BrightRED Study Guide

CfE HIGHER

MATHEMATICS

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

1 ALGEBRA

Quadratics: Completing the square

1. A
2. B
3. B
4. Method 1

   •^{1} $2(x^2 + 6x$ ... stated or implied by

   •^{2} $2[(x + 3)^2 -9]$
   $= 2(x + 3)^2 -18 +1$

   •^{3} $2(x + 3)^2 - 17$

   Method 2

   •^{1} $ax^2 + 2abx + ab^2 + c$

   •^{2} $a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$

   •^{3} $2(x + 3)^2 - 17$

5\.

Method 1	Method 1
•^{1} identify common factor	•^{1} $-1(x^2 - 2x$ stated or implied by •^{2}
•^{2} complete the square	•^{2} $-1(x - 1)^2 + 16$
Method 2	Method 2
•^{1} expand completed square form and equate coefficients	•^{1} $px^2 + 2pqx + pq^2 + r$ and $p = -1$,
•^{2} process for q and r and write in required form	•^{2} $q = -1$ and $r = 16$

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Quadratics: Solving quadratic inequations

1. (a) $2x^2 + 5x - 12 < 0 \Rightarrow (2x - 3)(x + 4) < 0$

   $-4 < x < \frac{3}{2}$

   (b) $x \leq -4,\ x \geq \frac{3}{2}$

2. $9x^2 - 12x + 4 = (3x - 2)^2$

   $(3x - 2) > 0$ for all x except $x = \frac{2}{3}$. $\left\{x\colon x \in \mathbb{R},\ x \neq \frac{2}{3}\right\}$

3. C $(3 + x)(4 - x) < 0$

   solution is either

   $-3 < x < 4$ or $x < -3,\ x > 4$

   $x = 0$ is FALSE so

   $x < -3$ and $x > 4$

4. B

5. B

6. B

7. $\bullet^1\ x(x - 2) < 15$

   $\bullet^2\ x^2 - 2x - 15 < 0$

   $\bullet^3\ (x - 5)(x + 3)$

   $\bullet^4\ 2 < x < 5$

8. C

Quadratics: The discriminant

1. B

2. D

3. C
   $(3 + x)(4 - x) < 0$

4. B

Polynomials 1

1. $x^3 + 3x^2 + 5x + 3 = (x + 1)(x^2 + 2x + 3)$

For $(x^2 + 2x + 3)$, $b^2 - 4ac = 2^2 - 4 \times 1 \times 3 < 0$, so does factorise, is irreducible.
Therefore the above expression is fully factorised.

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Polynomials 2

1. $\bullet^1$ If $(x - 2)$ is a factor of $f(x) = x^3 - 6x^2 + 4x + 8 = 0$ then $f(2) = 0$.

   $\bullet^2$ $f(2) = 2^3 - 6(2)^2 + 4(2) + 8 = 8 - 24 + 8 + 8 = 0 \Rightarrow x = 2$ is a root and $(x - 2)$ is a factor

2. $x^3 + 3x^2 + 2x + 3 = 2x + 3$ gives $x^3 + 3x^2 = 0 \Rightarrow x^2(x + 3) = 0$

   $x = 0$ (at A), $x = -3$ at B, when $x = -3$, $y = 2x + 3 = 2(-3) + 3 = -3$, B has coordinates $(-3, -3)$

3. (a) $\bullet^1$ If $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5 = 0$ then $f(1) = 0$.

   $\bullet^2$ $2 + 1 - 8 + 5 = 0$

   $\bullet^3$ $(x - 1)$ is a factor

   $\bullet^4$ $(x - 1)(2x^2 + 3x - 5)$

   $\bullet^5$ $(x - 1)(x - 1)(2x + 5)$

   (b) $x = 1$, (repeated root), $x = \frac{-5}{2}$

   (c) $2x^3 + x^2 - 6x + 2 = 2x - 3$ gives $2x^3 + x^2 - 8x + 5 = 0$ and since the line is a tangent at G there will be a repeated root so $x = 1$ and $y = 2(1) - 3 = -1$. G is $(1, -1)$.

Functions: Domain and range

1. A

2. (a) $-8 \leqslant f(x) \leqslant 6$; (remember $-1 \leqslant \cos x \leqslant 1$)

   (b) $g(x) \leqslant 5$; (c) $h(x) \leqslant 10$ in both (b) and (c) a value is being taken from 5 or 10, the smallest this value can be in both cases is zero.

   (d) $2 \leqslant k(x) \leqslant 5$, [here $0 \leqslant \sin^2 x \leqslant 1$]

Functions: Inverse functions

1. $f^{-1}(x) = \frac{x + 7}{4}$, $g^{-1}(x) = \frac{1}{2x}$

2. $y = \frac{2x - 3}{4 - x} \rightarrow y(4 - x) = 2x - 3 \rightarrow 4y + 3 = 2x + yx \rightarrow 4y + 3 = (2 + y)x \rightarrow x = \frac{4y + 3}{y + 2}$

   $\Rightarrow h^{-1}(x) = \frac{4x + 3}{x + 2}$, $x \neq 2$.

