$25^2 + 60^2 \stackrel{?}{=} 65^2$$
$$4{,}225 = 4{,}225$$

So, triangle *BCD* is a right triangle.

Lesson 7.2

1. Plot a point $X(-4, -3)$ to form the third vertex of a right triangle.

$PX = |3 - (-3)| = 6$ units
$QX = |4 - (-4)| = 8$ units

$$PX^2 + QX^2 = PQ^2$$
$$6^2 + 8^2 = PQ^2$$
$$36 + 64 = PQ^2$$
$$100 = PQ^2$$
$$PQ = \sqrt{100}$$
$$PQ = 10$$

The exact distance between points *P* and *Q* is 10 units.

2. a) Distance from *A* to *B*

$$= \sqrt{(0-3)^2 + [-4-(-2)]^2}$$
$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$
$$\approx 3.6 \text{ units}$$

b) Distance from *C* to *D*

$$= \sqrt{(4-2)^2 + [2-(-6)]^2}$$
$$= \sqrt{2^2 + 8^2}$$
$$= \sqrt{4+64}$$
$$= \sqrt{68}$$
$$\approx 8.2 \text{ units}$$

c) Distance from *E* to *F*

$$= \sqrt{[3-(-7)]^2 + (-3-8)^2}$$
$$= \sqrt{10^2 + 11^2}$$
$$= \sqrt{100+121}$$
$$= \sqrt{221}$$
$$\approx 14.9 \text{ units}$$

d) Distance from *G* to *H*

$$= \sqrt{[-1-(-2)]^2 + [-4-(-5)]^2}$$
$$= \sqrt{1^2 + 1^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2}$$
$$\approx 1.4 \text{ units}$$

Points *G* and *H* are closest to each other.