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

Functions: Composite functions

1. $g(f(x)) = g(x^2 + 1) = 3(x^2 + 1) - 4 = 3x^2 - 1$

2. (a) $f(g(x)) = f(x + 4) = (x + 4)^2 + 3 = x^2 + 8x + 19$

   (b) $g(f(x)) = g(x^2 + 3) = x^2 + 3 + 4 = x^2 + 7$

3. $f(g(x)) = f(x + 3) = (x + 3)((x + 3) - 1) + q = (x + 3)(x + 2) + q = x^2 + 5x + 6 + q$

4. $g\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}, f\left(g\left(\frac{\pi}{6}\right)\right) = f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

5 (a) $g(f(x)) = g(x^3 - 1) = 3(x^3 - 1) + 1 = 3x^3 - 2$

   (b) $g(f(x)) + xh(x) = 3x^3 - 2 + x(4x - 5) = 3x^3 + 4x^2 - 5x - 2$

6. (a) $f(g(x)) = f\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x^2}\right) + 3 = \frac{4}{x^2} + 3$

   (b) $y = \frac{4}{x^2} + 3 \Rightarrow y - 3 = \frac{4}{x^2} \Rightarrow x^2 = \frac{4}{y-3} \Rightarrow x = \sqrt{\frac{4}{y-3}}, f^{-1}(x) = \sqrt{\frac{4}{x-3}}, x \neq 3$

Functions: Graphs of functions 1

Working anti-clockwise: $y = \frac{1}{-x} = -\frac{1}{x}, y = \frac{1}{x}, y = -\frac{1}{x}$

Functions: Graphs of functions 2

1. $y = \log_3(x - 3)$

Functions: Exponentials and logarithmic functions

1. $\bullet^1$ $\log_5[(3 - 2x)(2 + x)] = 1$ stated or implied by $\bullet^2$

   $\bullet^2$ $(3 - 2x)(2 + x) = 5^1$
   $6 - x - 2x^2 = 5$

   $\bullet^3$ $2x^2 + x - 1 = 0$
   $(2x - 1)(x + 1) = 0$

   $\bullet^4$ $x = \frac{1}{2}, x = -1$

Functions: Exponentials and logarithmic graph transformations

The graph will be translated by 4 units left

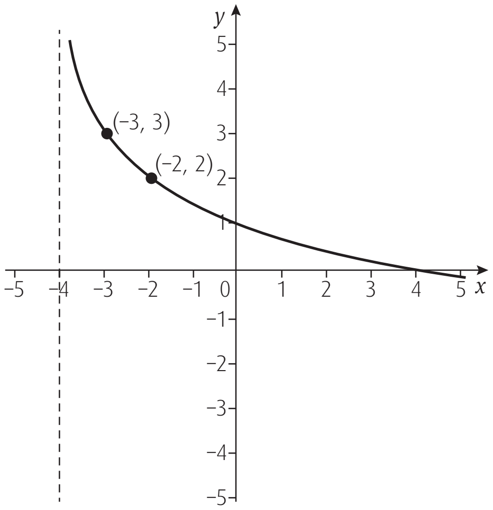
The graph will get closer and closer to the line given by $x = -4$ (the new asymptote instead of $x = 0$, the y-axis)

The graph will be reflected in the x-axis

The graph will be translated by 3 units upwards

The graph will need to be annotated:

choose at least 2 points from $(-3, 3)$,$(-2, 2)$, $(0, 1)$, $(4, 0)$.



Recurrence relations 1

1. A $u_2 = 3 \times 2 + 4 = 10$

   $\therefore u_3 = 3 \times 10 + 4 = 34$

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

Recurrence relations 2

1. (a) $L = kL + 5$ so $4 = 4k + 5 \Rightarrow k = -\frac{1}{4}$

2. (a) let u_{n+1} be the height at end of year., then

   $u_{n+1} = 0{\cdot}8u_n + 0{\cdot}5$

   A limit exists since $-1 < 0{\cdot}8 < 1$

   Let limit be L, then

   $L = 0{\cdot}8L + 0{\cdot}5$

   $\Rightarrow 0{\cdot}2L = 0{\cdot}5 \Rightarrow L = \frac{0{\cdot}5}{0{\cdot}2} = \frac{5}{2} = 2{\cdot}5$ metres

   (b) The limit, L needs to be $\leqslant 2$ metres

   So $L = pL + 0{\cdot}5$

   $\Rightarrow L - pL = 0{\cdot}5$

   $L(1 - p) = 0{\cdot}5$

   $L = \frac{0{\cdot}5}{1-p} \leqslant 2$

   $\Rightarrow 0{\cdot}5 \leqslant 2(1 - p)$

   $\Rightarrow 0{\cdot}25 \leqslant 1 - p$

   $\Rightarrow p \leqslant 0{\cdot}75$

   $0{\cdot}75$ represents the amount left, so a minimum of 25% must be trimmed.

2 CALCULUS

Differentiating functions

1. $\frac{dy}{dx} = -6x^{-4} + 4x^{\frac{1}{3}}$
2. $\frac{dV}{dr} = 4\pi r^2$. When $r = 2$, $\frac{dV}{dr} = 4\pi(2)^2 = 16\pi$
3. $f(x) = x^{-\frac{1}{5}}, f'(x) = -\frac{1}{5}x^{-\frac{6}{5}}$
4. $s'(t) = 2t - 5, s'(3) = 2(3) - 5 = 1$
5. $\frac{d}{dx}\left(\frac{1}{4}x^{-3}\right) = -\frac{3}{4}x^{-4} = \frac{-3}{4x^4}$
6. $\frac{dA}{dr} = 4\pi r + 6\pi$. When $r = 2$, $\frac{dA}{dr} = 4\pi(2) + 6\pi = 14\pi$

Differentiating using the chain rule

1. $\frac{d}{dx}\left((4 - 9x^4)^{\frac{1}{2}}\right) = \frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}} \times -36x^3 = -18x^3(4 - 9x^4)^{-\frac{1}{2}}$
2. $f'(x) = -\frac{1}{2}(4 - 3x^2)^{-\frac{3}{2}} \times -6x = 3x(4 - 3x^2)^{-\frac{3}{2}}$

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

Differentiating: Nature and properties of functions 1

1. $\frac{dy}{dx} = 4x^3 - 15x^2$, $m = 4(-1)^3 - 15(-1)^2 = -19$, $y = (-1)^4 - 5(-1)^3 + 6 = 12$

   $y - 12 = -19(x + 1)$, $y = -19x - 7$
2. $\frac{dy}{dx} = 2x - 5$, $m = 2(3) - 5 = 1$
3. $\frac{dy}{dx} = 3x^2 - 4$, $m = 3(2)^2 - 4 = 8$
4. $\frac{dy}{dx} = 20x^4 - 17$, $m = 20(1)^2 - 17 = 3$

Differentiating: Nature and properties of functions 2

1. Only statement (1) is correct.

   $f(x) < 0$ for $s < x < t$, since $f'(x) = 0$ when $x = 0$.
2. C Only statement (2) is correct.

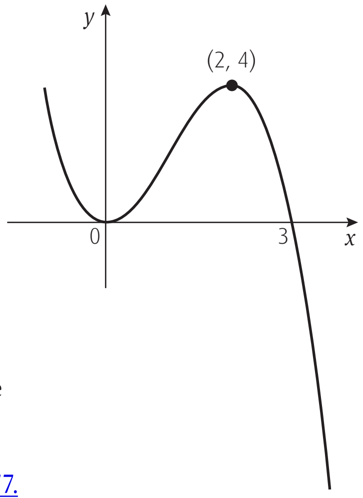
   $f'(1) = (1)^2 - 9 = -8$, $f'(-3) = (-3)^2 - 9 = 0$
3. $f'(x) = 3x^2 + 6x + 18 = 3(x^2 + 2x + 9)$

   $b^2 - 4ac = (2)^2 - 4(1)(9) < 0$ so no real solutions and a '$\cup$' shaped graph which is always positive so function is strictly increasing.
4. (a) $f'(x) = x^2 + x - 12 = (x + 4)(x - 3)$

   [$f'(x) = 0$ when $x = -4$ and when $x = 3$], '$\cup$' shaped graph so f is strictly decreasing ($f'(x) < 0$) when $-4 < x < 3$.

   (b) $-4 \leqslant x \leqslant 3$.

Differentiating: Nature and properties of functions 3



1. (a) $\frac{dy}{dx} = 6x - 3x^2$, for stationary points $\frac{dy}{dx} = 0$, $6x - 3x^2 = 0 \rightarrow 3x(2 - x) = 0$, $x = 0$ or $x = 2$

   stationary points (0, 0), (2, 4)

   $\frac{d^2y}{dx^2} = 6 - 6x$, when $x = 0$, $\frac{d^2y}{dx^2} = 6 > 0 \Rightarrow$ a minimum turning point at (0, 0)

   when $x = 2$, $\frac{d^2y}{dx^2} = -6 < 0 \Rightarrow$ a maximum turning point at (2, 4).

   (b) $3x^2 - x^3 = 0 \rightarrow x^2(3 - x) = 0 \rightarrow x = 0$, $x^2(3 - x) = 0 \rightarrow x = 3$. (0, 0) and (3, 0)
2. The following gives the marking instructions for this question rather than just the solution, remember using the marking instructions is a useful tool for learning where the marks are allocated and what you need to include in your solutions. Marking instructions can be found on the SQA website and also on http://maths777.weebly.com

   (a) *In marking instructions you will sometimes see letters which identify the type of mark being given such as 'ss' for selecting a strategy. Interpretation, selecting a strategy, processing and communication are all part of your solutions so it is important that you clearly show your working.*

   $\bullet^1$ ic interpret x intercept $\bullet^1$ (2, 0) (minimum response "(i) 2")

   $\bullet^2$ ic interpret y intercept $\bullet^2$ (0, −2) (minimum response "(ii) −2")

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

Notes

1. Candidates who obtain extra x-axis intercepts lose $\bullet^1$.
2. Candidates who obtain extra y-axis intercepts lose $\bullet^2$.
3. Candidates who interchange intercepts can gain at most one mark.

$\bullet^3$ ic	write in differentiable form	$\bullet^3$	$x^3 - 2x^2 + x - 2$	
$\bullet^4$ ss	know to and start to differentiate	$\bullet^4$	$3x^2\dots$ or $-4x\dots$	
$\bullet^5$ pd	complete derivative and equate to 0	$\bullet^5$	$3x^2 - 4x + 1$ **and** $f'(x) = 0$ **or** $3x^2 - 4x + 1 = 0$	
$\bullet^6$ pd	factorise derivative	$\bullet^6$	$(3x - 1)(x - 1)$	
$\bullet^7$ pd	process for x	$\bullet^7$	$\frac{1}{3}$ and 1	$x = \frac{1}{3}$ and $y = -\frac{50}{27}$
$\bullet^8$ pd	evaluate y-coordinates	$\bullet^8$	$-\frac{50}{27}$ and -2	$x = 1$ and $y = -2$
$\bullet^9$ ic	justify nature of stationary points	$\bullet^9$	$\begin{array}{c\|ccccc} x & \dots & \frac{1}{3} & \dots & 1 & \dots \\ \hline f'(x) & + & 0 & - & 0 & + \\ & & \text{max} & & \text{min} & \end{array}$	Accept a valid expression in lieu of $f'(x)$.
$\bullet^{10}$ ic	interpret and state conclusions	$\bullet^{10}$		

DON'T FORGET

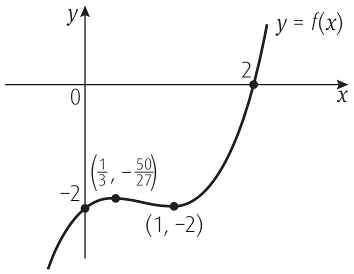
This question may be marked vertically. The dotted rectangle shows what is required for $\bullet^{10}$.

Notes

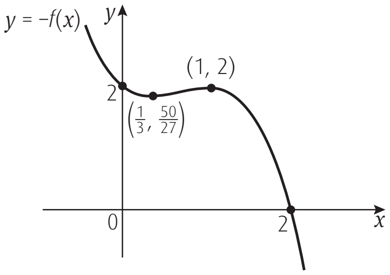
4. $\bullet^5$ is only available if "= 0" appears at or before $\bullet^6$ stage.
5. $\bullet^3$, $\bullet^4$ and $\bullet^5$ are the only marks available to candidates who solve $3x^2 - 4x = -1$.
6. At $\bullet$* the nature can be determined using the second derivative.
7. $\bullet^9$ is only available if the nature table is consistent with the candidate's derivative.
8. $\bullet^{10}$ is awarded for correct interpretation of the candidate's nature table in words.

Notice that other methods for a solution are sometimes given in the notes such as here for using the second derivative for determining the nature of the stationary points – note 6.

(c) (i)



(ii)



$\bullet^{11}$ ic	curve showing points from (a) and (b) without annotation	$\bullet^{11}$	sketch
$\bullet^{12}$ ic	**cubic** curve showing **all** intercepts and stationary points annotated	$\bullet^{12}$	sketch
$\bullet^{13}$ ic	curve from (i) reflected in x-axis	$\bullet^{13}$	reflected sketch

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

3. $\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 0$ for stationary points, $3(x - 3)(x + 1) \Rightarrow x = 3, -1$

   stationary points $(-1, 17)$, $(3, -15)$

   $\frac{d^2y}{dx^2} = 6x - 6$ when $x = -1$, $\frac{d^2y}{dx^2} = -12 < 0 \Rightarrow$ a maximum turning point at $(-1, 17)$,

   when $x = 3$, $\frac{d^2y}{dx^2} = 12 > 0 \Rightarrow$ a minimum turning point at $(3, -15)$.

Integrating functions

1. $\frac{1}{15}(5x + 7)^3 + c$
2. $2x^2 - x^3 + c$
3. $-\frac{1}{4}(1 - 8x)^{\frac{1}{2}} + c$
4. $\int\left(\frac{1}{6}x^{-2}\right)dx = -\frac{1}{6}x^{-1} + c = -\frac{1}{6x} + c$
5. $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + c$

Definite integrals and solving differential equations

1. $\bullet^1$ $\frac{1}{\frac{1}{2}}(\ldots)^{\frac{1}{2}}$

   $\bullet^2$ $\ldots \times \frac{1}{3}$

   $\bullet^3$ $\frac{2}{3}(3t + 4)^{\frac{1}{2}} - \frac{2}{3}(3(4) + 4)^{\frac{1}{2}} = 2 \rightarrow \frac{2}{3}(3t + 4)^{\frac{1}{2}} = 2 + \frac{8}{3}$

   $\bullet^4$ $(3t + 4)^{\frac{1}{2}} = 7 \rightarrow 3t + 4 = 49$

   $\bullet^5$ $t = 15$

Differentiating and integrating trigonometric functions

1. $2x\cos(x^2 - 3)$
2. $-12\cos^3 x \sin x$
3. $f'(x) = 12\cos 3x$, $f'(0) = 12\cos 3(0) = 12$
4. $-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$

Differentiation: Mathematical modelling and problem solving

1. (a) $L = 3x + 4y$

   $A = xy$, $xy = 24$, $y = \frac{24}{2x}$

   $L = 3x + 4 \times \frac{24}{2x}$ this line is essential

   $= 3x + \frac{48}{x}$

   (b) $L = 3x + 48x^{-1}$

   $\frac{dL}{dx} = 3 - 48x^{-2}$

   $3 - \frac{48}{x^2} = 0 \rightarrow x = 4$

   $\frac{d^2L}{dx^2} = 96x^{-3} = \frac{96}{x^3}$

   when $x = 4$, $\frac{d^2L}{dx^2} > 0 \Rightarrow$ a minimum

   $L = 3(4) + \frac{48}{(4)} = 24$

   Cost 24 × £8·25 = £198

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Integration: Mathematical modelling and problem solving

1.	$\bullet^1$	ss	know to integrate	$\bullet^1$	$\int$... **or** attempt integration	
	$\bullet^2$	ic	know to deal with areas on each side of y-axis	$\bullet^2$	Evidence of attempting to interpret the diagram to left of y-axis separately from diagram to the right.	
	$\bullet^3$	ic	interpret limits of one area	$\bullet^3$	$\int_{-2}^{0}$	e.g. $\int_{0}^{3}$ with no other
	$\bullet^4$	ic	use "upper–lower"	$\bullet^4$	$(x^3 - x^2 - 4x + 4) - (2x + 4)$	$(2x + 4) - (x^3 - x^2 - 4x + 4)$
	$\bullet^5$	pd	integrate	$\bullet^5$	$\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$	$3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
	$\bullet^6$	ic	substitute in limits	$\bullet^6$	$\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 - 3(-2)^2 = -\frac{16}{3}$	$3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4$
					Evidence for $\bullet^6$ may be implied by $\bullet^7$, but $\bullet^7$ must be consistent with $\bullet^5$.	
	$\bullet^7$	pd	evaluate the area on one side	$\bullet^7$	Hence area is $\frac{16}{3}$	$\frac{63}{4}$
	$\bullet^8$	ss	interpret integrand with limits of the other area	$\bullet^8$	$\int_{0}^{3} (2x + 4) - (x^3 - x^2 - 4x + 4)dx$	$\int_{-2}^{0} (x^3 - x^2 - 4x + 4) - (2x + 4)\, dx$
	$\bullet^9$	pd	evaluate the area on the other side	$\bullet^9$	$\frac{63}{4}$	$\frac{16}{3}$
	$\bullet^{10}$	ic	state total area	$\bullet^{10}$	$21\frac{1}{12}$ or $\frac{253}{12}$ or 21·1	$21\frac{1}{12}$ or $\frac{253}{12}$ or 21·1

2. Area = $\int_{-2}^{2}((14 - x^2) - (2x^2 + 2))dx = \int_{-2}^{2}(12 - 3x^2)\, dx$

$[12x - x^3]_{-2}^{2} = [16] - [-16] = 32$

Rates of change: Mathematical modelling and problem solving

1.	(a)	$\bullet^1$ ss	know to differentiate	$\bullet^1$	$a = v'(t)$
		$\bullet^2$ pd	differentiates trig. function	$\bullet^2$	$-8\sin\left(2t - \frac{\pi}{2}\right)$...
		$\bullet^3$ pd	applies chain rule	$\bullet^3$	... × 2 and complete $a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$
	(b)	$\bullet^4$ ss	know to and evaluate $a(10)$	$\bullet^4$	$a(10) = 6{\cdot}53$
		$\bullet^5$ ic	interpret result	$\bullet^5$	$a(10) > 0$ therefore increasing

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

3 GEOMETRY

Straight lines 1

1. Angle with positive direction of x-axis is $\frac{\pi}{3}$.
   $m_{GH} = \tan\frac{\pi}{3} = \sqrt{3}$.
2. $m = \tan 150° = -\frac{1}{\sqrt{3}}$.

Straight lines 2

1. $3y = 2x - 9 \rightarrow y = \frac{2}{3}x - 9 \Rightarrow m = \frac{2}{3} \Rightarrow$ perp. grad. $= -\frac{3}{2}$, $m_L = -\frac{3}{2}$.
2. $m_L = -\frac{5}{3}$, $y + 1 = -\frac{5}{3}(x + 2) \Rightarrow 3y + 5x + 13 = 0$ or $y = -\frac{5}{3}x - \frac{13}{3}$.
3. (a) For line BD, we know a point (B) and can use gradient of AB to find gradient of BD:

   $$m_{\text{AB}} = \frac{6-0}{-6+8} = \frac{6}{2} = 3$$

   $$\text{So } m_{\text{BD}} = -\frac{1}{3}$$

   Equation: $y - 6 = -\frac{1}{3}(x + 6)$

   $3y - 18 = -x - 6$

   $3y = 12 - x$ or $y = -\frac{1}{3}x + 4$.

   (b) The diagonals bisect each other, so the point of intersection will be the midpoint of BD (or AC, but BD is easier to find). D lies on $\mathbf{3y = 12 - x}$ and $y_{\text{D}} = 0$, so $x_{\text{D}} = 12$ (by substitution into the equation of the line).

   The midpoint of BD: B(−6, 6) D(12, 0) so midpoint P is **(3, 3)**

   (c) It's easy to find C in several ways, and you don't need any working written (but remember working is also for **your** benefit to avoid mistakes being made by carrying too much in your head).

   You could use the midpoint formula with P and AC.

   Or you could use the fact that BC is horizontal and equal in length to AD (20 units).

   Or use vectors $\overrightarrow{DC} = \overrightarrow{AB} = \binom{2}{6}$ … result **C(14, 6)**
4. (a) $m_{PQ} = -2$, $m_{QR} = \frac{1}{2}$, $y - 6 = \frac{1}{2}(x - 5) \rightarrow 2y = x + 7$ or $y = \frac{1}{2}x + \frac{7}{2}$

   (b) solve $x + 3y = 13$ and $\rightarrow x - 2y = -7$ simultaneously to get $x = 1$, $y = 4$, (1, 4).

   (c) use vectors or stepping out $\overrightarrow{QT} = \binom{-4}{-2}$, $R(-3, 2)$, $S(-1, -2)$
5. (a) midpoint (1, 3), $m_{PQ} = -3$, $m_{perp} = \frac{1}{3}$, eqn, perp. bisector: $y - 3 = \frac{1}{3}(x - 1) \rightarrow 3y = x + 8$ or $y = \frac{1}{3}x + \frac{8}{3}$

   (b) $m_{parallel} = -3$, $y - (-2) = -3(x - 1) \rightarrow y + 3x = 1$ or $y = -3x + 1$

   (c) solve simultaneously to get $x = -\frac{1}{2}$, $y = \frac{5}{2}$, $\left(-\frac{1}{2}, \frac{5}{2}\right)$.

Straight lines 3

1. midpoint$_{QR} = S(1, 5)$, $m_{PS} = \frac{5-(-2)}{1-(-3)} = \frac{7}{4}$.
2. (a) P = (−3, 0)

   (b) $m_{QR} = -2$, $m_{alt} = \frac{1}{2}$, eqn. alt: $y - 0 = \frac{1}{2}(x + 3) \Rightarrow 2y = x + 3$ or $y = \frac{1}{2}x + \frac{3}{2}$

   (c) eqn. QR: $y + 2 = -2(x - 8)$ or $y - 6 = -2(x - 4) \Rightarrow y + 2x = 14$

   solve simultaneously to get $x = 5$, $y = 4$.

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

Circles 1

1. C(2, −1), $r = \frac{3}{2}$

2. $r = \sqrt{\text{negative number}}$ which is not possible.

3. $C(2, 5)$, $r = 5$, distance$_{AP}$ = $\sqrt{5^2 + 4^2} = \sqrt{41} > 5$, hence A lies outside the circle.

   Alternatively, substitute A into the LHS of the circle equation

   $(-3)^2 + 1^2 - 4 \times (-3) - 10 \times 1 + 4 = 16, > 0$, so A lies outside the circle.

4. $\sqrt{p^2 + (2p)^2 - (2p + 2)} > 0$

   $\Rightarrow p^2 + (2p)^2 - (3p + 2) > 0$

   $\Rightarrow 5p^2 - 3p - 2 > 0$

   $\Rightarrow (5p + 2)(p - 1) > 0$

   $\Rightarrow p < -2/5, p > 1$

   (for more help on this final step see solving quadratic inequations)

Circles 2

1. (a) $2x - y + 5 = 0 \rightarrow y = 2x + 5$, substituting for y into the equation of the circle gives

   $x^2 + (2x + 5)^2 - 6x - 2(2x + 5) - 30 = 0$

   $x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$

   $5x^2 + 10x - 15 = 0 \Rightarrow 5(x + 3)(x - 1) = 0$

   giving $x = -3$, $x = 1$. Substituting into $y = 2x + 5$ gives P(−3, −1), Q(1, 7)

   (b) Circle has centre (3, 1), $r = \sqrt{40}$, midpoint $PQ(-1, 3)$.

   $\overrightarrow{Cmidpt_{PQ}} = \begin{pmatrix}-1\\3\end{pmatrix} - \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}$

   centre congruent circle: $\begin{pmatrix}-1\\3\end{pmatrix} + \begin{pmatrix}-4\\2\end{pmatrix} = \begin{pmatrix}-5\\5\end{pmatrix}$, C(−5, 5), $r = \sqrt{40}$

   Equation: $(x + 5)^2 + (y - 5)^2 = 40$.

Vectors 1

1. One possible journey, $S \rightarrow R \rightarrow W = -\mathbf{u} - \mathbf{v} - \mathbf{v}$ giving $-\mathbf{u} - 2\mathbf{v}$

Vectors 2

1. $\overrightarrow{PQ} = \begin{pmatrix}10\\-5\\20\end{pmatrix} \rightarrow \overrightarrow{PS} = \begin{pmatrix}20\\-10\\40\end{pmatrix} \rightarrow S(19, -5, 40)$

Vectors 3

1. $|\mathbf{u}| = \sqrt{(-3)^2 + 4^2} = 5$, the unit vector parallel to $\mathbf{u} = \begin{pmatrix}-3\\0\\4\end{pmatrix}$ is $\frac{1}{5}\begin{pmatrix}-3\\0\\4\end{pmatrix}$ *or* $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}$

2. $\mathbf{f} + \mathbf{g} = 5\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

   $|\mathbf{f} + \mathbf{g}| = \sqrt{5^2 + 4^2 + 5^2} = \sqrt{66}$

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

4 TRIGONOMETRY

Basics: Radians and trigonometric ratios

1. D

Basics: Exact values and basic trig graphs

1. B

Trigonometric graphs: Period, amplitude and graph transformations 1

1. A

Trigonometric graphs: Period, amplitude and graph transformations 2

1. B
2. A

The addition and double-angle formulae

1. A

Solving trigonometric equations 1

1. D, please note Option D should read $\frac{\pi}{6}$ and $\frac{11\pi}{6}$.
2. C

Solving trigonometric equations 2

1. (a)		**Method 1**: Using factorisation
	•¹ ss know to use double angle formula	•¹ $2\cos^2 x° - 1$ ... **stated, or implied by •²**
	•² ic express as a quadratic in $\cos x°$	•² $2\cos^2 x° - 3\cos x° + 1$ = 0 must appear at either of these lines to gain •².
	•³ ss start to solve	•³ $(2\cos x° - 1)(\cos x° - 1)$
		Method 2: Using quadratic formula
		•¹ $2\cos^2 x° - 1$...
		•² $2\cos^2 x° - 3\cos x° + 1 = 0$ **stated explicitly**
		•³ $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2}$
		In both methods:
	•⁴ pd reduce to equations in $\cos x°$ only	•⁴ $\cos x° = \frac{1}{2}$ and $\cos x° = 1$ Candidates who include 360 lose •⁴
	•⁵ ic process solutions in given domain	•⁵ 0, 60 and 300
		or
		•⁴ $\cos x° = 1$ and $x = 0$ Candidates who include 360 lose •⁴
		•⁵ $\cos x° = \frac{1}{2}$ and $x = 60$ or 300
(b)	•⁶ ic interpret relationship with (a)	•⁶ $2x = 0$ and 60 and 300
	•⁷ic interpret periodicity	•⁷ 0, 30, 150, 180, 210 and 330

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

2.				**Method 1**	
	$\bullet^1$ ss	use correct double angle formulae	$\bullet^1$	$2\sin x\cos x$	
	$\bullet^2$ ss	form correct equation	$\bullet^2$	$2\sin x\cos x - 2\cos^2 x = 0$	
	$\bullet^3$ ss	take out common factor	$\bullet^3$	$2\cos x(\sin x - \cos x) = 0$	
	$\bullet^4$ ic	proceed to solve	$\bullet^4$	$\cos x = 0$ and $\sin x = \cos x$	
				$\bullet^5$	$\bullet^6$
	$\bullet^5$ pd	find solutions	$\bullet^5$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$
	$\bullet^6$ pd	find remaining solutions	$\bullet^6$	$\frac{\pi}{4}$	$\frac{5\pi}{4}$
				Method 2	
	$\bullet^1$ ss	use double angle formulae	$\bullet^1$	$\cos 2x + 1$	
	$\bullet^2$ ss	form correct equation	$\bullet^2$	$\sin 2x - \cos 2x = 1$	
	$\bullet^3$ ss	express as a single trig function	$\bullet^3$	$\sqrt{2}\sin\left(2x - \frac{\pi}{4}\right) = 1$	
	$\bullet^4$ ic	proceed to solve	$\bullet^4$	$\sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	
				$\bullet^5$	$\bullet^6$
	$\bullet^5$ pd	find solutions	$\bullet^5$	$2x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{9\pi}{4}, \frac{11\pi}{4}$
	$\bullet^6$ pd	find solutions	$\bullet^6$	$x = \frac{\pi}{4}, \frac{\pi}{2}$	$x = \frac{5\pi}{4}, \frac{3\pi}{2}$

3.	$\bullet^1$ ss	use correct double angle formula	$\bullet^1$	$\sin x - 2(1 - 2\sin^2 x)$
				stated or implied by $\bullet^2$
	$\bullet^2$ ss	arrange in standard quadratic form	$\bullet^2$	$4\sin^2 x + \sin x - 3 = 0$
	$\bullet^3$ ss	start to solve	$\bullet^3$	$(4\sin x - 3)(\sin x + 1) = 0$
				OR
				$\frac{-1 \pm \sqrt{(1)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$
	$\bullet^4$ ic	reduce to equations in $\sin x$ only	$\bullet^4$	$\sin x = \frac{3}{4}$ and $\sin x = -1$
	$\bullet^5$ pd	process to find solutions in given domain	$\bullet^5$	$0{\cdot}848$, $2{\cdot}29$ and $\frac{3\pi}{2}$
				OR
			$\bullet^4$	$\sin x = \frac{3}{4}$ and $x = 0{\cdot}848, 2{\cdot}29$
			$\bullet^5$	$\sin x = -1$, and $x = \frac{3\pi}{2}$

The wave function 1

1.	(a)	$\bullet^1$ ss	use compound angle formula	$\bullet^1$	$R\sin x\cos a + R\cos x\sin a$	**stated explicitly**
		$\bullet^2$ ic	compare coefficients	$\bullet^2$	$R\cos a = 3$ and $R\sin a = -5$	stated explicitly
		$\bullet^3$ pd	process R	$\bullet^3$	$\sqrt{34}$ (Accept 5·8)	with or without working
		$\bullet^4$ pd	process a	$\bullet^4$	5·253 (Accept 5·3)	must be consistent with $\bullet^2$

ANSWERS TO 'THINGS TO DO AND THINK ABOUT'

	(b)	$\bullet^5$	pd	integrate given expression	$\bullet^5$	$3\sin x - 5\cos x$
		$\bullet^6$	ic	substitute limits	$\bullet^6$	$(3\sin t - 5cost) - (3\sin 0 - 5\cos 0)$
		$\bullet^7$	pd	process limits	$\bullet^7$	$3\sin t - 5\cos t + 5$
		$\bullet^8$	ss	know to use wave equation	$\bullet^8$	$\sqrt{34}\sin(t + 5{\cdot}3) + 5$
		$\bullet^9$	ic	write in standard format	$\bullet^9$	$\sin(t + 5{\cdot}3) = -\frac{2}{\sqrt{34}}$
		$\bullet^{10}$	ss	start to solve equation	$\bullet^{10}$	$t + 5{\cdot}3 = 3{\cdot}5$ and $5{\cdot}9$
		$\bullet^{11}$	pd	complete and state solution	$\bullet^{11}$	$t = 0{\cdot}6$

$\bullet^5$to $\bullet^{11}$ are available to candidates who chose to write this integrand as new wave function.

The wave function 2

1.	(a)	$\bullet^1$	ss	use compound angle formula	$\bullet^1$	$k\cos x\cos a - k\sin x\sin a$	**stated explicitly**
		$\bullet^2$	ic	compare coefficients	$\bullet^2$	$k\cos a = 1$ and $k\sin a = \sqrt{3}$	**stated explicitly**
		$\bullet^3$	pd	process k	$\bullet^3$	2 (do not accept $\sqrt{4}$)	
		$\bullet^4$	pd	process a	$\bullet^4$	$\frac{\pi}{3}$ **but** must be consistent with $\bullet^2$	
	(b)	$\bullet^5$	ic	interpret y-intercept	$\bullet^5$	1	
		$\bullet^6$	ss	strategy for finding roots	$\bullet^6$	e.g. $2\cos\left(x + \frac{\pi}{3}\right) = 0$ **or** $\sqrt{3}\sin x = \cos x$	
		$\bullet^7$	ic	state both roots	$\bullet^7$	$\frac{\pi}{6}, \frac{7\pi}{6}$	
2.	(a)	$\bullet^1$	ss	use compound angle formula	$\bullet^1$	$k\sin x^\circ\cos a^\circ - k\cos x^\circ\sin a^\circ$	**stated explicitly**
		$\bullet^2$	ic	compare coefficients	$\bullet^2$	$k\cos a^\circ = \sqrt{3}$ and $k\sin a^\circ = 1$	**stated explicitly**
		$\bullet^3$	pd	process for k	$\bullet^3$	2 (do not accept $\sqrt{4}$)	
		$\bullet^4$	pd	process for a	$\bullet^4$	30	
	(b)	$\bullet^5$	ic	interpret expression	$\bullet^5$	$4 - 5 \times 2\sin(x - 30)^\circ$	
		$\bullet^6$	pd	state maximum	$\bullet^6$	14	

3. D